On minimal λ_{rc}-open sets

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Abstract. We introduce and discuss the notions of minimal λ_{rc}-open sets in topological spaces. We investigate some its fundamental properties. We show that the notions of minimal open sets and minimal λ_{rc}-open sets are independent and finally we obtain some applications of a minimal λ_{rc}-open sets.
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1. Introduction

The study of semi open sets in topological spaces was initiated by Levine [10]. The complement of a subset A of X is denoted by $X \setminus A$. In 1937, M. Stone [22], defined regular closed set, a subset A is said to be regular-closed if $A = Cl(Int(A))$. The family of all regular-closed sets of (X, τ) is denoted by $RC(X)$. The concept of operation γ was initiated by Kasahara [4]. He also introduced γ-closed graph of a function. Using this operation, Ogata [21] introduced the concept of γ-open sets and investigated the related topological properties of the associated topology τ_γ and τ. He further investigated general operator approaches of closed graph of mappings. Further Ahmad and Hussain [1] continued studying the properties of γ -open(γ-closed) sets. In 2009, Hussain and Ahmad [3], introduced the concept of minimal γ-open sets. In 2011 [5] (resp. in 2013 [6]) Khalaf and Namiq, defined an operation γ called s-operation. They work in operation in topology in [14], [8], [9], [15], [16], [17],[18], [19]. They defined $\lambda_{\beta c}$-open set[13] by using s-operation and β-closed set and also investigated several properties of $\lambda_{\beta c}$-derived, $\lambda_{\beta c}$-interior and $\lambda_{\beta c}$-closure points in

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topological spaces. In 2017, Carpintero et al. [2], investigated the notions of minimal open sets in a generalized topological spaces and investigated its fundamental properties.
In this paper, we introduce and discuss minimal λ_{rc}-open sets in topological spaces and investigate some of their fundamental properties. We show that the notions of minimal λ_{rc}-open sets and minimal open sets are independent. Finally we obtain some applications of minimal λ_{rc}-open sets. First, we recall some definitions and results used in this paper.

2. Preliminaries

Throughout, X denotes a topological space. Let A be a subset of X, then the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a topological space (X,τ) is said to be semi open [10] if $A \subseteq Cl(Int(A))$. The complement of a semi open set is called semi closed [10]. The family of all semi open (resp. semi closed) sets in a topological space (X,τ) is denoted by $SO(X,\tau)$ or $SO(X)$ (resp. $SC(X,\tau)$ or $SC(X)$). We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda : SO(X) \to P(X)$ is called an s-operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V. It is assumed that $\lambda(\emptyset) = \emptyset$ and $\lambda(X) = X$ for any s-operation λ. Let X be a topological space and $\lambda : SO(X) \to P(X)$ be an s-operation, then a subset A of X is called a λ^*-open set [12] which is equivalent to λ-open set [5] and λ_s-open set [6] if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. The complement of a λ^*-open set is called λ^*-closed. The family of all λ^*-open (resp. λ^*-closed) subsets of a topological space (X,τ) is denoted by $SO_\lambda(X,\tau)$ or $SO_\lambda(X)$ (resp. $SC_\lambda(X,\tau)$ or $SC_\lambda(X)$).

Proposition 2.1 ([13]). *For a topological space X, $SO_\lambda(X) \subseteq SO(X)$.*

The following example shows that the contention of the above proposition may be strict..

Example 2.2 ([13]). Let $X = \{a,b,c\}$, and $\tau = \{\emptyset,\{a\},X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as $\lambda(A) = A$ if $b \in A$ and $\lambda(A) = X$ otherwise. Here, we have $\{a,c\}$ is semi open but it is not λ^*-open.

Definition 2.3 ([13]). *An s-operation λ on X is said to be s-regular which is equivalent to λ-regular [7] if for every semi open sets U and V of X containing the point $x \in X$, there exists a semi open set W containing x such that $\lambda(W) \subseteq \lambda(U) \cap \lambda(V)$.*

The proof of the following two propositions are in [7].

