Some generalized forms of soft compactness and soft Lindelöfness via soft α-open sets

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Abstract. By using a notion of soft α-open sets, we generalize the concepts of soft compact and soft Lindelöf spaces. We define the concepts of soft α-compact, soft α-Lindelöf, almost (approximately, mildly) soft α-compact and almost (approximately, mildly) soft α-Lindelöf spaces. We present two new kinds of the finite intersection property and utilize them to characterize almost soft α-compact and approximately soft α-compact spaces. To elucidate the relationships among the introduced spaces and to illustrate our main results, we supply several interesting examples. Also, we point out that the initiated spaces are preserved under soft α-irresolute mappings and we investigate certain of results which associate an extended soft topology with the introduced soft spaces. In the end, we conclude some findings which associate the introduced spaces with some soft topological notions such as soft α-connectedness, soft α-T_2-spaces, soft α-partition and soft subspaces.

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1. Introduction and preliminaries

In the year 1999, the Russian researcher Molodtsove [24] initiated and studied a new mathematical approach for solving problems associated with uncertainties, namely soft sets. He pointed out that there are no limited conditions to the description of objects, so researchers can choose the form of parameters they need, which simplifies the decision-making process and make the process more efficient in the case of incomplete information. Then Maji et al. [22] in 2003, carried out a systematic study to construct some basic operations between two soft sets. Despite of a lot of shortcomings of Maji et al's study, it formed the first block of the soft set theory. To remove this weakness and to define some soft operators in a way that preserves the crisp properties via the soft set theory, Ali et al. [3] formulated some new operators such as restricted union, restricted intersection and restricted difference of two soft sets and a relative complement of a soft set.

Shabir and Naz [28] in 2011, employed the idea of soft sets to define the soft topological spaces concept. They examined the main properties of soft closed operators and soft separation axioms. Later on, Min [23] showed that Example 9 of [28] does not satisfy a condition of soft T_2-spaces and proved that every soft T_3-space is soft T_2. Zorlutuna et al. [29] showed the connection between fuzzy sets and soft sets. Also, they initiated the first shape of soft point in order to study some properties of soft interior points and soft neighborhood systems. Aygünoğlu and Aygün [14] introduced a concept of soft compact spaces and investigated its main features. They also presented a notion of enriched soft topological spaces and illustrated its role to verify some results associated with constant soft mappings and soft compact spaces. The authors of [16, 25] simultaneously modified the first shape of soft point to be more effective for studying soft limit points and soft metric spaces. Chen [15] started studying generalized soft open sets by defining and investigating soft semi-open sets. Then Arockiarani and Lancy [13] presented a concept of soft pre-open sets and studied its fundamental properties. Akdag and Ozkan introduced a soft α-open sets notion in [1] and then carried out a detailed study on soft α-separation axioms in [2]. Ozturk and Bayramov [26] defined a soft compact-open topology concept and studied its main features. Kandil et al. [20] presented γ-operations and investigated their main properties. The authors of [4, 5, 7, 18] showed some alleged results on soft axioms and corrected them with the help of examples. Al-shami and Kočinac [12] proved the equivalence between the enriched and extended soft topologies and then they derived that $(int(H), K) = int(H, K)$ and $(cl(H), K) = cl(H, K)$, for any soft subset (H, K) of an extended soft topological space. Recently, Al-shami [6, 9] introduced a new class of generalized soft open sets, namely soft somewhere dense sets; and a newly soft mathematical structure, namely soft topological ordered spaces. Al-shami et al. [10, 11] studied new types of soft ordered maps by using soft α-open and soft β-open sets.

This paper is an attempt to open up the theoretical aspects of soft sets by extending the notions of soft compact and soft Lindelöf spaces to the framework of soft sets. We begin this work by presenting certain of concepts of soft set theory and soft topological spaces that we will need to demonstrate our new findings. The goal of this study is to employ soft α-open sets to initiate eight kinds of generalized soft compact spaces, namely soft α-compact, soft α-Lindelöf, almost (approximately, mildly) soft α-compact and almost (approximately, mildly) soft α-Lindelöf spaces. We characterize each one of these concepts and provide various examples to elucidate the relationships among these spaces. Moreover, we offer some soft topological concepts such as soft α-hyperconnectedness and soft α-partition spaces, and we establish some properties which associate them with the introduced generalized soft compact spaces. We demonstrate the relationships between an extended soft topology and the introduced soft spaces. The sufficient conditions for the eight initiated generalized soft compact spaces to be soft hereditary properties are investigated. last but not least, we point out that the soft α-irresolute maps preserve all of the given generalized soft compact spaces.

In order to investigate and discuss our new results, we recollect the following definitions and results which will be needed in the sequels. We shall write these definitions with respect to a fixed set of parameters because we only utilize them on the frame of soft topological spaces which defined on a fixed set of parameters.

Definition 1.1 ([24]). *A pair (G, K) is called a soft set over X provided that G is a mapping of a parameters set K into the family of all subsets of X. It can be expressed as follows: $(G, K) = \{(k, G(k)) : k \in K \text{ and } G(k) \in 2^X\}$.*

Definition 1.2 ([17, 28]). *Let (G, K) be a soft set over X. We say that:*

(i) *$x \in (G, K)$ if $x \in G(k)$ for each $k \in K$; and $x \notin (G, K)$ if $x \notin G(k)$ for some $k \in K$.*

(ii) *$x \Subset (G, K)$ if $x \in G(k)$ for some $k \in K$; and $x \not\Subset (G, K)$ if $x \notin G(k)$ for each $k \in K$.*

Definition 1.3 ([17]). *A soft subset (G, K) over X is called stable if there is a subset S of X such that $G(k) = S$, for each $k \in K$ and it is denoted by $\widetilde{S}$.*

Definition 1.4 ([3]). *We say that (G, K) is a soft subset of (H, K), denoted by $(G, K)\widetilde{\subseteq}(H, K)$, provided that $G(k) \subseteq H(k)$, for each $k \in K$.*

Definition 1.5 ([3]). *The relative complement of a soft set (G, K), denoted by $(G, K)^c$, is given by $(G, K)^c = (G^c, K)$, where a mapping $G^c : K \to 2^X$ is defined by $G^c(k) = X \setminus G(k)$, for each $k \in K$.*

Definition 1.6 ([3, 27]). *The soft union and intersection of two soft sets (G, K), (F, K) is given by the following rule:*

(i) $(G, K)\widetilde{\bigcup}(F, K) = (H, K)$, *where* $H(k) = G(k) \bigcup F(k)$ *for each* $k \in K$.

(ii) $(G, K)\widetilde{\bigcap}(F, K) = (H, K)$, *where* $H(k) = G(k) \bigcap F(k)$ *for each* $k \in K$.

Definition 1.7 ([22]). *A soft set (G, K) over X is called:*

(i) *An absolute soft set if $G(k) = X$ for each $k \in K$. It is denoted by $\widetilde{X}$;*

(ii) *A null soft set if $G(k) = \emptyset$ for each $k \in K$. It is denoted by $\widetilde{\emptyset}$.*

Definition 1.8 ([28]). *A collection τ of soft sets over X with a fixed set of parameters K is called a soft topology on X if it satisfies the following three axioms:*

(i) *The null soft set $\widetilde{\emptyset}$ and the absolute soft set $\widetilde{X}$ are members of τ;*

(ii) *τ is closed under an arbitrary soft union and*

(iii) *τ is closed under a finite intersection.*

The triple (X, τ, K) is called a soft topological space (For short, STS). Each soft set in τ is called soft open and its relative complement is called soft closed.

Proposition 1.9 ([28]). *Let (X, τ, K) be an STS. Then $\tau_k = \{G(k) : (G, K) \in \tau\}$ defines a topology on X, for each $k \in K$.*

Definition 1.10 ([14]). *A soft topology τ on X is said to be an enriched soft topology if axiom **(i)** of Definition (1.8) is replaced by the following condition: (G, K) is soft open if and only if $G(k) = X$ or $\emptyset$, for each $k \in K$. In this case, the triple (X, τ, K) is called an enriched STS over X.*

Proposition 1.11 ([25]). *Consider (X, τ, K) is an STS and τ_k is a topology on X as in the above proposition. Then $\tau^\star = \{(G, K) : G(k) \in \tau_k$, for each $k \in K\}$ is a soft topology on X finer than τ.*

Remark 1.12. The authors of [12] termed $\tau^\star$ an extended soft topology and demonstrated that the extended and enriched soft topologies are identical.

Theorem 1.13 ([12]). *We have the following two results, for any soft subset (H, K) of an extended soft topological space $(X, \tau^\star, K)$.*

(i) $(int(H), K) = int(H, K)$.

(ii) $(cl(H), K) = cl(H, K)$.

