Bipolar complex fuzzy sets and their properties

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Abstract. The primary motivation behind this paper is to present a brief overview of the bipolar complex fuzzy sets (in short BCFS) which is an extension of bipolar fuzzy set theory. New operations defined over the bipolar complex fuzzy sets some properties of these operations are discussed.

1. Introduction

Fuzzy sets are a sort of useful mathematical structure representing a vague collection of objects. There are various types of Fuzzy sets in the Fuzzy set theory, such as intuitional Fuzzy sets, valued Fuzzy sets, vague sets, etc.

Zhang [11] introduced bipolar fuzzy sets in 1998. Positive information in a bipolar fuzzy set is what is guaranteed to be possible, while negative information is what is impossible or forbidden or certainly false. Bipolar valued fuzzy set by Lee [3] introduced a further generalization of fuzzy sets in which the degree of membership between $[0,1]$ and $[-1,1]$ increased. In bipolar fuzzy sets, membership degree 0 means that elements are irrelevant to corresponding property, membership degree belongs to $(0,1]$ indicate that somewhat elements are sat-

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isfying the corresponding property and membership degree belongs to $[-1,0)$ indicate that somewhat elements are satisfying implicit counter property.

Ramot et al. [5] introduced a new innovative concept in 2002 and called it a complex fuzzy set (CFS). This approach is absolutely different from other researchers, where Ramot et al. extended the range of membership function to unit disc in the complex plane, unlike the others who limited to $[0,1]$. Hence, Ramot et al. [6] added an additional term called the phase term to solve the enigma in translating some complex-valued functions on physical terms to human language and vice versa.

We employ techniques similar to these used earlier by Abdallah Al-Husban and Abdul Razak Salleh ([13], [14]) and Abdallah Al-Husban et al. ([15], [**?**]).

In this work, we introduce some workable concepts about our (BCFS) concept, which are intersection, union and complement. Also some properties of the set theoretic operations of (BCFS).

2. Preliminaries

In this section, we remember the definitions and related results that this work requires.

Definition 2.1 ([9])**.** A fuzzy set (FS) A in a universe of discourse U is characterized by a membership function $\mu_A(x)$ that takes values in the interval $[0,1]$.

Definition 2.2 ([11])**.** Let X be a non-empty set. A bipolar fuzzy set (BFS) A in X is an objective having the form $A = \{(x, r_A^+(x), r_A^-(x)) : x \in X\}$ where $r^+ : X \to [0,1]$ and $r^- : X \to [-1,0]$ are mappings.

Definition 2.3 ([11])**.** Let X be every two bipolar fuzzy set.

$A = \{(x, r_A^+(x), r_A^-(x)) : x \in X\}$ and $B = \{(x, r_B^+(x), r_B^-(x)) : x \in X\}$ we define

i) $A \cap B = \{\min(r_A^+(x), r_B^+(x)), \max(r_A^-(x), r_B^-(x)) : x \in X\}$,

ii) $A \cup B = \{\max(r_A^+(x), r_B^+(x)), \min(r_A^-(x), r_B^-(x)) : x \in X\}$,

iii) $A^c = \{(x, 1 - r_A^+(x), -1 - r_A^-(x)) : x \in X\}$.

Definition 2.4 ([5])**.** A complex fuzzy set (CFS) A, defined on a universe of discourse U, is characterized by a membership function $\mu_A(x)$, that assigns to any element $x \in U$ a complex-valued grade of membership in A. By definition, the values of $\mu_A(x)$, may receive all lying within the unit circle in the complex plane, and are thus of the form $\mu_A(x) = r_A(x)e^{i\varpi_A(x)}$, where $i = \sqrt{-1}$, each of $r_A(x)$ and $\varpi_A(x)$ are both real-valued, and $r_A(x) \in [0,1]$. The CFS A may be represented as the set of ordered pairs

$$A = \{(x, \mu_A(x)) : x \in U\} = \left\{\left(x, r_A(x)e^{i\varpi_A(x)}\right) : x \in U\right\}.$$

Definition 2.5 ([5])**.** A complex fuzzy complement of A may be represented as follows:

$$A^c = \{(x, \mu_{A^c}(x)) : x \in U\} = \left\{\left(x, r_{A^c}(x)e^{i\varpi_{A^c}(x)}\right) : x \in U\right\},$$

where $r_{A^c}(x) = 1 - r_A(x)$ and $\varpi_{A^c}(x) = 2\pi - \varpi_A(x)$.