Proposition 2.4. *Let $\{A_\alpha\}_{\alpha\in I}$ be any collection of λ^*-open sets in a topological space (X,τ), then $\bigcup_{\alpha\in I} A_\alpha$ is a λ^*-open set.*

Proposition 2.5. *Let λ be semi-regular operation. If A and B are λ^*-open sets in X, then $A \cap B$ is also a λ^*-open set.*

Definition 2.6. *A λ^*-open [12] (λ-open [5], λ_s-open [6]) subset A of a topological space X is called λ_{rc}-open [20] if for each $x \in A$ there exists a regular closed set F such that $x \in F \subseteq A$. The complement of a λ_{rc}-open set is called λ_{rc}-closed. The family of all λ_{rc}-open (resp. λ_{rc}-closed) subsets of a topological space (X, τ) is denoted by $SO_{\lambda_{rc}}(X, \tau)$ or $SO_{\lambda_{rc}}(X)$(resp. $SC_{\lambda_{rc}}(X, \tau)$ or $SC_{\lambda_{rc}}(X)$).*

Definition 2.7. *Let X be a topological space and $\lambda : SO(X) \to P(X)$ be an s-operation, then a subset A of X is called a λ_{rc}-open neighbourhood of a point $x \in X$ if A is a λ_{rc}-open set and $x \in A$.*

Proposition 2.8 ([20]). *For a topological space X, $SO_{\lambda_{rc}}(X) \subseteq SO_{\lambda}(X) \subseteq SO(X)$.*

The following example shows that the contention of the above proposition may be strict.

Example 2.9. In Example 2.2, we have $\{a, c\}$ is semi open but it is not λ^*-open. And also $\{a, b\}$ is λ^*-open set but it is not λ_{rc} -open.

Definition 2.10 ([20]). *Let A be a subset of X. Then:*

1. *The λ_{rc}-closure of A ($\lambda_{rc}Cl(A)$) is the intersection of all λ_{rc}-closed sets containing A.*

2. *The λ_{rc}-interior of A ($\lambda_{rc}Int(A)$) is the union of all λ_{rc}-open sets of X contained in A.*

Proposition 2.11 ([20]). *For each point $x \in X$, $x \in \lambda_{rc}Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every $V \in SO_{\lambda_{rc}}(X)$ such that $x \in V$.*

Proposition 2.12 ([20]). *Let $\{A_\alpha\}_{\alpha \in I}$ be any collection of λ_{rc}-open sets in a topological space (X, τ), then $\bigcup_{\alpha \in I} A_\alpha$ is a λ_{rc}-open set.*

Proposition 2.13 ([20]). *Let λ be an s-regular operation. If A and B are λ_{rc}-open sets in X, then $A \cap B$ is also a λ_{rc}-open set.*

Definition 2.14 ([11]). *Let X be a space and $A \subseteq X$ be an open set. Then A is called a minimal open set if $\emptyset$ and A are the only open subsets of A.*

3. Minimal λ_{rc}-open sets

Definition 3.1. *Let X be a space and $A \subseteq X$ be a λ_{rc}-open set. Then A is called a minimal λ_{rc}-open set if $\emptyset$ and A are the only λ_{rc}-open subsets of A.*

The following example show that minimal open set and minimal λ_{rc}-open set are independent.

Example 3.2. Let $X = \{a, b, c\}$, and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as $\lambda(A) = A$ if $A \neq \{a\}, \{b\}$ and $\lambda(A) = \{a, b\}$ if $A = \{a\}$ or $\{b\}$. The λ_{rc}-open sets are $\emptyset, \{a, c\}, \{b, c\}$ and X. We have $\{a, c\}$ is minimal λ_{rc}-open set, but it is not minimal open set. And also $\{a\}$ is minimal open set, but it is not minimal λ_{rc}-open set.

Proposition 3.3. *Let A be a nonempty λ_{rc}-open subset of a space X. If $A \subseteq \lambda_{rc}Cl(C)$, then $\lambda_{rc}Cl(A) = \lambda_{rc}Cl(C)$, for any nonempty subset C of A.*

Proof. For any nonempty subset C of A,we have $\lambda_{rc}Cl(C) \subseteq \lambda_{rc}Cl(A)$. On the other hand, by supposition, we see $\lambda_{rc}Cl(A) \subseteq \lambda_{rc}Cl(\lambda_{rc}Cl(C)) = \lambda_{rc}Cl(C)$ implies $\lambda_{rc}Cl(A) \subseteq \lambda_{rc}Cl(C)$. Therefore we have $\lambda_{rc}Cl(A) = \lambda_{rc}Cl(C)$ for any nonempty subset C of A. □