Definition 1.14 ([12, 28]). *Let (F, K) be a soft subset of an STS (X, τ, K). Then:*

(i) *$(cl(F), K)$ is defined as $cl(F)(k) = cl(F(k))$, where $cl(F(k))$ is the closure of $F(k)$ in (X, τ_k), for each $k \in K$.*

(ii) *$(int(F), K)$ is defined as $int(F)(k) = int(F(k))$, where $int(F(k))$ is the interior of $F(k)$ in (X, τ_k), for each $k \in K$.*

Definition 1.15 ([25]). *Let (X, τ, K) be an STS and (Y, K) be a non-null soft subset of $\widetilde{X}$. Then $\tau_{(Y,K)} = \{(Y, K)\widetilde{\bigcap}(G, K) : (G, K) \in \tau\}$ is said to be a relative soft topology on (Y, K) and $((Y, K), \tau_{(Y,K)}, K)$ is called a soft subspace of (X, τ, K).*

Definition 1.16 ([16, 25]). *A soft subset (P, K) over X is called soft point if there is $k \in K$ and there is $x \in X$ such that $P(k) = \{x\}$ and $P(e) = \emptyset$, for each $e \in K \setminus \{k\}$. A soft point will be shortly denoted by P_k^x.*

Definition 1.17 ([16]). *A soft subset (H, K) of $\widetilde{X}$ is called a finite (resp. countable) soft set if $H(k)$ is finite (resp. countable) for each $k \in K$. A soft set is called an infinite (resp. uncountable) soft set if it is not finite (resp. countable).*

Definition 1.18. *A soft subset (A, K) of an STS (X, τ, K) is said to be:*

(i) *Soft α-open [1] if $(A, K)\widetilde{\subseteq}int(cl(int(A, K)))$.*

(ii) *Soft semi-open [15] if $(A, K)\widetilde{\subseteq}cl(int(A, K))$.*

(iii) *Soft pre-open [13] if $(A, K)\widetilde{\subseteq}int(cl(A, K))$.*

Proposition 1.19 ([1]). *The union of an arbitrary class of soft α-open sets is soft α-open and the intersection of an arbitrary class of soft α-closed sets is soft α-closed.*

Definition 1.20 ([1]). *For a soft subset (G, K) of (X, τ, K), we define the following two operators:*

(i) *$int_\alpha(G, K)$ is the soft union of all soft α-open sets contained in (G, K).*

(ii) *$cl_\alpha(G, K)$ is the soft intersection of all soft α-closed sets containing (G, K).*

Definition 1.21 ([2]). *An STS (X, τ, K) is said to be soft α T_2-space if for every $x \neq y$ in X, there are two disjoint soft α-open sets (G, K) and (F, K) such that $x \in (G, K)$ and $y \in (F, K)$.*

Proposition 1.22 ([8]). *Consider $((U, K), \tau_{(U,K)}, K)$ is a soft subspace of (X, τ, K) and let cl_U and int_U stand for the soft closure and soft interior operators, respectively, in $((U, K), \tau_{(U,K)}, K)$. Then:*

(i) *$cl_U(A, K) = cl(A, K)\widetilde{\bigcap}(U, K)$, for each $(A, K)\widetilde{\subseteq}(U, K)$.*

(ii) *$int(A, K) = int_U(A, K)\widetilde{\bigcap}int(U, K)$, for each $(A, K)\widetilde{\subseteq}(U, K)$.*

Throughout this work, the two notations (X, τ, K) and (Y, θ, K) stand for soft topological spaces and a notation S stands for a countable set.

2. Soft α-compact spaces

Definition 2.1. (i) *A family $\{(G_i, K) : i \in I\}$ of soft α-open sets is called a soft α-open cover of (X, τ, K) if $\widetilde{X} = \widetilde{\bigcup}_{i \in I}(G_i, K)$.*

(ii) *An STS (X, τ, K) is called soft α-compact (resp. soft α-Lindelöf) if every soft α-open cover of $\widetilde{X}$ has a finite (resp. countable) soft sub-cover of $\widetilde{X}$.*

For the purpose of brevity, we shall omit the proofs of the following three propositions.

Proposition 2.2. *Every soft α-compact space is soft α-Lindelöf.*

Proposition 2.3. *A finite (resp. countable) union of soft α-compact (resp. soft α-Lindelöf) subsets of (X, τ, K) is soft α-compact (resp. soft α-Lindelöf).*

Proposition 2.4. *Every soft α-compact (resp. soft α-Lindelöf) space is soft compact (resp. soft Lindelöf).*

The converse of Proposition (2.4) is incorrect as it is evident in the example below.

Example 2.5. Consider a set of parameters K is the set of irrational numbers $\mathcal{Q}^c$ and let a collection $\tau = \{\widetilde{\emptyset}, \widetilde{X}, (G, K)$ such that $G(k) = \{1\}$, for each $k \in K\}$ be a soft topology on $X = \{1, 2\}$. Obviously, (X, τ, K) is soft compact. On the other hand, a collection $\{(G, E)$: There exists $k \in K$ such that $G(k) = X$ and $G(k_j) = \{1\}$, for each $k_j \neq k\}$ forms a soft α-open cover of $\widetilde{X}$. Since this collection has not a countable sub-cover of $\widetilde{X}$, then (X, τ, K) is not soft α-Lindelöf.

Proposition 2.6. *Every soft α-closed subset (D, K) of a soft α-compact (resp. soft α-Lindelöf) space (X, τ, K) is soft α-compact (resp. soft α-Lindelöf).*

Proof. We will start with the proof for soft α-Lindelöf spaces, as the proof for soft α-compact spaces is analogous.

Let (D, K) be a soft α-closed subset of $\widetilde{X}$ and let $\{(H_i, K) : i \in I\}$ be a soft α-open cover of (D, K). Then (D^c, K) is soft α-open and $(D, K) \widetilde{\subseteq} \widetilde{\bigcup}_{i \in I}(H_i, K)$. Therefore $\widetilde{X} = \widetilde{\bigcup}_{i \in I}(H_i, K) \widetilde{\bigcup}(D^c, K)$. Since $\widetilde{X}$ is soft α-Lindelöf, then $\widetilde{X} = \widetilde{\bigcup}_{i \in S}(H_i, K) \widetilde{\bigcup}(D^c, K)$. This implies that $(D, K) \widetilde{\subseteq} \widetilde{\bigcup}_{i \in S}(H_i, K)$. Hence (D, K) is soft α-Lindelöf. □

Corollary 2.7. *If (G, K) is soft α-compact (resp. soft α-Lindelöf) and (D, K) is soft α-closed subsets of $\widetilde{X}$, then their soft intersection is soft α-compact (resp. soft α-Lindelöf).*

To show that the converse of the above proposition is not necessarily correct, we give the following example.

Example 2.8. Take $K = \{k_1, k_2\}$ and assume that τ is the same as in Example (2.5). Then (X, τ, K) is soft α-compact. Take a soft set (G, K) such that $G(k_1) = \{1\}$ and $G(k_2) = \emptyset$. Then (G, K) is a soft α-compact, but it is not soft α-closed.

Theorem 2.9. *An STS (X, τ, K) is soft α-compact (resp. soft α-Lindelöf) if and only if every soft collection of soft α-closed subsets of (X, τ, K), satisfying the finite (resp. countable) intersection property, has, itself, a non-null soft intersection.*

Proof. We only prove the theorem when (X, τ, K) is soft α-Lindelöf, the other case can be made similarly.

Let $\Lambda = \{(F_i, K) : i \in I\}$ be a soft α-closed subsets of $\widetilde{X}$. Suppose that $\widetilde{\bigcap}_{i\in I}(F_i, K) = \widetilde{\emptyset}$. Then $\widetilde{X} = \widetilde{\bigcup}_{i\in I}(F_i^c, K)$. As (X, τ, K) is soft α-Lindelöf, then $\widetilde{\bigcup}_{i\in S}(F_i^c, K) = \widetilde{X}$. Therefore $\widetilde{\bigcap}_{i\in S}(F_i, K) = \widetilde{\emptyset}$.

Conversely, let $\Lambda = \{(H_i, K) : i \in I\}$ be a soft α-open cover of $\widetilde{X}$. Suppose that Λ has no a countable soft sub-collection which cover $\widetilde{X}$. Then $\widetilde{\bigcup}_{i\in S}(H_i, K) \neq \widetilde{X}$, for any countable set S. Now, $\widetilde{\bigcap}_{i\in S}(H_i^c, K) \neq \widetilde{\emptyset}$ implies that $\{(H_i^c, K) : i \in I\}$ is a soft collection of soft α-closed subsets of $\widetilde{X}$ which has the countable intersection property. Thus $\widetilde{\bigcap}_{i\in I}(H_i^c, K) \neq \widetilde{\emptyset}$. This implies that $\widetilde{X} \neq \widetilde{\bigcup}_{i\in I}(H_i, K)$. But this contradicts that Λ is a soft α-open cover of $\widetilde{X}$. Hence (X, τ, K) is soft α-Lindelöf. □

Definition 2.10. *A soft mapping $g : (X, \tau, K) \to (Y, \theta, K)$ is called soft α-irresolute if the inverse image of each soft α-open subset of $\widetilde{Y}$ is a soft α-open subset of $\widetilde{X}$.*

We investigate the following theorem which will be useful to prove Theorem (3.11) and Theorem (4.15).

