Asymptotic stability analysis of nonlinear systems with impulsive effects and disturbance input

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Abstract. In this paper, a sufficient condition for asymptotic stability of nonlinear systems with impulse time window is derived, which avoids solving linear matrix inequalities. For the system with disturbance input and bounded gain error due to limit of equipment and technology in practical applications, another sufficient condition is also obtained. Numerical examples are carried out to validate effectiveness of the proposed results.
Keywords: impulse time window, disturbance input, bounded gain error, impulsive control, asymptotic stability, Chua's oscillator.

1. Introduction

Customarily, R^n is an $n-$ dimensional real Euclidean space with norm $\|\cdot\|$. $R^{m\times n}$ denotes the set of all $m \times n-$ dimensional real matrices. A^T, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the transpose, the maximal and the minimal eigenvalue of a real matrix A, respectively. $A > 0$ means the matrix A is symmetric

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and positive definite. I is the identity matrix of proper dimension. Define $f\left(x\left(t_0^-\right)\right)=\lim_{t\to t_0^-} f\left(x\left(t\right)\right)$.

Impulsive control theory has received considerable attention and many scholars have been researching on this topic because it can be applied in many fields. For instance, HIV prevention model [3], chaotic systems [6], neural networks [17], etc. A lot of results of impulsive control and its applications have been reported, see [9, 10, 14, 16] and reference therein.

In the previous literature of impulsive control [2, 13], the assumption of impulses occur at fixed times. Recently, Feng, Li and Huang [4] discussed the following nonlinear impulsive control systems with impulse time window:

$$\begin{cases} \dot{x}(t)=Ax\left(t\right)+f\left(x\left(t\right)\right), & mT\le t< mT+\tau_m,\\ x(t)=x\left(t^-\right)+Jx\left(t^-\right), & t=mT+\tau_m,\\ \dot{x}(t)=Ax\left(t\right)+f\left(x\left(t\right)\right), & mT+\tau_m< t<(m+1)T, \end{cases} \tag{1.1}$$

where $x(t)\in R^n$ is the state vector, $f: R^n\to R^n$ is said to be a continuous nonlinear function if $f(0)=0$, there exists a constant $l\ge 0$ such that $\left\|f\left(x\right)\right\|\le l\left\|x\right\|$, $A, J\in R^{n\times n}$ are constant matrices, $T>0$ is the control period, τ_m is unknown within impulse time window $(mT,(m+1)\,T)$. Since the impulsive effects can be stochastically occurred in an impulse time window in system (1.1), which is more general and more applicable than ones impulses occurred at fixed times. Some results related to impulse time window can be found in [8, 15].

Zou et al. [18] considered system (1.1) with bounded gain error and parameter uncertainty. The corresponding system was described as

$$\begin{cases} \dot{x}(t)=(A+\Delta A)x\left(t\right)+f\left(x\left(t\right)\right), & mT\le t< mT+\tau_m,\\ x(t)=(J_m+\Delta J_m)x\left(t^-\right), & t=mT+\tau_m,\\ \dot{x}(t)=Ax\left(t\right)+f\left(x\left(t\right)\right), & mT+\tau_m< t<(m+1)T, \end{cases} \tag{1.2}$$

where $\Delta A=DG\left(t\right)E$ is the parameter uncertainty, $\Delta J_m=mF(t)J_m$ is gain error which is often time-varying and bounded.

In many practical applications, we can not guarantee the input and impulses without any error due to the limit of equipment and technology. In what follows, we will consider system (1.1) with disturbance input and bounded gain error. System (1.1) can be rewritten as follows:

$$\begin{cases} \dot{x}(t)=Ax\left(t\right)+Bw\left(t\right)+f\left(x\left(t\right)\right), & mT\le t< mT+\tau_m,\\ x(t)=x\left(t^-\right)+Jx\left(t^-\right)+\phi\left(x\left(t^-\right)\right), & t=mT+\tau_m,\\ \dot{x}(t)=Ax\left(t\right)+Bw\left(t\right)+f\left(x\left(t\right)\right), & mT+\tau_m< t<(m+1)T, \end{cases} \tag{1.3}$$