Proposition 3.4. *Let A be a nonempty λ_{rc}-open subset of a space X. If $\lambda_{rc}Cl(A) = \lambda_{rc}Cl(C)$, for any nonempty subset C of A, then A is a minimal λ_{rc}-open set.*

Proof. Suppose that A is not a minimal λ_{rc}-open set. Then there exists a nonempty λ_{rc}-open set B such that $B \subsetneq A$ and hence there exists an element $x \in A$ such that $x \notin B$. Then we have $\lambda_{rc}Cl(\{x\}) \subseteq X \setminus B$ implies that $\lambda_{rc}Cl(\{x\})$ is a proper subset of $\lambda_{rc}Cl(A)$. And the result follows. □

Remark 3.5. For simplify, we assume that λ is an s-regular operation in the remainder of this section three, such as in Proposition 3.7, 3.8, 3.9, 3.10, Corollary 3.11, 3.12 and Theorem 3.13. Observe that if the condition of λ is not an s-regular operation, then the intersection of two λ_{rc}-open sets not necessarily is a λ_{rc}-open set, as we can see in the following Example.

Example 3.6. Let $X = \{a, b, c\}$, and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. We define an s-operation $\lambda : SO(X) \to P(X)$ as $\lambda(A) = A$ if $A \neq \{a\}, \{b\}$ and $\lambda(A) = \{a, b\}$ if $A = \{a\}$ or $\{b\}$.
$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
$SO_{\lambda}(X) = \{\emptyset, X, \{a, b\}, \{a, c\}, \{b, c\}\}$.
$SO_{\lambda_{rc}}(X) = \{\emptyset, X, \{a, c\}, \{b, c\}\}$.
Clearly λ is not a s-regular operation and the intersection of the λ_{rc}-open sets $\{a, c\}$ and $\{b, c\}$ is not a λ_{rc}-open.

Proposition 3.7. *The following statements are true:*

1. *If A is a minimal λ_{rc}-open set and B a λ_{rc}-open set, By proposition 2.13, $A \cap B$ is a λ_{rc}-open set. Then $A \cap B = \emptyset$ or $A \subseteq B$.*

2. *If B and C are minimal λ_{rc}-open sets. Then $B \cap C = \emptyset$ or $B = C$.*

Proof. (1) Let B be a λ_{rc}-open set such that $A \cap B \neq \emptyset$. Since A is a minimal λ_{rc}-open set and $A \cap B \subseteq A$, we have $A \cap B = A$. Therefore $A \subseteq B$.
(2) If $A \cap B \neq \emptyset$, then by (1), we have $B \subseteq C$ and $C \subseteq B$. Therefore, $B = C$. □

Proposition 3.8. *Let A be a minimal λ_{rc}-open set. If $x \in A$, then $A \subseteq B$ for any λ_{rc}-open neighborhood B of x.*

Proof. Let B be a λ_{rc}-open neighborhood of x such that A is not contained in B. Since λ is a s-regular operation, then $\emptyset \neq A \cap B$ is a λ_{rc}-open set. This contradicts our assumption that A is a minimal λ_{rc}-open set. □

Proposition 3.9. *Let A be a minimal λ_{rc}-open set. Then for any $x \in A$, $A = \cap\{B : B \text{ is } \lambda_{rc} - \text{open neighborhood of } x\}$.*

Proof. By Proposition 3.4 and the fact that A is λ_{rc}-open neighborhood of x, we have $A = \cap\{B : B \text{ is } \lambda_{rc} - \text{open neighborhood of } x\} \subseteq A$. Therefore, the result follows. □

Proposition 3.10. *If A is a minimal λ_{rc}-open set in X not containing $x \in X$. Then for any λ_{rc}-open neighborhood C of x, either $C \cap A = \emptyset$ or $A \subseteq C$.*

Proof. Since C is a λ_{rc}-open set, we have the result by Proposition 3.3. □

Corollary 3.11. *If A is a minimal λ_{rc}-open set in X not containing a point $x \in X$. If $A_x = \cap\{B : B \text{ is } \lambda_{rc} - \text{open neighborhood of } x\}$. Then either $A_x \cap A = \emptyset$ or $A \subseteq A_x$.*