Theorem 2.11. *The following five statements are equivalent for a soft mapping $g : (X, \tau, K) \to (Y, \theta, K)$:*

(i) *g is soft α-irresolute;*

(ii) *The inverse image of each soft α-closed subset of $\widetilde{Y}$ is a soft α-closed subset of $\widetilde{X}$;*

(iii) *$cl_\alpha(g^{-1}(A, K)) \subseteq g^{-1}(cl_\alpha(A, K))$, for each soft subset (A, K) of $\widetilde{Y}$;*

(iv) *$g(cl_\alpha(E, K)) \subseteq cl_\alpha(g(E, K))$, for each soft subset (E, K) of $\widetilde{X}$;*

(v) *$g^{-1}(int_\alpha(A, K)) \subseteq int_\alpha(g^{-1}(A, K))$, for each soft subset (A, K) of $\widetilde{Y}$.*

Proof. (i) ⇒ (ii): Suppose that (F, K) is a soft α-closed subset of $\widetilde{Y}$. Then (F^c, K) is soft α-open. Therefore $g^{-1}(F^c, K)$ is a soft α-open subset of $\widetilde{X}$. It is well known that $g^{-1}(F^c, K) = X - g^{-1}(F, K)$. Hence $g^{-1}(F, K)$ is a soft α-closed subset of $\widetilde{X}$.

(ii) $\Rightarrow$ (iii): For any soft subset (A, K) of $\widetilde{Y}$, we get that $cl_\alpha(A, K)$ is a soft α-closed subset of $\widetilde{Y}$. Since $g^{-1}(cl_\alpha(A, K))$ is a soft α-closed subset of $\widetilde{X}$, then $cl_\alpha(g^{-1}(A, K))\widetilde{\subseteq}cl_\alpha(g^{-1}(cl_\alpha\ (A, K))) = g^{-1}(cl_\alpha(A, K))$.

(iii) $\Rightarrow$ (iv): For any soft subset (E, K) of $\widetilde{X}$, we know that $cl_\alpha(E, K)\widetilde{\subseteq}cl_\alpha(g^{-1}(g(E, K))$. By **(iii)**, we find that $cl_\alpha(g^{-1}(g(E, K)))\widetilde{\subseteq}g^{-1}(cl_\alpha(g(E, K)))$.

Hence $g(cl_\alpha(E, K))\widetilde{\subseteq}g(g^{-1}\ (cl_\alpha(g(E, K))))\widetilde{\subseteq}cl_\alpha(g(E, K))$.

(iv) $\Rightarrow$ (v): Let (A, K) be any soft subset of $\widetilde{Y}$.

Then $g(cl_\alpha(X - g^{-1}(A, K)))\widetilde{\subseteq}cl_\alpha(g(X - g^{-1}(A, K)))$. Therefore $g(X - int_\alpha(g^{-1}(A, K))) = g(cl_\alpha(X - g^{-1}(A, K)))\widetilde{\subseteq}cl_\alpha(\widetilde{Y} - (A, K)) = \widetilde{Y} - int_\alpha(A, K)$. Thus $\widetilde{X} - int_\alpha(g^{-1}(A, K))\widetilde{\subseteq}g^{-1}(\widetilde{Y} - int_\alpha(A, K)) = g^{-1}(\widetilde{Y}) - g^{-1}(int_\alpha(A, K))$. Hence $g^{-1}(int_\alpha(A, K))\widetilde{\subseteq}int_\alpha(g^{-1}(A, K))$.

(v) $\Rightarrow$ (i): Suppose that (A, K) is any soft α-open subset of $\widetilde{Y}$. Since $g^{-1}(int_\alpha(A, K))\ \widetilde{\subseteq}int_\alpha(g^{-1}(A, K))$, then $g^{-1}(A, K)\widetilde{\subseteq}int_\alpha(g^{-1}(A, K))$. Since $int_\alpha(g^{-1}(A, K))\widetilde{\subseteq}g^{-1}(A, K)$, then $g^{-1}(A, K) = int_\alpha(g^{-1}(A, K))$. Therefore $g^{-1}(A, K)$ is a soft α-open set. Hence g is a soft α-irresolute map. □

Proposition 2.12. *The soft α-irresolute image of a soft α-compact (resp. soft α-Lindelöf) set is soft α-compact (resp. soft α-Lindelöf).*

Proof. For the proof, let $g : X \to Y$ be a soft α-irresolute mapping and let (D, K) be a soft α-Lindelöf subset of $\widetilde{X}$. Suppose that $\{(H_i, K) : i \in I\}$ is a soft α-open cover of $g(D, K)$. Then $g(D, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in I}(H_i, K)$. Now, $(D, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in I}g^{-1}(H_i, K)$ and $g^{-1}(H_i, K)$ is soft α-open, for each $i \in I$. By hypotheses, (D, K) is soft α-Lindelöf, then $(D, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in S}g^{-1}(H_i, K)$. Therefore $g(D, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in S}g(g^{-1}(H_i, K)) \widetilde{\subseteq} \widetilde{\bigcup}_{i\in S}(H_i, K)$. Thus $g(D, K)$ is soft α-Lindelöf.

A similar proof is given in case of a soft α-compact space. □

Proposition 2.13. *A soft subset (H, K) of (X, τ, K) is soft α-open if and only if there exists a soft open set (G, K) such that $(G, K)\widetilde{\subseteq}(H, K)\widetilde{\subseteq}int(cl((G, K))$.*

Proof. Necessity: Consider (H, K) is a soft α-open set.

Then $int(H, K)\widetilde{\subseteq}(H, K)\widetilde{\subseteq}int\ (cl(int(H, K)))$. Taking $int(H, K) = (G, K)$. Hence $(G, K)\widetilde{\subseteq}(H, K)\widetilde{\subseteq}int(cl((G, K))$.

Sufficiency: Suppose that (H, K) is a soft set such that there exists a soft open set (G, K) satisfies that $(G, K)\widetilde{\subseteq}(H, K)\widetilde{\subseteq}int(cl(G, K))$.

Then $cl(G, K)\widetilde{\subseteq}cl(int(H, K))$. So $int(cl(G, K))\widetilde{\subseteq}int(cl(int(H, K)))$. By our assumption, $(H, K)\widetilde{\subseteq}int(cl((G, K))$. Thus $(H, K)\widetilde{\subseteq}int(cl(int(H, K)))$. Hence the proof is complete. □

Corollary 2.14. *A soft subset (H, K) of (X, τ, K) is soft α-closed if and only if there exists a soft closed set (F, K) such that $cl(int(F, K))\widetilde{\subseteq}(H, K)\widetilde{\subseteq}(F, K)$.*

In what follows, we list some properties of an extended soft topology (Definition (1.11)) and its relationship with soft α-compact and soft α-Lindelöf spaces.

Proposition 2.15. *If H is an α-open subset of (X, τ_k), then there exists a soft α-open subset (F, K) of an extended soft topological space (X, τ, K) such that $F(k) = H$.*

Proof. Suppose that H is an α-open subset of (X, τ_k). Then there exists an open subset $G(k)$ of (X, τ_k) such that $G(k) \subseteq H \subseteq int(cl(G(k)))$. Since $G(k)$ is an open subset of (X, τ_k), then a soft set (F, K), which is defined as $F(k) = G(k)$ and $F(k_i) = \emptyset$, for each $k_i \neq k$, is soft open. Also, we define a soft set (L, K) as $L(k) = H$ and $L(k_i) = \emptyset$, for each $k_i \neq k$. So we infer that $(G, K)\widetilde{\subseteq}(L, K)\widetilde{\subseteq}int(cl(G, K))$. From Proposition (2.13), we obtain that (L, K) is soft α-open. □

Theorem 2.16. *If (X, τ, K) is an extended soft α-compact (resp. extended soft α-Lindelöf) space, then (X, τ_k) is α-compact (resp. α-Lindelöf), for each $k \in K$.*

Proof. We prove the theorem in case of an extended soft α-Lindelöf space and the other case is proven similarly.

Let $\{H_j(k) : j \in J\}$ be an α-open cover of (X, τ_k). We construct a soft α-open cover of (X, τ, K) consisting of the following soft sets:

(i) From the above proposition, we can choose all soft α-open sets (F_j, K) in which $F_j(k) = H_j(k)$, for each $j \in J$.

(ii) Since (X, τ, K) is extended, then we take a soft open set (G, K) which satisfies that $G(k) = \emptyset$ and $G(k_i) = X$, for all $k_i \neq k$.

Obviously, $\{(F_j, K)\widetilde{\bigcup}(G, K) : j \in J\}$ is a soft α-open cover of (X, τ, K). As (X, τ, K) is soft α-Lindelöf, then $\widetilde{X} = \bigcup_{j \in S}(F_j, K)\widetilde{\bigcup}(G, K)$. So $X = \bigcup_{j \in S} F_j(k) = \bigcup_{j \in S} H_j(k)$. Hence (X, τ_k) is an α-Lindelöf space. □

To show that the converse of the above theorem fails, we consider the example below.

Example 2.17. Let a set of parameters be the set of irrational numbers $\mathcal{Q}^c$ and τ be a soft discrete topology on $X = \{1, 2, 3\}$. A collection Λ which consists of all soft points of $\widetilde{X}$ forms a soft open cover of $\widetilde{X}$. Obviously, Λ has not a countable subcover. So $\widetilde{X}$ is not soft α-Lindelöf. But (X, τ_k) is soft α-compact, for each $k \in \mathcal{Q}^c$.