Definition 2.6 ([5])**.** Let A and B be two complex fuzzy sets on U where

$$A = \left\{\left\langle x, \mu_A(x) = r_A(x)e^{i\arg_{r_A}(x)}\right\rangle : x \in U\right\},$$
$$B = \left\{\left\langle x, \mu_B(x) = r_B(x)e^{i\arg_{r_B}(x)}\right\rangle : x \in U\right\}.$$

The complex fuzzy intersection of A and B denoted by $A \oplus B$, is specified by $A \oplus B = \{\langle x, \mu_{A\oplus B}(x)\rangle : x \in U\}$, where $\mu_{A\oplus B}(x) = r_{A\oplus B}(x)e^{i\arg_{r_{A\oplus B}}(x)} = \max(r_A(x), r_B(x))\, e^{i\max\left(\arg_{r_A}(x),\arg_{r_B}(x)\right)}$.

The complex fuzzy union of A and B denoted by $A \otimes B$, is specified by

$$A \otimes B = \{\langle x, \mu_{A\otimes B}(x)\rangle : x \in U\},$$

where

$$\mu_{A\otimes B}(x) = r_{A\otimes B}(x)e^{i\arg_{r_{A\otimes B}}(x)} = \min(r_A(x), r_B(x))\, e^{i\min\left(\arg_{r_A}(x),\arg_{r_B}(x)\right)}.$$

Definition 2.7 ([5])**.** Let A and B be two complex fuzzy sets on X, $\mu_A(x) = r_A(x)e^{i\arg_A(x)}$ and $\mu_B(x) = r_B(x)e^{i\arg_B(x)}$ their membership functions, respectively. We say that A is greater than B, denoted by $A \supseteq B$ or $B \supseteq A$, if for any $x \in X$, $r_A(x) \geq r_B(x)$ and $\arg_A(x) \geq \arg_B(x)$.

3. Bipolar complex fuzzy sets

In this work, we start with the introduction of a (BCFS) definition and some related properties are discussed.

Definition 3.1. Let X is a non-empty set. A bipolar complex fuzzy set (BCFS) A in X is an objective having the form $A = \{(x, r_A^+e^{i\theta_A^+}, r_A^-e^{i\theta_A^-}) : x \in X\}$. where $r_A^+ : X \to [0,1]$ and $r_A^- : X \to [-1,0]$ are mappings. $r_A^+e^{i\theta_A^+}$ the positive complex membership degree and $r_A^-e^{i\theta_A^-}$ the negative complex membership degree. Also the phase term of bipolar complex positive membership function and bipolar complex negative membership function belongs to $(0, 2\pi]$ and $r_A^+ \in [0,1]$, $r_A^- \in [-1,0]$.

Example 3.1. Let $A = \left\{ \begin{array}{c} \left(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}\right), \left(b, 0.8e^{1.3\pi i}, -0.2e^{0\pi i}\right), \\ \left(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}\right) \end{array} \right\}$ is a (BCFS) of $X = \{a, b, c\}$.

Definition 3.2. The complement of a (BCFS) $A = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}) : x \in X\}$ is denoted by A^c and defined by $A^c = \{(x, 1 - r_A^+ e^{i(2\pi - \theta_A^+)}, -1 - r_A^- e^{i(2\pi - \theta_A^-)}) : x \in X\}$.

Example 3.2. Let $X = \{a, b, c\}$ be a universe of discourse

$$A = \left\{ \begin{array}{c} \left(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}\right), \left(b, 0.8e^{1.3\pi i}, -0.2e^{0\pi i}\right), \\ \left(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}\right) \end{array} \right\} \text{ is a (BCFS).}$$

Then

$$A^c = \left\{ \begin{array}{c} (a, (1-0.2)e^{i(2\pi - 2\pi)}, (-1+0.4)e^{i(2\pi + 1.2\pi)}), \\ (b, (1-0.8)e^{i(2\pi - 1.3\pi)}, (-1+0.2)e^{i(2\pi - 0\pi)}), \\ (c, (1-0.3)e^{i(2\pi - \pi)}, (-1+0.4)e^{i(2\pi + 1.5\pi i}) \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \left(a, 0.8e^{0i}, -0.6e^{3.2\pi i}\right), \left(b, 0.2e^{0.7\pi i}, -0.8e^{2\pi i}\right), \\ \left(c, 0.7e^{i\pi}, -0.6e^{3.5\pi i}\right) \end{array} \right\}.$$

