Modules closed full large extensions of cyclic submodules are summands

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Abstract. This paper introduced a new generalization of extending modules, namely modules in which every closed full large extension of a cyclic submodule is a direct summand, introduced a new generalization of the concept of injective modules. In fact, we give and study the properties of the concept of full-LE-Cy-injective modules. Although full-LE-Cy-injective modules are far from injective modules, they are exactly the same on some kind of rings. Then we make use of relatively full-LE-Cy-injectivity on modules to study direct sums of two (C_1-LE-Cy)-modules. We show that a direct sum of two relatively full-LE-Cy-injective modules is a (C_1-LE-Cy)-module if and only if each one of them is a (C_1-LE-Cy)-module. Examples are provided to illustrate and delimit the theory.

Keywords: (C_1-LE-Cy)-modules, full LE-Cy-modules, full-LE-Cy-injective modules

1. Introduction

all rings are associative with unity, R denotes such a ring, and all modules considered are unitary right R-modules. A module M is said to be an extending module (or module with the condition (C_1)), if every closed submodule C of M is a direct summand of M. The notion of extending modules was generalized recently by many authors see ([1], [7], [3] and [9]). Some of such generaliza-

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tions were named in [4] by principally extending modules, in [6] by generalized extending modules.

In [8], Nicholson and Yousif have introduced and studied the structure of principally injective rings, and have given some characterizations of such rings in terms of the internal properties of these rings. They defined a module M over a ring R to be principally injective if every R-homomorphism from a principal right ideal of R to M can be extended to R. In [4], Kamal and El-mnophy adopt the concept of principally injective rings, in [8], and generalize it to modules. They also introduced the concept of principally extending, (denoted by P-extending). A module M is called a P-extending module if every cyclic submodule is large in a direct summand of M, or equivalently, every EC-closed submodule of M is a direct summand. A submodule N of M is called an EC-submodule of M if there exists m in M such that mR is large in N.

The present paper studies the concept of modules with the condition that every full LE-Cy-submodule is large in a direct summand. This new concept, in turn, generalizes the concept of extending modules. A module M is a large extension of cyclic (denoted by LE-Cy-module) if $mR \leq^L M$ for some $m \in M$. A module M is said to be a full large extension of cyclic module (denoted by full LE-Cy-module) if every submodule of M is a LE-Cy-module. A full LE-Cy-submodule N of M is said to be full LE-Cy-closed in M if N has no proper full LE-Cy-large extensions in M. Consider the following condition on a module M: (C_1-LE-Cy): Every full LE-Cy-submodule of M is large in a direct summand of M. A module M which satisfies the condition (C_1-LE-Cy) is called a (C_1-LE-Cy)-module, equivalently, every full LE-Cy-closed submodule of M is a direct summand of M. Let $M = M_1 \oplus M_2$. It is well known that M_1 is M_2-injective if and only if for every submodule K of M with $K \cap M_1 \neq 0$, there exists a submodule L of M such that $K \leq L$, and $M = M_1 \oplus L$. In analogue, we introduce here the concept of full large extensions of cyclic injectivity (relative full large extensions of cyclic injectivity) which is one of the generalizations of the concept of injectivity (relative injectivity). This generalization is extremely useful in analyzing the structure of direct sums of (C_1-LE-Cy)-modules. We show that if $M = M_1 \oplus M_2$, then M_1 is M_2-full-LE-Cy-injective if and only if for every full LE-Cy-submodule K of M with $K \cap M_1 = 0$, there exists a submodule $M^\backprime$ of M such that $K \subseteq M^\backprime$, and $M = M_1 \oplus M^\backprime$. We also show that if $M = M_1 \oplus M_2$, M_i is M_j-full-LE-Cy-injective ($i \neq j$), then M is a (C_1-LE-Cy)-module if and only if M_i is a (C_1-LE-Cy)-module (i= 1, 2).