Proof. If $A \subseteq B$ for any λ_{rc}-open neighborhood B of x, then $A \subseteq \cap\{B : B \text{ is } \lambda_{rc}-\text{open neighborhood of } x\}$. Therefore $A \subseteq A_x$. Otherwise there exists a λ_{rc}-open neighborhood B of x such that $B \cap A = \emptyset$. Then we have $A_x \cap A = \emptyset$. □

Corollary 3.12. *If A is a nonempty minimal λ_{rc}-open set of X, then for a nonempty subset C of A, $A \subseteq \lambda_{rc}Cl(C)$.*

Proof. Let C be any nonempty subset of A. Let $y \in A$ and B be any λ_{rc}-open neighborhood of y. By Proposition 3.4, we have $A \subseteq B$ and $C = A \cap C \subseteq B \cap C$. Thus we have $B \cap C \neq \emptyset$ and hence $y \in \lambda_{rc}Cl(C)$. This implies that $A \subseteq \lambda_{rc}Cl(C)$. □

Combining Corollary 3.12 and Propositions 3.3 and 3.4, we have:

Theorem 3.13. *Let A be a nonempty λ_{rc}-open subset of space X. Then the following are equivalent:*

1. *A is minimal λ_{rc}-open set, where λ is s-regular.*
2. *For any nonempty subset C of A, $A \subseteq \lambda_{rc}Cl(C)$.*
3. *For any nonempty subset C of A, $\lambda_{rc}Cl(A) = \lambda_{rc}Cl(C)$.*

4. Finite λ_{rc}-open sets

In this section, we study some properties of minimal λ_{rc}-open sets in finite λ_{rc}-open sets and λ_{rc}-locally finite spaces.

Proposition 4.1. *Let (X, τ) be a topological space and $\emptyset \neq B$ a finite λ_{rc}-open set in X. Then there exists at least one (finite) minimal λ_{rc}-open set A such that $A \subseteq B$.*

Proof. Suppose that B is a finite λ_{rc}-open set in X. Then we have the following two possibilities:

1. B is a minimal λ_{rc}-open set.
2. B is not a minimal λ_{rc}-open set.

In case (1), if we choose $A = B$, then the proposition is proved. If the case (2) is true, then there exists a nonempty (finite) λ_{rc}-open set B_1 which is properly contained in B. If B_1 is minimal λ_{rc}-open, we take $A = B_1$. If B_1 is not a minimal λ_{rc}-open set, then there exists a nonempty (finite)λ_{rc}-open set B_2 such that $B_2 \subseteq B_1 \subseteq B$. We continue this process and have a sequence of λ_{rc}-open sets $\subseteq B_m \subseteq ... \subseteq B_2 \subseteq B_1 \subseteq B$. Since B is a finite, this process will end in a finite number of steps. That is, for some natural number k, we have a minimal λ_{rc}-open set B_k such that $A = B_k$. This completes the proof. □

Definition 4.2. *A space X is said to be a λ_{rc}-locally finite space, if for each $x \in X$ there exists a finite λ_{rc}-open set A in X such that $x \in A$.*

Definition 4.3. *Let $X = \mathbb{R}$ and $\tau = P(\mathbb{R})$. We define an s-operation $\lambda : SO(\mathbb{R}) \to P(\mathbb{R})$ as $\lambda(A) = A$ for every subset A of $\mathbb{R}$. Then $(\mathbb{R}, \tau)$ is a λ_{rc}-locally finite space*

Corollary 4.4. *Let X be a λ_{rc}-locally finite space and B a nonempty λ_{rc}-open set. Then there exists at least one (finite) minimal λ_{rc}-open set A such that $A \subseteq B$, where λ is s-regular.*

Proof. Since B is a nonempty set, there exists an element x of B. Since X is a λ_{rc}-locally finite space, we have a finite λ_{rc}-open set B_x such that $x \in B_x$. Since $B \cap B_x$ is a finite λ_{rc}-open set, we get by Proposition 4.1, a minimal λ_{rc}-open set A such that $A \subseteq B \cap B_x \subseteq B$. □