Now, we give a condition which guarantees the converse of the above theorem holds.

Proposition 2.18. *Let (X, τ_k) be extended and K be finite (resp. countable). Then (X, τ, K) is soft α-compact (resp. soft α-Lindelöf) space iff (X, τ_k) is α-compact (resp. α-Lindelöf), for each $k \in K$.*

Proof. Necessity: It is obtained from the theorem above.

Sufficiency: Let $\{(G_j, K) : j \in J\}$ be a soft α-open cover of (X, τ, K) and $| K |= m$. Then $X = \bigcup_{j\in J} G_j(k)$, for each $k \in K$.

It follows, from Proposition (2.13), that there exists a soft open set (H_j, K) such that $(H_j, K)\tilde{\subseteq}(G_j, K)\tilde{\subseteq}int(cl((H_j, K))$. By Theorem (1.13), we find that $int(cl((H_j, K)) = (int(cl(H_j)), K)$. So $G_j(k)$ is soft α-open subset of (X, τ_k), for each $j \in J$. As (X, τ_k) is α-compact for each $k \in K$, then $X = \bigcup_{j=1}^{j=n_1} G_j(k_1)$, $X = \bigcup_{j=n_1+1}^{j=n_2} G_j(k_2)$,..., $X = \bigcup_{j=n_{m-1}+1}^{j=n_m} G_j(k_m)$. Therefore $\widetilde{X} = \widetilde{\bigcup}_{j=1}^{j=n_m}(G_j, K)$. Thus (X, τ, K) is soft α-compact.

A similar proof can be given for the case between parentheses. □

Proposition 2.19. *If (U, K) is soft open and (H, K) is soft α-open subsets of (X, τ, K), then $(U, K)\widetilde{\bigcap}\ (H, K)$ is a soft α-open subset of $((U, K), \tau_{(U,K)}, K)$.*

Proof. Since (U, K) is soft open and (H, K) is soft α-open subsets of (X, τ, K), then
$(U, K)\widetilde{\bigcap}(H, K)\tilde{\subseteq}\ (U, K)\widetilde{\bigcap}int(cl(int(H, K)))\tilde{\subseteq}int_U[(U, K))\widetilde{\bigcap}cl(int(H, K)]$
$\tilde{\subseteq}int_U(cl(U, K))\ \widetilde{\bigcap}int(H, K)$.

So $(U, K)\widetilde{\bigcap}(H, K)\ \tilde{\subseteq}int_U[cl(U, K))\widetilde{\bigcap}int(H, K)]\widetilde{\bigcap}(U, K) = int_U[cl(U,\ K))\widetilde{\bigcap}$ $int(H, K)\widetilde{\bigcap}(U, K)] = int_U[cl_U[(U, K))\widetilde{\bigcap}int(H, K)]]$. Since (U, K) is soft open, then $int_U[cl_U[(U, K))\widetilde{\bigcap}int(H, K)]] = int_U[cl_U[int[(U, K))\widetilde{\bigcap}(H, K)]]]\tilde{\subseteq}$ $int_U[cl_U[int_U[(U, K))\widetilde{\bigcap}\ (H, K)]]]$. Hence the proof is complete. □

Proposition 2.20. *For each soft open set (A, K) and soft set (B, K) in (X, τ, K), we have $(A, K)\widetilde{\bigcap}\ cl_\alpha(B, K)\tilde{\subseteq}cl_\alpha((A, K)\widetilde{\bigcap}(B, K))$.*

Proof. Let $P_k^x \in (A, K)\widetilde{\bigcap}cl_\alpha(B, K)$. Then $P_k^x \in (A, K)$ and $P_k^x \in cl_\alpha(B, K)$. Therefore for each soft α-open set (U, K) containing P_k^x, we have $(U, K)\widetilde{\bigcap}(B, K)$ $\neq \widetilde{\emptyset}$. Since $(U, K)\widetilde{\bigcap}(A, K)$ is a non-null soft α-open set and $P_k^x \in (U, K)\widetilde{\bigcap}(A, K)$, then $((U, K)\widetilde{\bigcap}(A, K))\ \widetilde{\bigcap}(B, K) \neq \widetilde{\emptyset}$. Now, $(U, K)\widetilde{\bigcap}((A, K)\widetilde{\bigcap}(B, K)) \neq \widetilde{\emptyset}$ implies that $P_k^x \in cl_\alpha((A, K)\widetilde{\bigcap}(B, K))$.

Therefore $(A, K)\widetilde{\bigcap}cl_\alpha(B, K)\tilde{\subseteq}\ cl_\alpha((A, K)\widetilde{\bigcap}(B, K))$. □

Lemma 2.21. *If (U, K) is a soft open subset of (X, τ, K) and (H, K) is soft α-open subset of $((U, K), \tau_{(U,K)}, K)$, then (H, K) is soft α-open subset of (X, τ, K).*

Proof. Since (H, K) is soft α-open subset of $((U, K), \tau_{(U,K)}, K)$, then
$(H, K)\tilde{\subseteq}int_U[\ cl_U[int_U(H, K)]]\ \widetilde{\bigcap}(U, K) = int[cl_U[int_U(H, K)]]$
$\tilde{\subseteq}int[cl[int_U(H, K)]] = int[cl[int_U\ [(H, K)\widetilde{\bigcap}(U, K)]]] = int[cl[int(H, K)]]$. So (H, K) is a soft α-open subset of (X, τ, K). □

Now, we are in a position to verify the following result.

Theorem 2.22. *A soft open subset (A,K) of (X,τ,K) is soft α-compact (resp. soft α-Lindelöf) if and only if a soft open subspace $((A,K),\tau_{(A,K)},K)$ is soft α-compact (resp. soft α-Lindelöf).*

Proof. We prove the theorem in case of soft α-compactness and the proof of the case between parentheses is made similarly.

Necessity: Let $\{(H_i,K) : i \in I\}$ be a soft α-open cover of $((A,K),\tau_{(A,K)},K)$. Since (A,K) is soft open containing (H_i,K), then it follows, by the above lemma, that (H_i,K) is soft α-open subsets of (X,τ,K).

By hypotheses, $(A,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n}(H_i,K)$.

Thus a soft subspace $((A,K),\tau_{(A,K)},K)$ is soft α-compact.

Sufficiency: Let $\{(G_i,K) : i \in I\}$ be a soft α-open cover of (A,K) in (X,τ,K). Now, $(A,K)\widetilde{\bigcap}(G_i,K)$ is a soft α-open subset of (X,τ,K).

By Proposition (2.19), we find that $(A,K)\widetilde{\bigcap}(G_i,K)$ is soft α-open subset of $((A,K),\tau_{(A,K)},K)$. As a soft subspace $((A,K),\tau_{(A,K)},K)$ is soft α-compact, then $(A,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n}((A,K)\widetilde{\bigcap}(G_i,K))$. So $(A,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n}(G_i,K)$. Thus (A,K) is a soft α-compact subset of (X,τ,K). □

Definition 2.23. *An STS (X,τ,K) is said to be soft α T_2'-space if for every two distinct soft points P_k^x and P_k^y, there are two disjoint soft α-open sets (G,K) and (F,K) such that $P_k^x \in (G,K)$ and $P_k^y \in (F,K)$.*

Lemma 2.24. *The soft intersection of a finite family of soft α-open sets is soft α-open.*

Proposition 2.25. *If (A,K) is a soft α-compact subset of a soft α T_2'-space (X,τ,K), then (A,K) is soft α-closed.*

Proof. Let the given conditions be satisfied and let $P_k^x \in (A,K)^c$. Then for each $P_k^y \in (A,K)$, there are two disjoint soft α-open sets (G_i,K) and (W_i,K) such that $P_k^x \in (G_i,K)$ and $P_k^y \in (W_i,K)$. It follows that $\{(W_i,K) : i \in I\}$ forms a soft α-open cover of (A,K). Consequently, $(A,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n}(W_i,K)$. By the above lemma, it follows that $\widetilde{\bigcap}_{i=1}^{i=n}(G_i,K) = (H,K)$ is a soft α-open set and since $(H,K)\widetilde{\bigcap}[\widetilde{\bigcup}_{i=1}^{i=n}(W_i,K)] = \widetilde{\emptyset}$, then $(H,K)\widetilde{\subseteq}(A,K)^c$. Thus $(A,K)^c$ is a soft α-open set. Hence (A,K) is soft α-closed. □

Corollary 2.26. *If (A,K) is a stable soft α-compact subset of a soft α T_2-space (X,τ,K), then (A,K) is soft α-closed.*

Proof. Since (A,K) is stable, then $P_k^x \in (A,K)$ if and only if $x \in (A,K)$. So by using similar technique of the above proof, the corollary holds. □

3. Almost soft α-compact spaces

Definition 3.1. *An STS (X, τ, K) is called almost soft α-compact (resp. almost soft α-Lindelöf) if every soft α-open cover of $\widetilde{X}$ has a finite (resp. countable) soft sub-collection in which the soft α-closures of whose members cover $\widetilde{X}$.*

Definition 3.2. *A soft set (F, K) is said to be:*

(i) *Soft α-clopen provided that it is soft α-open and soft α-closed.*

(ii) *Soft α-dense set provided that $cl_\alpha(F, K) = \widetilde{X}$.*

For the purpose of brevity, we shall omit the proofs of the following three propositions.