Definition 3.3. The union of two (BCFS) as follows:

Let A and B be two (BCFS) in X, where $A = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}) : x \in X\}$ and $B = \{(x, r_B^+ e^{i\theta_B^+}, r_B^- e^{i\theta_B^-}) : x \in X\}$. Then the union of A and B is denoted as $A \cup B$ and is given as:

$$A \cup B = \{(\max(r_A^+, r_B^+) e^{i \max(\theta_A^+, \theta_B^+)}, \min(r_A^-, r_B^-) e^{i \min(\theta_A^-, \theta_B^-)}) : x \in X\}.$$

Example 3.3. Let $X = \{a, b, c\}$ be a universe of discourse. Let A and B be two (BCFS). Let

$$A = \left\{ \begin{array}{c} \left(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}\right), \left(b, 0.8e^{1.3\pi i}, -0.2e^{0\pi i}\right), \\ \left(c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i}\right) \end{array} \right\}$$

and

$$B = \left\{ \begin{array}{c} \left(a, 0.2e^{\pi i}, -0.3e^{-1.4\pi i}\right), \left(b, 0.7e^{1.5\pi i}, -0.1e^{1.5\pi i}\right), \\ \left(c, 0.1e^{\pi i}, -0.3e^{-1.5\pi i}\right) \end{array} \right\},$$

then

$$(A \cup B) = \left\{ \begin{array}{c} \left(a, 0.2e^{2\pi i}, -0.3e^{-1.2\pi i}\right), \left(b, 0.8e^{1.5\pi i}, -0.1e^{0\pi i}\right), \\ \left(c, 0.3e^{\pi i}, -0.3e^{-1.5\pi i}\right) \end{array} \right\}.$$

Definition 3.4. The intersection of two (BCFS) as follows: Let A and B be two (BCFS) in X, where $A = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}) : x \in X\}$ and $B = \{(x, r_B^+ e^{i\theta_B^+}, r_B^- e^{i\theta_B^-}) : x \in X\}$. Then the intersection of A and B is denoted as $A \cap B$ and is given as:
$(A \cap B)(x) = \{(\min(r_A^+, r_B^+) e^{i \min(\theta_A^+, \theta_B^+)}, \max(r_A^-, r_B^-) e^{i \max(\theta_A^-, \theta_B^-)})\}$.

Example 3.4. Let $X = \{a, b, c\}$ be a universe of discourse. Let A and B be two (BCFS). Let
$A = \{(a, 0.2e^{2\pi i}, -0.4e^{-1.2\pi i}), (b, 0.8e^{1.3\pi i}, -0.2e^{0\pi i}), (c, 0.3e^{\pi i}, -0.4e^{-1.5\pi i})\}$ and
$B = \{(a, 0.2e^{\pi i}, -0.3e^{-1.4\pi i}), (b, 0.7e^{1.5\pi i}, -0.1e^{1.5\pi i}), (c, 0.1e^{\pi i}, -0.3e^{-1.5\pi i})\}$,
then

$$(A \cap B) = \{(a, 0.2e^{\pi i}, -0.3e^{-1.2\pi i}), (b, 0.7e^{1.3\pi i}, -0.2e^{0\pi i}), (c, 0.1e^{\pi i}, -0.3e^{-1.5\pi i})\}.$$

Definition 3.5. If A and B are (BCFSs) in a universe of discourse X , where $A = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}) : x \in X\}$ and $B = \{(x, r_B^+ e^{i\theta_B^+}, r_B^- e^{i\theta_B^-}) : x \in X\}$, then

1) $A \subset B$ if and only if $r_A^+ < r_B^+$ and $r_A^- > r_B^-$ for amplitude terms and the phase terms (arguments) $\theta_A^+ < \theta_B^+$ and $\theta_A^- > \theta_B^-$ for all $x \in X$.

2) $A = B$ if and only if $r_A^+ = r_B^+$ and $r_A^- = r_B^-$ for amplitude terms and the phase terms (arguments) $\theta_A^+ = \theta_B^+$ and $\theta_A^- = \theta_B^-$ for all $x \in X$.