Proposition 4.5. *Let X be a space and for any $\alpha \in I$, B_α a λ_{rc}-open set and $\emptyset \neq A$ a finite λ_{rc}-open set. Then $A \cap (\bigcap_{\alpha \in I} B_\alpha)$ is a finite λ_{rc}-open set, where λ is s-regular.*

Proof. We see that there exists an integer n such that $A \cap (\bigcap_{\alpha \in I} B_\alpha) = A \cap (\bigcap_{i=1}^{n} B_{\alpha_i}))$ and hence we have the result. □

Using Proposition 4.5, we can prove the following:

Theorem 4.6. *Let X be a space and for any $\alpha \in I$, B_α is a λ_{rc}-open set and for any $\beta \in I$, B_β is a nonempty finite λ_{rc}-open set. Then $(\bigcup_{\beta\in I} B_\beta) \cap (\bigcap_{\alpha\in I} B_\alpha)$ is a λ_{rc}-open set, where λ is s-regular.*

It is important to know that the notions of λ_{rc}-locally finite space and locally finite space are independent, because the family of λ_{rc}-open sets and the family of open set in a topological (X, τ) with an s-operation $\lambda : SO(X) \to P(X)$ are independent.

5. More properties

Let A be a nonempty finite λ_{rc}-open set. It is clear, by Proposition 3.3 and 4.1, that if λ is s-regular, then there exists a natural number m such that$\{A_1, A_2, \dots, A_m\}$ is the class of all minimal λ_{rc}-open sets in A satisfying the following two conditions:

1. For any i, n with $1 \leq i, n \leq m$ and $i \neq n$, $A_i \cap A_n = \emptyset$.
2. If C is a minimal λ_{rc}-open set in A, then there exists i with $1 \leq i \leq m$ such that $C = A_i$.

Theorem 5.1. *Let X be a space and $A \neq \emptyset$ be a finite λ_{rc}-open set which is not a minimal λ_{rc}-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{rc}-open sets in A and $y \in A \setminus (\bigcup_{i=1}^{m} A_i)$. Define $A_y = \cap\{B_y\}$, where B_y is a λ_{rc}-open neighborhood of y. Then there exists a natural number $k \in \{1, 2, 3, \dots, m\}$ such that A_k is contained in A_y, where λ is s-regular.*

Proof. Suppose on the contrary that for any natural number $k \in \{1, 2, 3, \dots, m\}$, A_k is not contained in A_y. By Proposition 3.7, for any minimal λ_{rc}-open set A_k in A, $A_k \cap A_y = \emptyset$. By Proposition 4.5, $\emptyset \neq A_y$ is a finite λ_{rc}-open set. Therefore by Proposition 4.1, there exists a minimal λ_{rc}-open set C such that $C \subseteq A_y$. Since $C \subseteq A_y \subseteq A$, we have C is a minimal λ_{rc}-open set in A. By supposition, for any minimal λ_{rc}-open set A_k, we have $A_k \cap C \subseteq A_k \cap A_y = \emptyset$. Therefore, for any natural number $k \in \{1, 2, 3, \dots, m\}$, $C \neq A_k$. This contradicts our assumption. Hence the proof. □

Proposition 5.2. *Let X be a space and $A \neq \emptyset$ be a finite λ_{rc}-open set which is not a minimal λ_{rc}-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{rc}-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$. Then there exists a natural number $k \in \{1, 2, 3, \dots, m\}$, such that for any λ_{rc}-open neighborhood B_y of y, A_k is contained in B_y, where λ is s-regular.*

Proof. This follows from Theorem 5.1. □

Theorem 5.3. *Let X be a space and $A \neq \emptyset$ be a finite λ_{rc}-open set which is not a minimal λ_{rc}-open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{rc}-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$. Then there exists a natural number $k \in \{1, 2, 3, \dots, m\}$, such that $y \in \lambda_{rc}Cl(A_k)$. where λ is s-regular.*

Proof. It follows from Proposition 5.2, that there exists a natural number $k \in \{1, 2, 3, \ldots, m\}$ such that $A_k \subseteq B$ for any λ_{rc}-open neighborhood B of y. Therefore $\emptyset \neq A_k \subseteq A_k \cap B$ and then, $y \in \lambda_{rc}Cl(A_k)$. □