In Section 2, Example 2.1., shows that there are LE-Cy-modules, which are not full LE-Cy-modules. We consider connections between relative full large extensions of cyclic injectivity and relative injectivity, Theorem 2.1., gives an equivalent condition of a module M to be full large extensions of cyclic injective relative to a module N in module decompositions. In Section 3, we introduce the concept of modules with the condition that every full LE-Cy-submodule is large in a direct summand. This new concept, in turn, generalizes the concept of extending modules. Example 3.1., shows that a (C_1-LE-Cy)-module not necessary to be an extending module, this example also shows that a direct sum of two (C_1-LE-Cy)-modules not necessary to be a (C_1-LE-Cy)-module. In Proposition 3.3., we show that a direct sum of two relatively full-LE-Cy-injective modules is a (C_1-LE-Cy)-module if and only if each one of them is a (C_1-LE-Cy)-module.

Let R be a ring and M, N be R-modules and ϕ be an R-homomorphism from M into N. If $N \subseteq M$, then $N \leq M$, $N \leq^L M$, $N \leq^c M$, $N \leq^{\oplus} M$, $Z(M)$, $Z_2(M)$ and $\langle\phi\rangle = \{m + \phi(m) : m \in M\}$ denote N is a submodule of M, N is a large submodule of M, N is a closed submodule of M, N is a direct summand submodule of M, the singular submodule of M, the second singular submodule of M, and the graph of a module homomorphism $\phi : M \to N$, respectively.

2. Full LE-Cy-injective modules

In this section, we introduce the concept of full large extensions of cyclic injectivity (relative full large extension of cyclic injectivity) which is one of the generalizations of injectivity (relative injectivity). This generalization is extremely useful in analyzing the structure of modules whose closed full large extension of cyclic are summands.

Definition 2.1. *A module M is a large extension of cyclic (denoted by LE-Cy-module) if $mR \leq^L M$ for some $m \in M$. A module M is said to be a full large extension of cyclic module (denoted by full LE-Cy-module) if every submodule of M is a LE-Cy-module.*

Remark 2.1.

1. Every uniform module is a full LE-Cy-module.
2. Let R be a principal right ideal ring, then R_R is a full LE-Cy-module.
3. There are semisimple modules, which are full LE-Cy-modules, for example $\mathbb{Z}_n$, where $n = p_1p_2...p_n$ (for distinct primes) as a $\mathbb{Z}$-module is full LE-Cy-module. It is clear that $\mathbb{Z}_n$ as a $\mathbb{Z}$-module for each nonzero n in $\mathbb{Z}$ is a full LE-Cy-module.
4. Every non-Noetherian semisimple module is not a full LE-Cy-module.
5. There are LE-Cy-modules, which are not full LE-Cy-modules, for example let S be the set of all functions: $\mathbb{R} \to \mathbb{R}$ ($\mathbb{R}$ is the set of real numbers). S is a commutative ring with +,. defined by, $(f + g)(r) = f(r) + g(r), (f.g)(r) = f(r).g(r))$ for all $f, g \in S$ and $r \in \mathbb{R}$. Hence S_S is an $LE - Cy$-module, and not full LE-Cy; for $I = \{f \in S : f(r) = 0$ for all $| r |> n$ (for some positive integer $n \in \mathbb{Z}$)$\}$ is not an LE-Cy-module.

Definition 2.2. *Let M and N be modules. We say that M is full large extension of cyclic injective relative to N (for short M is N-full-LE-Cy-injective) if, for each monomorphism $\alpha : K \longrightarrow N$, with K a full LE-Cy-module, and each homomorphism $\beta : K \longrightarrow M$, there exists a homomorphism $\phi : N \longrightarrow M$ such* *that $\phi\alpha = \beta$. M is called a full- LE-Cy-injective module, if M is N-full-LE-Cy-injective for every module N.*

Fully cyclic large extensions injectivity is one of the generalizations of injectivity. We are going to give some properties of such modules.