Proposition 3.3. *Every almost soft α-compact space is almost soft α-Lindelöf.*

Proposition 3.4. *A finite (resp. countable) union of almost soft α-compact (resp. almost soft α-Lindelöf) subsets of (X, τ, K) is almost soft α-compact (resp. almost soft α-Lindelöf).*

Proposition 3.5. *Every soft α-compact (resp. soft α-Lindelöf) space is almost soft α-compact (resp. almost soft α-Lindelöf).*

The converse of Proposition (3.5) is incorrect as it is evident in the example below.

Example 3.6. We illustrate that the given soft topological space (X, τ, K) in Example (2.5) is not soft α-Lindelöf. On the other hand, we can note that any soft α-open subset of (X, τ, K) must contain a soft open set (G, K). Since (G, K) is soft α-dense, then any soft α-open set is soft α-dense. So (X, τ, K) is almost soft α-compact.

Proposition 3.7. *Every soft α-clopen subset (D, K) of an almost soft α-compact (resp. almost soft α-Lindelöf) space (X, τ, K) is almost soft α-compact (resp. almost soft α-Lindelöf).*

Proof. Let us prove the proposition in case of (X, τ, K) is almost soft α-compact, the case between parentheses can be achieved similarly.

Let (D, K) be a soft α-clopen subset of $\widetilde{X}$ and let $\{(H_i, K) : i \in I\}$ be a soft α-open cover of (D, K). Then (D^c, K) is soft α-clopen. Therefore $\widetilde{X} = \widetilde{\bigcup}_{i \in I}(H_i, K)\widetilde{\bigcup}(D^c, K)$. Since $\widetilde{X}$ is almost soft α-compact, then $\widetilde{X} = \widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(H_i, K)\ \widetilde{\bigcup}(D^c, K)$. This implies that $(D, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(H_i, K)$. Hence (D, K) is almost soft α-compact. □

Corollary 3.8. *If (G, K) is an almost soft α-compact (resp. almost soft α-Lindelöf) subset of $\widetilde{X}$ and (D, K) is a soft α-clopen subset of $\widetilde{X}$, then (G, K) $\widetilde{\bigcap}(D, K)$ is almost soft α-compact (resp. almost soft α-Lindelöf).*

In Example (3.6), let (H,K) be a soft subset of (X,τ,K), where $H(k_1) = \{1,4\}$ and $H(k_i) = \{5\}$, for each $i \neq 1$. Then a soft set (H,K) is almost soft α-compact, but it is not soft α-clopen. So the converse of the above proposition is not necessarily correct.

Definition 3.9. *A collection $\Lambda = \{(F_i,K) : i \in I\}$ of soft sets is said to have the first type of finite (resp. countable) α-intersection property if $\widetilde{\bigcap}_{i=1}^{i=n} int_\alpha(F_i,K) \neq \widetilde{\emptyset}$, for any $n \in \mathcal{N}$ (resp. $\widetilde{\bigcap}_{i\in S} int_\alpha(F_i,K) \neq \widetilde{\emptyset}$, for any countable set S).*

It is clear that any collection satisfies the first type of finite (resp. countable) α-intersection property is also satisfies the finite (resp. countable) intersection property.

Theorem 3.10. *An STS (X,τ,K) is almost soft α-compact (resp. almost soft α-Lindelöf) if and only if every soft collection of soft α-closed subsets of (X,τ,K), satisfying the first type of finite (resp. countable) α-intersection property, has, itself, a non-null soft intersection.*

Proof. We will start with the proof for almost soft α-compactness, because the proof for almost soft α-Lindelöfness is analogous.

Let $\Lambda = \{(F_i,K) : i \in I\}$ be a soft α-closed subsets of $\widetilde{X}$. Suppose that $\widetilde{\bigcap}_{i\in I}(F_i,K) = \widetilde{\emptyset}$. Then $\widetilde{X} = \widetilde{\bigcup}_{i\in I}(F_i^c,K)$. As (X,τ,K) is almost soft α-compact, then $\widetilde{X} = \widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(F_i^c,K)$. Therefore $\widetilde{\emptyset} = (\widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(F_i^c,K))^c = \widetilde{\bigcap}_{i=1}^{i=n} int_\alpha(F_i,K)$. Hence the necessary condition holds.

Conversely, let Λ be a soft α-closed subsets of $\widetilde{X}$ which satisfies the first type of finite α-intersection property. Then it also satisfies the finite intersection property. Since Λ has a non-null soft intersection, then (X,τ,K) is a soft α-compact space. It follows, by Proposition (3.5), that (X,τ,K) is almost soft α-compact. □

Theorem 3.11. *The soft α-irresolute image of an almost soft α-compact (resp. almost soft α-Lindelöf) set is almost soft α-compact (resp. almost soft α-Lindelöf).*

Proof. For the proof, let $g : X \to Y$ be a soft α-irresolute mapping and (D,K) be an almost soft α-Lindelöf subset of $\widetilde{X}$. Suppose that $\{(H_i,K) : i \in I\}$ is a soft α-open cover of $g(D,K)$. Then $g(D,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in I}(H_i,K)$. Now, $(D,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in I} g^{-1}(H_i,K)$ and $g^{-1}(H_i,K)$ is soft α-open, for each $i \in I$. By hypotheses, (D,K) is almost soft α-Lindelöf, then $(D,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in S} cl_\alpha(g^{-1}(H_i,K))$. So $g(D,K)\widetilde{\subseteq}\ \widetilde{\bigcup}_{i\in S} g(cl_\alpha(g^{-1}(H_i,K)))$. From item **(iv)** of Theorem (2.11), we obtain that $g(cl_\alpha(g^{-1}(H_i,K)))\widetilde{\subseteq} cl_\alpha(g(g^{-1}\ (H_i,K)))\widetilde{\subseteq} cl_\alpha(H_i,K)$.

Thus $g(D,K)\widetilde{\subseteq}\widetilde{\bigcup}_{i\in S} cl_\alpha(H_i,K)$. Hence $g(D,K)$ is almost soft α-Lindelöf.

A similar proof is given in case of an almost soft α-compact space. □

Definition 3.12. *An STS (X, τ, K) is said to be soft α-hyperconnected if it does not contain disjoint soft α-open sets.*

Proposition 3.13. *Every soft α-hyperconnected space is almost soft α-compact.*

Proof. Since any soft α-open set in a soft α-hyperconnected space is soft α-dens, then the space is almost soft α-compact. □

The converse of the above proposition need not be correct in general, as the following example shall illustrates.

Example 3.14. Let $K = \{k_1, k_2\}$ be a set of parameters and consider $\tau = \{\widetilde{\emptyset}, \widetilde{X}, (G, K), (H, K), (L, K)$ be a soft topology on $X = \{33, 44\}$ such that:
$(G, K) = (k_1, \{33\}), (k_2, \emptyset)$;
$(H, K) = (k_1, \emptyset), (k_2, \{44\})$ and
$(L, K) = (k_1, \{33\}), (k_2, \{44\})\}$.

Obviously, (X, τ, K) is almost soft α-compact. On the other hand, the soft sets (G, K) and (H, K) are two disjoint soft α-open sets. Then (X, τ, K) is not soft α-hyperconnected.

Theorem 3.15. *If (X, τ, K) is an extended almost soft α-compact (resp. extended almost soft α-Lindelöf) space, then (X, τ_k) is almost α-compact (resp. almost α-Lindelöf), for each $k \in K$.*

Proof. We prove the theorem in case of an almost soft α-compact space and the other proof follows similar lines.

Let $\{H_j(k) : j \in J\}$ be an α-open cover for (X, τ_k). We construct a soft α-open cover for $\widetilde{X}$ like the introduced soft α-open cover in the proof of Theorem (2.16). Now, (X, τ, K) is almost soft α-compact implies that $\widetilde{X} = \bigcup_{j=1}^{j=n} cl_\alpha[(F_j, K)\widetilde{\bigcup}(G, K)] = \bigcup_{j=1}^{j=n}[(cl_\alpha(F_j), K)\ \widetilde{\bigcup}(G, K)]$.

Therefore, $X = \bigcup_{j=1}^{j=n} cl_\alpha(F_j(k)) = \bigcup_{j=1}^{j=n} cl_\alpha(H_j(k))$. Hence (X, τ_k) is an almost α-compact space. □

It can be seen from Example (2.17) that the converse of the above theorem need not be true in general.

Proposition 3.16. *Let (X, τ_k) be extended and K be finite (resp. countable). Then (X, τ, K) is almost soft α-compact (resp. almost soft α-Lindelöf) space iff (X, τ_k) is almost α-compact (resp. almost α-Lindelöf), for each $k \in K$.*

Proof. Necessity: It is obtained from the theorem above.