where $w\left(t\right)\in R^r$ is the disturbance input, $\phi\left(x\left(t\right)\right)$ is gain error, $\phi: R^n\to R^n$ is a continuous nonlinear function satisfying $\phi(0)=0$, $B\in R^{n\times r}$ is a constant matrix. Without loss of generality, we assume that

$$\left\|w\left(t\right)\right\|\le l_1\left\|x\left(t\right)\right\|, \left\|\phi\left(x\left(t\right)\right)\right\|\le l_2\left\|x\left(t\right)\right\|,$$

where $l_1, l_2 \geq 0$ are two constants. In fact, for the case of parameter uncertainties, they are commonly assumed that $w(t) = \Delta Ax(t) = DG(t)Ex(t)$ and $\phi(x(t)) = \Delta J_m x(t) = mF(t)Jx(t)$, where $\|G(t)\| \leq 1$ and $\|F(t)\| \leq 1$, while D and E are appropriate known matrices, $m > 0$ is a known constant. At the same time, we have

$$\begin{aligned} \|w(t)\|^2 &= (DG(t)Ex(t))^T (DG(t)Ex(t)) \\ &= x^T(t)E^TG^T(t)D^TDG(t)Ex(t) \\ &\leq \|D\|^2\|E\|^2\|x(t)\|^2 \end{aligned}$$

and

$$\begin{aligned} \|\phi(x(t))\|^2 &= (mF(t)Jx(t))^T (mF(t)Jx(t)) \\ &= x^T(t)J^TF^T(t)mmF(t)Jx(t) \\ &\leq m^2\|J\|^2\|x(t)\|^2. \end{aligned}$$

We can choose $l_1 = \|D\|\|E\|$ and $l_2 = m\|J\|$. Clearly, (1.3) is more general than (1.1) and (1.2).

The aim of this paper is to present a new sufficient condition for the asymptotic stability of system (1.1). Compared with the results shown in [4, 5], our result is simpler. At the same time, we investigate system (1.3) and establish a new sufficient condition for the asymptotic stability of system (1.3). Furthermore, numerical examples are given to demonstrate effectiveness of our theoretical results.

2. Main results

We begin this section with the following lemmas.

Lemma 2.1 ([7]). *Let $x, y \in R^n$, then*

$$\left|x^Ty\right| \leq \|x\|\|y\|.$$

Lemma 2.2 ([7]). *Let A be a real symmetrical matrix. Then for any $x \in R^n$,*

$$\lambda_{\min}(A)x^Tx \leq x^TAx \leq \lambda_{\max}(A)x^Tx.$$

Theorem 2.1. *Let $0 < P \in R^{n\times n}$ such that*

$$gT + \ln\beta < 0,$$

where $\beta = \lambda_{\max}(P^{-1}(I+J)^TP(I+J))$, $\beta_1 = \lambda_{\max}(P^{-1}(PA + A^TP))$, $\beta_2 = \lambda_{\max}(P)$, $\beta_3 = \lambda_{\min}(P)$, $g = \beta_1 + 2l\sqrt{\frac{\beta_2}{\beta_3}}$, then system (1.1) is asymptotically stable at origin.

Proof. Consider the following Lyapunov function:

$$V(x(t)) = x^T(t)Px(t).$$

If $t \in [mT, mT+\tau_m)$, then by Lemmas 2.1 and 2.2, we have

$$\begin{aligned} D^+(V(x(t))) &= 2x^T(t)P(Ax(t)+f(x(t))) \\ &= 2x^T(t)PAx(t)+2x^T(t)Pf(x(t)) \\ &= x^T(t)\left(PA+A^TP\right)x(t)+2x^T(t)P^{\frac{1}{2}}P^{\frac{1}{2}}f(x(t)) \\ &\leq \beta_1 x^T(t)Px(t)+2\sqrt{x^T(t)Px(t)f^T(x(t))Pf(x(t))} \\ &\leq \beta_1 x^T(t)Px(t)+2\sqrt{x^T(t)Px(t)\beta_2 f^T(x(t))f(x(t))} \\ &\leq \beta_1 x^T(t)Px(t)+2\sqrt{x^T(t)Px(t)\beta_2 l^2 x^T(t)x(t)} \\ &\leq \beta_1 x^T(t)Px(t)+2l\sqrt{x^T(t)Px(t)\frac{\beta_2}{\beta_3}x^T(t)Px(t)} \\ &= gV(x(t)), \end{aligned}$$

which implies that

$$V(x(t)) \leq V(x(mT))e^{g(t-mT)}. \tag{2.1}$$

Similarly, if $t \in (mT+\tau_m,(m+1)T)$, we also have

$$D^+(V(x(t))) \leq gV(x(t)),$$

which leads to

$$V(x(t)) \leq V(x(mT+\tau_m))e^{g(t-mT-\tau_m)}. \tag{2.2}$$

If $t = mT+\tau_m$, then we have

$$\begin{aligned} V(x(t)) &= \left(x(t^-)+Jx(t^-)\right)^T P\left(x(t^-)+Jx(t^-)\right) \\ &= x^T(t^-)(I+J)^T P(I+J)x(t^-) \\ &= x^T(t^-)P^{\frac{1}{2}}P^{-\frac{1}{2}}(I+J)^T P(I+J)P^{-\frac{1}{2}}P^{\frac{1}{2}}x(t^-) \\ &\leq \beta V(x(t^-)). \end{aligned} \tag{2.3}$$

It follows from (2.2) and (2.3) that

$$V(x(t)) \leq \beta V\left(x\left((mT+\tau_m)^-\right)\right)e^{g(t-mT-\tau_m)}, \tag{2.4}$$

where $t \in [mT+\tau_m,(m+1)T)$.

When $m=0$, if $t \in [0,\tau_0)$, then by (2.1) we have

$$V(x(t)) \leq V(x(0))e^{gt}$$

and so

$$V\left(x\left(\tau_0^-\right)\right) \leq V(x(0))e^{g\tau_0}. \tag{2.5}$$

If $t \in [\tau_0, T)$, then by (2.4) and (2.5) we have

$$V(x(t)) \leq \beta V(x(0))e^{gt}$$

and so

$$V\left(x\left(T\right)\right)\leq\beta V\left(x\left(0\right)\right)e^{gT}. \tag{2.6}$$

When $m=1$, if $t\in[T,T+\tau_1)$, then by (2.1) and (2.6) we have

$$\begin{aligned} V\left(x\left(t\right)\right) &\leq V\left(x\left(T\right)\right)e^{g(t-T)} \\ &\leq \beta V\left(x\left(0\right)\right)e^{gt}. \end{aligned} \tag{2.7}$$

If $t\in[T+\tau_1,2T)$, then by (2.4) and (2.7) we have

$$\begin{aligned} V\left(x\left(t\right)\right) &\leq \beta V\left(x\left(\left(T+\tau_1\right)^{-}\right)\right)e^{g(t-T-\tau_1)} \\ &\leq \beta^2 V\left(x\left(0\right)\right)e^{gt}. \end{aligned}$$

By induction, when $m=k,\ k=0,1,\cdots$, if $t\in[kT,kT+\tau_k)$, then we have

$$\begin{aligned} V\left(x\left(t\right)\right) &\leq \beta^k V\left(x\left(0\right)\right)e^{gt} \\ &\leq \begin{cases} V\left(x\left(0\right)\right)e^{gT+k(gT+\ln\beta)}, & g>0, \\ V\left(x\left(0\right)\right)e^{k(gT+\ln\beta)}, & g\leq 0. \end{cases} \end{aligned} \tag{2.8}$$