Proposition 3.6. *Let A, B and R be any three (BCFS) over U. Then the following holds:*

i. $A \cup A = A$,
ii. $A \cap A = A$,
iii. $A \cup B = B \cup A$,
iv. $A \cap B = B \cap A$,
v. $A \cup (B \cap R) = (A \cup B) \cap (A \cup R)$,
vi. $A \cap (B \cup R) = (A \cap B) \cup (A \cap R)$,
vii. $A \cup (B \cup R) = (A \cup B) \cup R$,
viii. $A \cap (B \cap R) = (A \cap B) \cap R$.

Proof. Let A, B and R are three (BCFS) given as: $A = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}) : x \in X\}$, $B = \{(x, r_B^+ e^{i\theta_B^+}, r_B^- e^{i\theta_B^-}) : x \in X\}$ and $R = \{(x, r_R^+ e^{i\theta_R^+}, r_R^- e^{i\theta_R^-}) : x \in X\}$.

So, to prove (i) we need to recall 3.3, then we have:

$A \cup A = \{(\max(r_A^+, r_A^+) e^{i \max(\theta_A^+, \theta_A^+)}, \min(r_A^-, r_A^-) e^{i \min(\theta_A^-, \theta_A^-)}) : x \in X\} = \{(x, r_A^+ e^{i\theta_A^+}, r_A^- e^{i\theta_A^-}) : x \in X\} = A$.

Analogously to (i), we can prove (ii) with recalling 3.4.

To prove (iii) we need to recall 3.3, and then we have

$A \cup B = \{(\max(r_A^+, r_B^+) e^{i \max(\theta_A^+, \theta_B^+)}, \min(r_A^-, r_B^-) e^{i \min(\theta_A^-, \theta_B^-)}) : x \in X\} =$
$\{(\max(r_B^+, r_A^+) e^{i \max(\theta_B^+, \theta_A^+)}, \min(r_B^-, r_A^-) e^{i \min(\theta_B^-, \theta_A^-)}) : x \in X\} = B \cup A$.

Analogously to (iii), we can prove (iv) with recalling 3.4.

To prove (v) we need to recall both 3.3 and 3.4, and then we have

$$A \cup (B \cap R) = \left\{ \left\langle \begin{array}{c} x, \max\left(r_A^+(x), r_{B\cap R}^+(x)\right) e^{i \max\left(\theta_A^+(x), \theta_{B\cap R}^+(x)\right)}, \\ \min\left(r_A^-(x), r_{B\cap R}^-(x)\right) e^{i \min\left(\theta_A^-(x), \theta_{B\cap R}^-(x)\right)} \end{array} \right\rangle : x \in X \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \max[r_A^+(x), \\ \min(r_B^+(x), r_R^+(x)) e^{i \min(\theta_A^+(x), \min(\theta_B^+(x), \theta_R^+(x))}], \\ \min[r_A^-(x), \max(r_B^-(x), r_R^-(x)) e^{i \max(\theta_A^-, (x) \max(\theta_B^-(x), \theta_R^-(x))}] \end{array} \right\rangle : x \in X \right\}$$

$$= \left\{ \begin{array}{c} x, \min[\max(r_A^+(x), r_B^+(x)), \\ \left\langle \begin{array}{c} \max(r_A^+(x), r_R^+(x))] e^{i \min[\max(\theta_A^+(x), \theta_B^+(x)), \max(\theta_A^+(x), \theta_B^+(x))]}, \\ \max[\min(r_A^-(x), r_B^-(x)), \end{array} \right\rangle \\ \min(r_A^-(x), r_R^-(x))] e^{i \max[\min(\theta_A^-(x), \theta_B^-(x)), \min(\theta_A^-(x), \theta_B^-(x))]} \\ : x \in X \end{array} \right\}$$

$= (A \cup B) \cap (A \cup R)$.

Analogously to prove (v), we can prove (vi).