Sufficiency: Let $\{(G_j, K) : j \in J\}$ be a soft α-open cover of (X, τ, K). By similar discussion of the proof of the sufficient part of of Proposition (2.18), we obtain that $X = \bigcup_{j=1}^{j=n_1} cl_\alpha(G_j(k_1))$, $X = \bigcup_{j=n_1+1}^{j=n_2} cl_\alpha(G_j(k_2))$,..., $X =$

$\bigcup_{j=n_{m-1}+1}^{j=n_m} cl_\alpha(G_j(k_m))$. Therefore $\widetilde{X} = \widetilde{\bigcup}_{j=1}^{j=n_m} cl_\alpha(G_j, K)$. Since τ is extended, then it follows from Theorem (1.13), that $\widetilde{X} = \widetilde{\bigcup}_{j=1}^{j=n_m}(cl_\alpha(G_j), K) = \widetilde{\bigcup}_{j=1}^{j=n_m} cl_\alpha(G_j, K)$ Hence (X, τ, K) is almost soft α-compact.

A similar proof is given for the case between parentheses. □

Remark 3.17. If (X, τ, K) is an extended almost soft α-compact (resp. extended almost soft α-Lindelöf) space, then K is finite (resp. countable).

Proposition 3.18. *Consider $((U, K), \tau_{(U,K)}, K)$ is a soft subspace of (X, τ, K). Let cl_α and int_α stand for the soft α-closure and soft α-interior operators, respectively, in (X, τ, K) and let $cl_{\alpha U}$ and $int_{\alpha U}$ stand for the soft α-closure and soft α-interior operators, respectively, in $((U, K), \tau_{(U,K)}, K)$. Then:*

(i) *$cl_{\alpha U}(A, K) = cl_\alpha(A, K)\widetilde{\bigcap}(U, K)$, for each $(A, K)\widetilde{\subseteq}(U, K)$.*

(ii) *$int_\alpha(A, K) = int_{\alpha U}(A, K))$, for each $(A, K)\widetilde{\subseteq}(U, K)$.*

Theorem 3.19. *A soft open subset (A, K) of (X, τ, K) is almost soft α-compact (resp. almost soft α-Lindelöf) if and only if a soft open subspace $((A, K), \tau_{(A,K)}, K)$ is almost soft α-compact (resp. almost soft α-Lindelöf).*

Proof. Necessity: Let $\{(H_i, K) : i \in I\}$ be a soft α-open cover of $((A, K), \tau_{(A,K)}, K)$. Since (A, K) is soft open containing (H_i, K), then it follows, from Lemma (2.21), that (H_i, K) is soft α-open subsets of (X, τ, K).

By hypotheses, $(A, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(H_i, K) = \widetilde{\bigcup}_{i=1}^{i=n}[cl_\alpha(H_i, K)\widetilde{\bigcap}(A, K)$ $= \widetilde{\bigcup}_{i=1}^{i=n} cl_{\alpha U}(H_i, K)$. Thus a soft open subspace $((A, K), \tau_{(A,K)}, K)$ is almost soft α-compact.

Sufficiency: Let $\{(G_i, K) : i \in I\}$ be a soft α-open cover of (A, K) in (X, τ, K). Now, $(A, K)\widetilde{\bigcap}(G_i, K)$ is a soft α-open subset of (X, τ, K). By Proposition (2.19), we find that $(A, K)\widetilde{\bigcap}(G_i, K)$ is a soft α-open subset of $((A, K), \tau_{(A,K)}, K)$. As a soft open subspace $((A, K), \tau_{(A,K)}, K)$ is almost soft α-compact, then $(A, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n} cl_{\alpha U}[(A, K)\ \widetilde{\bigcap}(G_i, K)]\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n} cl_{\alpha U}(G_i, K)$.

So, $(A, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(G_i, K)$. Thus (A, K) is an almost soft α-compact subset of (X, τ, K).

A case between parentheses can be proven similarly. □

Proposition 3.20. *If (A, K) is an almost soft α-compact subset of a soft α T_2'-space (X, τ, K), then (A, K) is soft α-closed.*

Proof. Let the given conditions be satisfied and let $P_k^x \in (A, K)^c$. Then for each $P_k^y \in (A, K)$, there are two disjoint soft α-open sets (G_i, K) and (W_i, K) such that $P_k^x \in (G_i, K)$ and $P_k^y \in (W_i, K)$. It follows that $\{(W_i, K) : i \in I\}$ forms a soft α-open cover of (A, K). Consequently, $(A, K)\widetilde{\subseteq}\widetilde{\bigcup}_{i=1}^{i=n} cl_\alpha(W_i, K)$.

By Lemma (2.24), we obtain $\widetilde{\bigcap}_{i=1}^{i=n}(G_i,K) = (H,K)$ is a soft α-open set and since $(H,K)\widetilde{\bigcap}[\widetilde{\bigcup}_{i=1}^{i=n}(W_i,K)] = \widetilde{\emptyset}$, then $(H,K)\widetilde{\bigcap}[\widetilde{\bigcup}_{i=1}^{i=n}cl_\alpha(W_i,K)] = \widetilde{\emptyset}$. So $(H,K)\widetilde{\subseteq}(A,K)^c$. Thus $(A,K)^c$ is a soft α-open set. Hence (A,K) is soft α-closed. □

Corollary 3.21. *If (A,K) is an almost soft α-compact stable subset of a soft α T_2-space (X,τ,K), then (A,K) is soft α-closed.*

4. Approximately soft α-compact spaces

Definition 4.1. *An STS (X,τ,K) is called approximately soft α-compact (resp. approximately soft α-Lindelöf) space if every soft α-open cover of $\widetilde{X}$ has a finite (resp. countable) soft sub-collection in which its soft α-closure cover $\widetilde{X}$.*

Proposition 4.2. *Every approximately soft α-compact space is approximately soft α-Lindelöf*

Proof. Straightforward. □

We give an example below in order to show that the converse of the above proposition is not correct in general.

Example 4.3. Consider $(\mathcal{R},\tau,K)$ is a soft topological space such that $K = \{k_1,k_2\}$ is a set of parameters and $\tau = \{\widetilde{\emptyset},(G_i,K)\widetilde{\subseteq}\mathcal{R}$ such that for each $k \in K$, $G_i(k) = \{n\}$ or their soft union$\}$. Then any soft set (G,K) is soft α-open if and only if there exists $n \in \mathcal{N}$ such that $n \in (G_i,K)$. We define a soft α-open cover Λ of $\widetilde{X}$ as follows, $\Lambda = \{(G,K) : G(k) = \{1,x\}$, for each $k \in K\}$. This soft α-open cover has not a finite sub-cover in which its soft α-closure cover $\widetilde{X}$, hence $(\mathcal{R},\tau,K)$ is not approximately soft α-compact. On the other hand, for any soft α-open cover of $\widetilde{\mathcal{R}}$, we can find countable soft α-open subsets of Λ contains a soft open set $\{(G(k_1),\mathcal{N}),(G(k_2),\mathcal{N})\}$. This a soft α-open set is soft α-dense, hence $(\mathcal{R},\tau,K)$ is approximately soft α-Lindelöf.

Proposition 4.4. *A finite (resp. countable) union of approximately soft α-compact (resp. approximately soft α-Lindelöf) subsets of (X,τ,K) is approximately soft α-compact (resp. approximately soft α-Lindelöf).*

Proof. Let $\{(A_s,K) : s \in S\}$ be approximately soft α-Lindelöf subsets of (X,τ,K) and let $\{(G_i,K) : i \in I\}$ be a soft α-open cover of $\widetilde{\bigcup}_{s\in S}(A_s,K)$. Then there exist countable sets M_s such that $(A_1,K)\widetilde{\subseteq}cl_\alpha(\widetilde{\bigcup}_{i\in M_1}(G_i,K))$, $\dots$, $(A_n,K)\widetilde{\subseteq}cl_\alpha(\widetilde{\bigcup}_{i\in M_n}(G_i,K))$, $\dots$.

Therefore, $\widetilde{\bigcup}_{s\in S}(A_s,K)\widetilde{\subseteq}cl_\alpha(\widetilde{\bigcup}_{i\in M_1}(G_i,K))\widetilde{\bigcup}\dots\widetilde{\bigcup}cl_\alpha(\widetilde{\bigcup}_{i\in M_n}(G_i,K))\widetilde{\bigcup}\dots$ $\widetilde{\subseteq}cl_\alpha$ $(\widetilde{\bigcup}_{i\in\bigcup_{s\in S}M_s}(G_i,K))$. Since $\bigcup_{s\in S}M_s$ is a countable set, then the desired result is proved.

A similar proof is given in case of an approximately soft α-compact space. □

Proposition 4.5. *Every almost soft α-compact (resp. almost soft α-Lindelöf) space is approximately soft α-compact (resp. approximately soft α-Lindelöf).*

Proof. The proof is obtained directly from the fact that
$\widetilde{\bigcup}_{i\in I} cl_\alpha(G_i,K)\widetilde{\subseteq} cl_\alpha(\widetilde{\bigcup}_{i\in I}(G_i,K))$. □

Corollary 4.6. *Every soft α-hyperconnected space is approximately soft α-Lindelöf.*

We give an example below in order to show that the converse of the above proposition is not correct in general.