If $t\in[kT+\tau_k,(k+1)T)$, we obtain

$$\begin{aligned} V\left(x\left(t\right)\right) &\leq \beta^{k+1} V\left(x\left(0\right)\right)e^{gt} \\ &\leq \begin{cases} V\left(x\left(0\right)\right)e^{(k+1)(gT+\ln\beta)}, & g>0, \\ \beta V\left(x\left(0\right)\right)e^{k(gT+\ln\beta)}, & g\leq 0. \end{cases} \end{aligned} \tag{2.9}$$

It follows from (2.8), (2.9), and $gT+\ln\beta<0$ that

$$\lim_{t\to\infty}V\left(x\left(t\right)\right)=0.$$

This completes the proof. □

Remark 2.1. The computation amount of solving linear matrix inequalities is not small [1]. Compared with the results shown in [4], Theorem 2.1 avoids solving linear matrix inequalities.

Theorem 2.2. *Let $0<P\in R^{n\times n}$ such that*

$$hT+\ln\gamma<0,$$

where

$$\beta=\lambda_{\max}(P^{-1}(I+J)^TP(I+J)),\beta_1=\lambda_{\max}(P^{-1}(PA+A^TP)),\beta_2=\lambda_{\max}(P),$$
$$\beta_3=\lambda_{\min}(P),\beta_4=\lambda_{\max}(B^TPB),\beta_5=\lambda_{\max}(B^TB),\beta_6=\lambda_{\max}((I+J)^T(I+J)),$$
$$h=\beta_1+2\sqrt{\frac{\beta_4l_1^2+\beta_2l^2+2ll_1\beta_2\sqrt{\beta_5}}{\beta_3}},$$
$$\gamma=\beta+\frac{l_2\beta_2(2\sqrt{\beta_6}+l_2)}{\beta_3},$$

then system (1.3) is asymptotically stable at origin.

Proof. Consider the following Lyapunov function:

$$V\left(x\left(t\right)\right)=x^{T}\left(t\right)Px\left(t\right).$$

By Lemmas 2.1 and 2.2, we have

$$\begin{aligned}\left(Bw\left(t\right)\right)^{T}Pf\left(x\left(t\right)\right) &\leq \sqrt{\left(Bw\left(t\right)\right)^{T}Bw\left(t\right)\left(Pf\left(x\left(t\right)\right)\right)^{T}Pf\left(x\left(t\right)\right)}\\ &\leq \sqrt{\beta_5\beta_2^2w^{T}\left(t\right)w\left(t\right)f^{T}\left(x\left(t\right)\right)f\left(x\left(t\right)\right)}\\ &\leq \sqrt{\beta_5\beta_2^2l_1^2l^2x^{T}\left(t\right)x\left(t\right)}\\ &\leq \tfrac{ll_1\beta_2}{\beta_3}\sqrt{\beta_5}x^{T}\left(t\right)Px\left(t\right).\end{aligned}\tag{2.10}$$

Similarly, we have

$$w^{T}\left(t\right)B^{T}PBw\left(t\right)\leq\frac{\beta_4l_1^2}{\beta_3}x^{T}\left(t\right)Px\left(t\right)\tag{2.11}$$

and

$$f^{T}\left(x\left(t\right)\right)Pf\left(x\left(t\right)\right)\leq\frac{\beta_2l^2}{\beta_3}x^{T}\left(t\right)Px\left(t\right).\tag{2.12}$$

By Lemmas 2.1, 2.2, (2.10), (2.11) and (2.12), we have

$$\begin{aligned}&x^{T}\left(t\right)P\left(Bw\left(t\right)+f\left(x\left(t\right)\right)\right)=x^{T}\left(t\right)P^{\frac{1}{2}}P^{\frac{1}{2}}\left(Bw\left(t\right)+f\left(x\left(t\right)\right)\right)\\ &\leq\sqrt{x^{T}\left(t\right)Px\left(t\right)\left(Bw\left(t\right)+f\left(x\left(t\right)\right)\right)^{T}P\left(Bw\left(t\right)+f\left(x\left(t\right)\right)\right)}\end{aligned}\tag{2.13}$$

$$\begin{aligned}&=\sqrt{x^T(t)Px(t)(w^T(t)B^TPBw(t)+2w^T(t)B^TPf(x(t))+f^T(x(t))Pf(x(t)))}\\ &\leq\sqrt{\frac{\beta_4l_1^2+\beta_2l^2+2ll_1\beta_2\sqrt{\beta_5}}{\beta_3}}x^{T}\left(t\right)Px\left(t\right).\end{aligned}$$