To prove (vii) we need to recall 3.3, and then we have

$$A \cup (B \cup R) = \left\{ \left\langle \begin{array}{c} x, \max(r^+_A(x), r^+_{B\cup R}(x))e^{i\max(\theta^+_A(x),\theta^+_{B\cup R}(x))}, \\ \min(r^-_A(x), r^-_{B\cap R}(x))e^{i\min(\theta^-_A(x),\theta^-_{B\cup R}(x))} \end{array} \right\rangle : x \in X \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \max[r^+_A(x), \max(r^+_B(x), r^+_R(x))e^{i\max(\theta^+_A(x),\min(\theta^+_B(x),\theta^+_R(x))}], \\ \min[r^-_A(x), \min(r^-_B(x), r^-_R(x))e^{i\min(\theta^-_A,(x)\max(\theta^-_B(x),\theta^-_R(x))}] \end{array} \right\rangle : x{\in}X \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \max[(r^+_A(x), r^+_B(x), r^+_R(x))]e^{i\max(\theta^+_A(x),\theta^+_B(x),\theta^+_R(x))}, \\ \min[r^-_A(x), r^-_B(x), r^-_R(x))]e^{i\min(\theta^-_A(x),\theta^-_B(x),\theta^-_R(x))} \end{array} \right\rangle : x{\in}X \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \max[\max(r^+_A(x), r^+_B(x)), r^+_R(x)]e^{i\max(\max(\theta^+_A(x),\theta^+_B(x)),\theta^+_R(x))}, \\ \min[\min(r^-_A(x), r^-_B(x)), r^-_R(x))]e^{i\min(\min(\theta^-_A(x),\theta^-_B(x)),\theta^-_R(x))} \end{array} \right\rangle : x \in X \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \max[r^+_{A\cup B}(x), r^+_R(x)]e^{i\max(\theta^+_{A\cup B}(x),\theta^+_R(x))}, \\ \min[r^-_{A\cup B}(x), r^-_R(x))]e^{i\min(\theta^-_{A\cup B}(x),\theta^-_R(x))} \end{array} \right\rangle : x \in X \right\} = (A \cup B) \cup R$$

Analogously to prove (vii), we can prove (viii). □

Proposition 3.7. *Let A and B be BCFSs over universe of discourse U. Then*

i. $(A^c)^c = A$,

ii. $(A \cup B)^c = A^c \cap B^c$,

iii. $(A \cap B)^c = A^c \cup B^c$.

Proof. Let A and B are two (BCFS) given as:

$A = \{(x, r^+_A(x)e^{i\theta^+_A(x)}, r^-_A(x)e^{i\theta^-_A(x)}) : x \in X\}$ and

$B = \{(x, r^+_B(x)e^{i\theta^+_B(x)}, r^-_B(x)e^{i\theta^-_B(x)}) : x \in X\}$.

So, to prove (i) we need to apply 3.2 twice. Then, we have

$A^c = \{(x, (1 - r^+_A(x))e^{i(2\pi-\theta^+_A(x))}, (-1 - r^-_A(x))e^{i(2\pi-\theta^-_A(x))}) : x \in X\}$, thus

$$(A^c)^c = \left\{ \left(\begin{array}{c} x, \left(1 - \left(1 - r^+_A(x)\right)\right) e^{i\left(2\pi-(2\pi-\theta^+_A(x))\right)}, \\ \left(-1 - \left(-1 - r^-_A(x)\right)\right) e^{i\left(2\pi-(2\pi-i\theta^-_A(x))\right)} \end{array} \right) : x \in X \right\}$$

$$= \left\{ \left(x, r^+_A(x)e^{i\theta^+_A(x)}, r^-_A(x)e^{i\theta^-_A(x)}\right) : x \in X \right\} = A$$

To prove (ii) we need to apply 3.2 and 3.3. Then, we have

$(A \cup B)^c$ =complement$(A \cup B)$

$$=\text{complement}\left\{ \left\langle \begin{array}{c} x, \max\left(r^+_A(x), r^+_B(x)\right) e^{i\max\left(\theta^+_A(x),\theta^+_B(x)\right)}, \\ \min\left(r^-_A(x), r^-_B(x)\right) e^{i\min\left(\theta^-_A(x),\theta^-_B(x)\right)} \end{array} \right\rangle : x \in X \right\}$$

$$= \left\{ \left\langle \begin{array}{c} x, \min\left(1 - r^+_A(x), 1 - r^+_B(x)\right) e^{i\min\left(\theta^+_A(x),\theta^+_B(x)\right)-2\pi}, \\ \max\left(-1 - r^-_A(x), -1 - r^-_B(x)\right) e^{i\max\left(\theta^-_A(x),\theta^-_B(x)\right)-2\pi} \end{array} \right\rangle : x \in X \right\}$$

$= A^c \cap B^c$.

Analogously to prove (ii), we can prove (iii). □

4. Conclusion

In this work, introduced (BCFS), also a new operations defined over the (BCFS) some properties of this operations are discussed. These properties illustrate the relationship between the basic set theoretical operations such as: commutative laws, associative laws, distributive laws, and De Morgan's laws.

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