Example 4.7. Consider $(\mathcal{R},\tau,K)$ is a soft topological space such that $K=\{k_1,k_2\}$ and $\tau=\{\widetilde{\emptyset},\widetilde{\mathcal{R}},(G_1,K),(G_2,K),(G_3,K)$, where

$$(G_1,K)=\{(k_1,\{1\}),(k_2,\{1\}\};$$
$$(G_2,K)=\{(k_1,\{2\}),(k_2,\{2\}\};$$
$$(G_3,K)=\{(k_1,\{1,2\}),(k_2,\{1,2\}\}.$$

Then any soft set (G,K) is soft α-open if and only if $1\in(G,K)$ or $2\in(G,K)$. We define a soft α-open cover Λ of $\widetilde{X}$ as follows, $\Lambda=\{$for each $k\in K$, $(G,K):G(k)=\{1,x\}:x\neq 2$ and $(H,K):H(k)=\{2\}\}$. This soft α-open cover has not a countable sub-cover in which its soft α-closure of whose members cover $\widetilde{X}$, hence $(\mathcal{R},\tau,K)$ is not almost soft α-Lindelöf. On the other hand, any soft α-open cover contains a soft α-open set (G_3,K). A soft α-open set (G_3,K) is soft α-dense, hence $(\mathcal{R},\tau,K)$ is approximately soft α-compact.

Definition 4.8. *A collection $\Lambda=\{(F_i,K):i\in I\}$ of soft sets is said to have the second type of finite (resp. countable) α-intersection property if*
$int_\alpha[\widetilde{\bigcap}_{i=1}^{i=n}(F_i,K)]\neq\widetilde{\emptyset}$, for any $n\in\mathcal{N}$ (resp. $int_\alpha[\widetilde{\bigcap}_{i\in S}(F_i,K)]\neq\widetilde{\emptyset}$, for any countable set S).

It is clear that any collection satisfies the second type of finite (resp. countable) α-intersection property is also satisfies the first type of finite (resp. countable) α-intersection property.

Theorem 4.9. *An STS (X,τ,K) is approximately soft α-compact (resp. approximately soft α-Lindelöf) if and only if every soft collection of soft α-closed subsets of (X,τ,K), satisfying the second type of finite (resp. countable) α-intersection property, has, itself, a non-null soft intersection.*

Proof. We only prove the theorem when (X,τ,K) is approximately soft α-compact, the other case can be made similarly.

Let $\Lambda=\{(F_i,K):i\in I\}$ be a soft α-closed subsets of $\widetilde{X}$. Suppose that $\widetilde{\bigcap}_{i\in I}(F_i,K)=\widetilde{\emptyset}$. Then $\widetilde{X}=\widetilde{\bigcup}_{i\in I}(F_i^c,K)$. As (X,τ,K) is approximately soft α-compact, then $\widetilde{X}=cl_\alpha(\widetilde{\bigcup}_{i=1}^{i=n}(F_i^c,K))$. Therefore $\widetilde{\emptyset}=(cl_\alpha(\widetilde{\bigcup}_{i=1}^{i=n}(F_i^c,K)))^c=int_\alpha(\widetilde{\bigcap}_{i=1}^{i=n}(F_i,K))$. Hence the necessary condition holds.

Conversely, Let Λ be a soft α-closed subsets of $\widetilde{X}$ which satisfies the second type of finite α-intersection property. Then it also satisfies the first type of finite α-intersection property. Since Λ has a non-null soft intersection, then (X, τ, K) is an almost soft α-compact space. It follows, by Proposition (4.5), that (X, τ, K) is approximately soft α-compact. □

Definition 4.10. *A topological space (X, τ) is called approximately α-compact (resp. approximately α-Lindelöf) space if every α-open cover of X has a finite (resp. countable) sub-cover in which its α-closure cover X.*

Theorem 4.11. *A soft open subset (A, K) of (X, τ, K) is approximately soft α-compact (resp. approximately soft α-Lindelöf) if and only if a soft subspace $((A, K),\ \tau_{(A,K)}, K)$ is approximately soft α-compact (resp. approximately soft α-Lindelöf).*

Proof. The proof is similar of that Theorem (3.19). □

Definition 4.12. *An STS (X, τ, E) is called soft α-separable provided that it contains a countable α-dense soft set.*

Proposition 4.13. *If there exists a finite (resp. countable) soft α-dense subset of an STS (X, τ, K) such that K is finite (resp. countable), then (X, τ, K) is approximately soft α-compact (resp. approximately soft α-Lindelöf).*

Proof. Let $\{(G_i, K) : i \in I\}$ be a soft α-open cover of (X, τ, K) and let (B, K) be a finite (countable) soft α-dense subset of (X, τ, K). Then for each $P_{k_s}^{x_s} \in (B, K)$, there exists (G_{x_s}, K) containing $P_{k_s}^{x_s}$. This implies that $\widetilde{X} = cl_\alpha[\widetilde{\bigcup}(G_{x_s}, K)]$. Since (B, K) and K are finite (countable), then the collection $\{(G_s, K)\}$ is finite (countable). Hence the proof is complete. □

Corollary 4.14. *Every soft α-separable with a countable set of parameters K is approximately soft α-Lindelöf.*

Theorem 4.15. *The soft α-irresolute image of an approximately soft α-compact (resp. approximately soft α-Lindelöf) set is approximately soft α-compact (resp. approximately soft α-Lindelöf).*

Proof. We prove the theorem by using a similar technique of the proof of Theorem (3.11) and employing item **(iii)** of Theorem (2.11). □

Proposition 4.16. *If (A, K) is an approximately soft α-compact subset of a soft α T_2'-space (X, τ, K), then (A, K) is soft α-closed.*

Proof. The proof is similar of that Proposition (3.20). □

Corollary 4.17. *If (A, K) is an approximately soft α-compact stable subset of a soft α T_2-space (X, τ, K), then (A, K) is soft α-closed.*

5. Mildly soft α-compact spaces

Definition 5.1. *An STS (X, τ, K) is called mildly soft α-compact (resp. mildly soft α-Lindelöf) if every soft α-clopen cover of $\widetilde{X}$ has a finite (resp. countable) soft subcover.*

The proofs of the next two propositions are easy and will be omitted.

Proposition 5.2. *A finite (resp. countable) union of mildly soft α-compact (resp. mildly soft α-Lindelöf) subsets of (X, τ, K) is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Proposition 5.3. *Every mildly soft α-compact space is mildly soft α-Lindelöf.*

It can be seen from Example (4.3) that the converse of above proposition fails.

Proposition 5.4. *Every almost soft α-compact (resp. almost soft α-Lindelöf) space (X, τ, K) is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Proof. We only prove the proposition in case of (X, τ, K) is almost soft α-Lindelöf, the other case can be achieved similarly.

Let $\Lambda = \{(H_i, K) : i \in I\}$ be a soft α-clopen coverof (X, τ, K). Then $\widetilde{X} = \widetilde{\bigcup}_{s \in S} cl_\alpha(H_i, K)$. Now, $cl_\alpha(H_i, K) = (H_i, K)$. Therefore (X, τ, K) is mildly soft α-Lindelöf. □

Corollary 5.5. *Every soft α-compact (resp. soft α-Lindelöf) space is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Corollary 5.6. *If (X, τ, K) is soft α-hyperconnected, then the following six concepts are equivalent:*

(i) *Almost soft α-compact;*

(ii) *Almost soft α-Lindelöf;*

(iii) *Approximately soft α-compact;*

(iv) *Approximately soft α-Lindelöf;*

(v) *Mildly soft α-compact;*

(vi) *Mildly soft α-Lindelöf.*

Proposition 5.7. *Every soft α-connected space (X, τ, E) is mildly soft α-compact.*

Proof. Because (X, τ, K) is soft α-connected, then the only soft α-clopen subsets of (X, τ, K) are $\widetilde{X}$ and $\widetilde{\emptyset}$. Therefore (X, τ, K) is mildly soft α-compact.

□

One can be easily seen from Example (3.14) that the two soft sets (M, K) and (V, K), where $M(k_1) = M(k_2) = \{33\}$ and $V(k_1) = M(k_2) = \{44\}$, are disjoint soft α-open and their soft union is $\widetilde{X}$. So the converse of the above proposition is not always true.

In the next example, we illuminate that an approximately soft α-compact space need not be mildly soft α-Lindelöf.

Example 5.8. Assume that $(\mathcal{R}, \tau, K)$ is the same as in Example (4.7). We illustrated that $(\mathcal{R}, \tau, K)$ is an approximately soft α-Lindelöf space. The given soft collection Λ forms an α-clopen cover of $\mathcal{R}$. Since Λ has not a countable sub-cover, then $(\mathcal{R}, \tau, K)$ is not a mildly soft α-Lindelöf space.