If $t\neq mT+\tau_m$, by (2.13) we have

$$\begin{aligned}D^{+}\left(V\left(x\left(t\right)\right)\right) &= 2x^{T}\left(t\right)P\left(Ax\left(t\right)+Bw\left(t\right)+f\left(x\left(t\right)\right)\right)\\ &= 2x^{T}\left(t\right)PAx\left(t\right)+2x^{T}\left(t\right)P\left(Bw\left(t\right)+f\left(x\left(t\right)\right)\right)\\ &\leq (\beta_1+2\sqrt{\frac{\beta_4l_1^2+\beta_2l^2+2ll_1\beta_2\sqrt{\beta_5}}{\beta_3}})x^{T}\left(t\right)Px\left(t\right)\\ &= hV\left(x\left(t\right)\right),\end{aligned}$$

which implies that

$$V\left(x\left(t\right)\right)\leq V\left(x\left(mT\right)\right)e^{h(t-mT)}.$$

If $t = mT + \tau_m$, then we have

$$
\begin{aligned}
V(x(t)) &= ((I+J)x(t^-) + \phi(x(t^-)))^T P((I+J)x(t^-) + \phi(x(t^-))) \\
&= x^T(t^-)(I+J)^T P(I+J)x(t^-) + 2x^T(t^-)(I+J)^T P\phi(x(t^-)) \\
&+ \phi^T(x(t^-))P\phi(x(t^-)) \qquad (2.14) \\
&\le \beta x^T(t^-)Px(t^-) + \frac{2l_2\beta_2\sqrt{\beta_6}}{\beta_3}x^T(t^-)Px(t^-) + \frac{\beta_2 l_2^2}{\beta_3}x^T(t^-)Px(t^-) \\
&= (\beta + \frac{l_2\beta_2(2\sqrt{\beta_6}+l_2)}{\beta_3})V(x(t^-)).
\end{aligned}
$$

The rest of proof is same as that of Theorem 2.1, so we omit it here for simplicity. This completes the proof. □

Remark 2.2. If we choose $B = 0$ and $\phi(x(t)) = 0$, Theorem 2.2 is reduced into Theorem 2.1.

3. Numerical examples

In this section, we illustrate the effectiveness of our theoretical results employing the Chua's oscillator.

Example 3.1. The Chua's oscillator [12] is described by

$$
\begin{cases}
\dot{x}_1 = \alpha(x_2 - x_1 - g(x_1)), \\
\dot{x}_2 = x_1 - x_2 + x_3, \\
\dot{x}_3 = -\eta x_2,
\end{cases} \qquad (3.1)
$$

where α and η are parameters and $g(x_1)$ is the piecewise linear characteristics of the Chua's diode, which is defined by $g(x_1) = bx_1 + 0.5(a-b)(|x_1+1| - |x_1-1|)$, where $a < b < 0$ are two constants.

By decomposing the linear and nonlinear parts of the system in (3.1), we can rewrite it into the following form, $\dot{x}(t) = Ax + f(x)$, where

$$
A = \begin{bmatrix} -\alpha - \alpha b & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\eta & 0 \end{bmatrix}, f(x) = \begin{bmatrix} -0.5\alpha(a-b)(|x_1+1| - |x_1-1|) \\ 0 \\ 0 \end{bmatrix}.
$$

By simple calculation, we can choose $l^2 = \alpha^2(a-b)^2$.