Proposition 5.4. *Let $A \neq \emptyset$ be a finite λ_{rc}-open set in a space X and for each $k \in \{1, 2, 3, \ldots, m\}$, A_k is a minimal λ_{rc}-open set in A. If the class $\{A_1, A_2, \ldots, A_m\}$ contains all minimal λ_{rc}-open sets in A, then for any $\emptyset \neq B_k \subseteq A_k$, $A \subseteq \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$, where λ is s-regular.*

Proof. If A is a minimal λ_{rc}-open set, then this is the result of Theorem 3.13(2). Otherwise, when A is not a minimal λ_{rc}-open set. If x is any element of $A \setminus (A_1 \cup A_2 \cup A_3 \cup ... \cup A_m)$, then by Theorem 5.3, $x \in \lambda_{rc}Cl(A_1) \cup \lambda_{rc}Cl(A_2) \cup ....\lambda_{rc}Cl(A_m)$. Therefore, by Theorem 3.13 (3), we obtain that $A \subseteq \lambda_{rc}Cl(A_1) \cup \lambda_{rc}Cl(A_2) \cup ....\lambda_{rc}Cl(A_m) = \lambda_{rc}Cl(B_1) \cup \lambda_{rc}Cl(B_2) \cup ....\lambda_{rc}Cl(B_m) = \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$. □

Proposition 5.5. *Let $A \neq \emptyset$ be a finite λ_{rc}-open set and for each $k \in \{1, 2, 3, \ldots, m\}$, A_k is a minimal λ_{rc}-open set in A. If for any $\emptyset \neq B_k \subseteq A_k$, $A \subseteq \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$, then $\lambda_{rc}Cl(A) = \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$.*

Proof. For any $\emptyset \neq B_k \subseteq A_k$, with $k \in \{1, 2, 3, \ldots, m\}$, we have $\lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m) \subseteq \lambda_{rc}Cl(A)$. Also, we have $\lambda_{rc}Cl(A) \subseteq \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m) = \lambda_{rc}Cl(B_1) \cup \lambda_{rc}Cl(B_2) \cup \lambda_{rc}Cl(B_3) \cup ... \cup \lambda_{rc}Cl(B_m)$. Therefore, $\lambda_{rc}Cl(A) = \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$, for any nonempty subset B_k of A_k with $k \in \{1, 2, 3, \ldots, m\}$. □

Proposition 5.6. *Let $A \neq \emptyset$ be a finite λ_{rc}-open set and for each $k \in \{1, 2, 3, \ldots, m\}$, A_k is a minimal λ_{rc}-open set in A. If for any $\emptyset \neq B_k \subseteq A_k$, $\lambda_{rc}Cl(A_k) = \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$, then the class $\{A_1 \cup A_2 \cup A_3 \cup ... \cup A_m\}$ contains all minimal λ_{rc}-open sets in A.*

Proof. Suppose that C is a minimal λ_{rc}-open set in A and $C \neq A_k$ for $k \in \{1, 2, 3, \ldots, m\}$. Then we have $C \cap \lambda_{rc}Cl(A_k) = \emptyset$ for each $k \in \{1, 2, 3, \ldots, m\}$. It follows that any element of C is not contained in $\lambda_{rc}Cl(A_1 \cup A_2 \cup A_3 \cup ... \cup A_m)$. This is a contradiction to the fact that $C \subseteq A \subseteq \lambda_{rc}Cl(A) = \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$. This completes the proof. □

Combining Propositions 5.4, 5.5 and 5.6, we have the following theorem:

Theorem 5.7. *Let A be a nonempty finite λ_{rc}-open set and A_k a minimal λ_{rc}-open set in A for each $k \in \{1, 2, 3, \ldots, m\}$. Then the following three conditions are equivalent:*

1. *The class $\{A_1, A_2, \ldots, A_m\}$ contains all minimal λ_{rc}-open sets in A.*
2. *For any $\emptyset \neq B_k \subseteq A_k$, $A_k \subseteq \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$.*
3. *For any $\emptyset \neq B_k \subseteq A_k$, $\lambda_{rc}Cl(A_k) = \lambda_{rc}Cl(B_1 \cup B_2 \cup B_3 \cup ... \cup B_m)$, where λ is s-regular.*

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