Theorem 5.9. *An STS (X, τ, K) is mildly soft α-compact (resp. mildly soft α-Lindelöf) if and only if every soft collection of soft clopen subsets of (X, τ, K), satisfying the finite (resp. countable) intersection property, has, itself, a non-null soft intersection.*

Proof. The proof is similar to that of Theorem (2.9). □

Proposition 5.10. *The soft α-irresolute image of a mildly soft α-compact (resp. mildly soft α-Lindelöf) set is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Proof. By using a similar technique of the proof of Proposition (2.12), the proposition holds. □

For the sake of economy, the proofs of the following two results will be omitted.

Proposition 5.11. *If (D, K) is a soft α-clopen subset of a mildly soft α-compact (resp. mildly soft α-Lindelöf) space (X, τ, K), then (D, K) is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Corollary 5.12. *If (G, K) is a mildly soft α-compact (resp. mildly soft α-Lindelöf) subset of $\widetilde{X}$ and (D, K) is a soft α-clopen subset of $\widetilde{X}$, then $(G, K)\widetilde{\bigcap}(D, K)$ is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Definition 5.13. *An STS (X, τ, K) is said to be soft α-partition provided that a soft set is soft α-open if and only if it is soft α-closed.*

Theorem 5.14. *Let (X, τ, K) be a soft α-partition topological space. Then the following four statements are equivalent.*

(i) *(X, τ, K) is soft α-Lindelöf (resp. soft α-compact);*

(ii) *(X, τ, K) is almost soft α-Lindelöf (resp. almost soft α-compact);*

(iii) *(X, τ, K) is approximately soft α-Lindelöf (resp. approximately soft α-compact);*

(iv) *(X, τ, K) is mildly soft α-Lindelöf (resp. mildly soft α-compact).*

Proof. **(i) $\rightarrow$ (ii)**: It follows from Proposition (3.5).

(ii) $\rightarrow$ (iii): It follows from Proposition (4.5).

(iii) $\rightarrow$ (iv): Let $\{(G_i, K) : i \in I\}$ be a soft α-clopen cover of $\widetilde{X}$. As (X, τ, K) is approximately soft α-Lindelöf, then $\widetilde{X}\widetilde{\subseteq}cl_\alpha(\widetilde{\bigcup}_{s\in S}(G_i, K))$ and as (X, τ, K) is soft α-partition, then $cl_\alpha(\widetilde{\bigcup}_{s\in S}(G_i, K)) = \widetilde{\bigcup}_{s\in S}(G_i, K)$. Therefore (X, τ, K) is mildly soft α-Lindelöf.

(iv) $\rightarrow$ (i): Let $\{(G_i, K) : i \in I\}$ be a soft α-open cover of $\widetilde{X}$. As (X, τ, K) is soft α-partition, then $\{(G_i, K) : i \in I\}$ is a soft α-clopen cover of $\widetilde{X}$ and as (X, τ, K) is mildly soft α-Lindelöf, then $\widetilde{X} = \widetilde{\bigcup}_{s\in S}(G_i, K)$.

A similar proof can be given for the case between parentheses. □

Lemma 5.15. *If H is an α-clopen subset of (X, τ_k), then there exists a soft α-clopen subset (F, K) of an extended soft topological space (X, τ, K) such that $F(k) = H$.*

Proof. Suppose that H is an α-clopen subset of (X, τ_k). Then $cl(int(cl(H))) \subseteq H \subseteq int(cl(int(H)))$. Now, we define a soft set (L, K) as $L(k) = H$ and $L(k_i) = \emptyset$, for each $k_i \neq k$.

Since (X, τ, K) is extended, then we can conclude that $(cl(int(cl(L))), K) = cl(int(cl(L, K)))\widetilde{\subseteq}(L, K)\widetilde{\subseteq}(int(cl(int(L))), K) = int(cl(int(L, K)))$.

Hence (L, K) is a soft α-clopen subset of (X, τ, K) □

Theorem 5.16. *If (X, τ, K) is an extended mildly soft α-compact (resp. extended mildly soft α-Lindelöf) space, then (X, τ_k) is mildly α-compact (resp. mildly α-Lindelöf), for each $k \in K$.*

Proof. We prove the theorem in case of an extended mildly soft α-Lindelöf space and the other case is proven similarly.

Let $\{H_j(k) : j \in J\}$ be an α-clopen cover of (X, τ_k). We construct a soft α-open cover of (X, τ, K) consisting of the following soft sets:

(i) From the above lemma, we can choose all soft α-clopen sets (F_j, K) in which $F_j(k) = H_j(k)$, for each $j \in J$.

(ii) Since (X, τ, K) is extended, then we take a soft clopen set (G, K) which satisfies that $G(k) = \emptyset$ and $G(k_i) = X$, for all $k_i \neq k$.

Obviously, $\{(F_j, K)\widetilde{\bigcup}(G, K) : j \in J\}$ is a soft α-clopen cover of (X, τ, K). As (X, τ, K) is mildly soft α-Lindelöf, then $\widetilde{X} = \bigcup_{j\in S}(F_j, K)\widetilde{\bigcup}(G, K)$. So $X = \bigcup_{j\in S} F_j(k) = \bigcup_{j\in S} H_j(k)$. Hence (X, τ_k) is a mildly α-Lindelöf space. □

It can be seen from Example (2.17) that the converse of the above theorem need not be true in general.

Proposition 5.17. *Let (X, τ, K) be extended and K be finite (resp. countable). Then (X, τ, K) is soft mildly α-compact (resp. extended soft mildly α-Lindelöf) space iff (X, τ_k) is mildly α-compact (resp. mildly α-Lindelöf), for each $k \in K$.*

Proof. Necessity: It is obtained from the theorem above.

Sufficiency: Let $\{(G_j, K) : j \in J\}$ be a soft α-clopen cover of (X, τ, K). Then $X = \bigcup_{j\in J} G_j(k)$ for each $k \in K$. It follows, from Lemma (5.15), that there exists a soft clopen set (H_j, K) such that $H_j(k) = G_j(k)$ and $H_j(k_i) = X$, for each $k_i \neq k$. As (X, τ_k) is mildly α-compact, for each $k \in K$, then $X = \bigcup_{j=1}^{j=n_1} G_j(k_1)$, $X = \bigcup_{j=n_1+1}^{j=n_2} G_j(k_2), \ldots$, $X = \bigcup_{j=n_{m-1}+1}^{j=n_m} G_j(k_m)$. Therefore $\widetilde{X} = \widetilde{\bigcup}_{j=1}^{j=n_m}(G_j, K)$. Thus (X, τ, K) is mildly soft α-compact.

A similar proof can be given for the case between parentheses. □

Remark 5.18. If (X, τ, K) is an extended mildly soft α-compact (resp. extended mildly soft α-Lindelöf) space, then K is finite (resp. countable).

Definition 5.19. *A collection β of soft α-open sets is called soft α-base of (X, τ, K) if every soft α-open subset of $\widetilde{X}$ can be written as a soft union of members of β*

Theorem 5.20. *Consider (X, τ, K) has a soft α-base consists of soft α-clopen sets. Then (X, τ, K) is soft α-compact (resp. soft α-Lindelöf) if and only if it is mildly soft α-compact (resp. mildly soft α-Lindelöf).*

Proof. The necessary condition is obvious.

To verify the sufficient condition, assume that Λ is a soft α-open cover of a mildly soft α-compact space (X, τ, K). Since $\widetilde{X}$ is a soft union of members of the soft α-base and $\widetilde{X}$ is mildly soft α-compact, then we can find a finite member (H_s, K) of the soft α-base satisfies that $\widetilde{X} = \widetilde{\bigcup}_{s=1}^{s=n}(H_s, K)$. So for each member (G_s, K) of Λ, there exists a member (H_s, K) of the soft α-base such that $(H_s, K)\widetilde{\subseteq}(G_s, K)$. Thus $\widetilde{X} = \widetilde{\bigcup}_{s=1}^{s=n}(G_s, K)$. Hence (X, τ, K) is soft α-compact.

The proof in case of a mildly soft α-Lindelöf space is similar. □

Proposition 5.21. *If (A, K) is a mildly soft α-compact subset of a soft α T_2'-space (X, τ, K), then (A, K) is soft α-closed.*

Proof. The proof is similar to that of Proposition (2.25). □

Corollary 5.22. *If (A, K) is a mildly soft α-compact stable subset of a soft α T_2-space (X, τ, K), then (A, K) is soft α-closed.*

Conclusion

The purpose of the present study is to establish and introduce eight generalized forms of soft compactness and soft Lindelöfness, namely soft α-compactness, soft α-Lindelöfness, almost (approximately, mildly) soft α-compactness and almost (approximately, mildly) soft α-Lindelöfness. With the help of illustrative examples, the relationships among these concepts are shown and the image of these spaces under soft α-irresolute maps is investigated. Some properties of soft α-open sets which enable us to prove certain of our results are studied and verified. The relationships of some of the introduced spaces with soft α T_2-spaces and soft α T_2'-spaces are given. We study the equivalent conditions for all of the initiated spaces and illustrate under what conditions the four types of soft α-compact (the four types of soft α-Lindelöf) spaces are equivalent. The eight introduced concepts are compared in relation with many soft topological notions such as soft α-connectedness, soft subspaces and soft α-partition. The concepts presented in this study are fundamental for further researches and will open a way to improve more applications on soft topology.

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