In the initial condition $x(0) = (0.5, 0.3, -0.5)^T$, Chua's circuit (3.1) has chaotic phenomenon when $\alpha = 9.2156$, $\eta = 15.9946$, $a = -1.24905$, $b = -0.75735$, as shown in Figure 1. Meanwhile, for the sake of convenience, we can choose $P = I$, $J = diag(-0.5, -0.5, -0.5)$. A small calculations show that $\beta = 0.25$, $\beta_1 = 16.5498$, $l = 4.5313$ and $g = 25.6124$. By the condition of Theorem 2.1, we have $T < 0.0541$. Thus by Theorem 2.1 we know that the

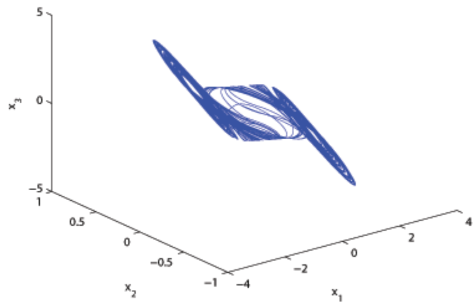

Figure 1: The chaotic phenomenon of Chua's oscillator with the initial condition $x(0) = (0.5, 0.3, -0.5)^T$.

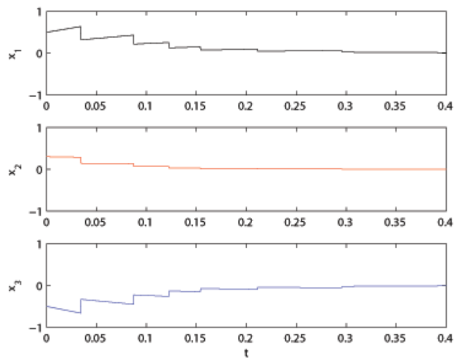

Figure 2: Time response curves of controlled Chua's oscillator with the initial condition $x(0) = (0.5, 0.3, -0.5)^T$.

origin of the system (1.1) is asymptotically stable. The simulation results with $T = 0.0500$ are shown in Figure 2.

Example 3.2. In this example, the coefficient matrix A and the impulsive control gain matrix J are same as Example 3.1. Suppose that $w(t) = 0.05x(t)cost$, $\phi(t) = 0.05x(t)sintJ$, $B = I$. Simple calculations show that $l_1 = 0.05$, $l_2 = 0.025\sqrt{3}$, $\beta = 0.25$, $\beta_1 = 16.5498$, $\beta_2 = \beta_3 = \beta_4 = \beta_5 = 1$, $\beta_6 = 0.25$, $h = 25.7124$ and $\gamma = 0.2952$. By the condition of Theorem 2.2, we have $T < 0.0475$. Thus by Theorem 2.2 we know that the origin of the system (1.3) is asymptotically stable. The simulation results with $T = 0.0400$ are shown in Figure 3.

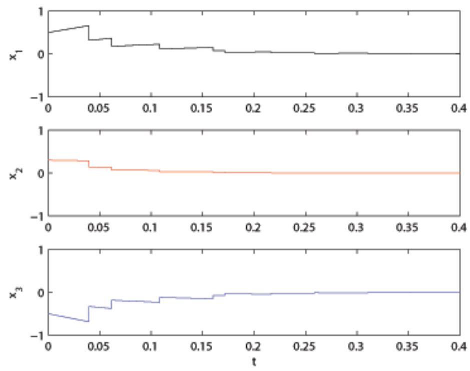

Figure 3: Time response curves of controlled Chua's oscillator with the initial condition $x(0) = (0.5, 0.3, -0.5)^T$.

4. Conclusions

In this paper, we discuss asymptotic stability of nonlinear impulsive control systems with impulse time window. The stability conditions avoid solving linear matrix inequalities. At the same time, we consider the nonlinear impulsive control systems with impulse time window, disturbance input and bounded gain error. Obviously, system (1.3) is more general and more applicable than [4, 5, 11, 18]. Finally, numerical examples demonstrate the effectiveness of the theoretical results.

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Accepted: 15.05.2019