Lemma 2.1. *Isomorphic copy of a full LE-Cy-module is a full LE-Cy-module.*

Proof. Let M be a full *LE-Cy*-module, and $\alpha : M \longrightarrow N$ be an R-isomorphism. Let L be a nonzero submodule of N. Since M is a full *LE-Cy*-module, there exists m in M such that mR is large in $\alpha^{-1}(L)$. It is easy to check that $\alpha(m)R$ is large in L. Therefore, N is a full *LE-Cy*-module. □

Proposition 2.1. *Let M and N be R-modules. Then the following are equivalent:*

1) *M is N-full-LE-Cy-injective.*

2) *For each full LE-Cy-submodule K of N each homomorphism $\beta : K \longrightarrow M$ can be extended to N.*

Proof. 1) ⇒ 2) It is clear.

2) ⇒ 1) Let K be a full *LE-Cy*-module, and $\alpha : K \longrightarrow N$ be an R-monomorphism and $\beta : K \longrightarrow M$ be an R-homomorphism. Since $K \cong \alpha(K)$ and K is an *LE-Cy*-module. By Lemma 2.1., we have $\alpha(K)$ is an *LE-Cy*-module. By assumption; there exists an R-homomorphism $\phi : N \longrightarrow M$ such that $\phi\alpha(x) = \beta(x)$ for all $x \in K$. □

Remark 2.2. 1) N-full-*LE-Cy*-injectivity and N-injectivity are equivalent, whenever N be a full *LE-Cy*-module.

2) Full-*LE-Cy*-injectivity and injectivity are the same for modules over principal right ideal rings. In particular, injectivity and full-*LE-Cy*-injectivity are the same for $\mathbb{Z}$-modules.

3) Let $M = M_1 \oplus M_2$ be an R-module, and $\alpha : M_1 \longrightarrow M_2$ is a homomorphism. Then the following are well known:

i) $\langle\alpha\rangle = \{m_1 + \alpha(m_1) : m_1 \in M_1\}$ is a complementary summand of M_2 in M.

ii) $\langle\alpha\rangle \cong M_1$.

iii) If α is an monomorphism, then $\langle\alpha\rangle \cap M_1 = 0$.

Proposition 2.2.

1. *If M is N-full-LE-Cy-injective, then M is N'-full-LE-Cy-injective; for each submodule N' of M.*
2. *If M is N-full-LE-Cy-injective and $M' \leq^{\oplus} M$, then M' is N-full-LE-Cy-injective.*

Proof. It is clear. □

Theorem 2.1. *Let M_1 and M_2 be an R-modules and let $M = M_1 \oplus M_2$. Then the following are equivalent:*

1) *M_1 is M_2-full-LE-Cy-injective.*

2) *For every full LE-Cy-submodule H of M such that $H \cap M_1 = 0$, there exists a submodule M_3 of M such that $M = M_1 \oplus M_3$, and $H \leq M_3$.*

Proof. 1) $\Rightarrow$ 2) Let H be a full *LE-Cy*-submodule of M such that $H \cap M_1 = 0$. Let $\pi_i : M \longrightarrow M_i (i = 1, 2)$ be the projections. Observe that $\pi_2|_H : H \longrightarrow M_2$ is an monomorphism. Since M_1 is full M_2-*LE-Cy*-injective, there exists a R-homomorphism $\alpha : M_2 \longrightarrow M_1$ such that $\alpha \circ \pi_2|_H = \pi_1|_H$. Take $M_3 = \langle \alpha \rangle$, thus, by Remark 2.1., we have $M = M_1 \oplus M_3$. Now, for all $h \in H$, $h = \pi_1(h) + \pi_2(h) = \alpha \circ \pi_2(h) + \pi_2(h) \in M_3$. Therefore, $H \leq M_3$.

2) $\Rightarrow$ 1) Let K be a full *LE-Cy* submodule of M_2, $g : K \longrightarrow M_1$ be R-homomorphism. By Remark 2.1., we have $\langle g \rangle = \{k - g(k) : k \in K\} \cong K$. Thus, by Lemma 2.1., $\langle g \rangle$ is a full *LE-Cy*-submodule of M. Since $\langle g \rangle \cap M_1 = 0$, there exists a submodule M_3 of M such that $M = M_1 \oplus M_3$ and $\langle g \rangle \leq M_3$. Let $\pi_1 : M_1 \oplus M_3 \longrightarrow M_1$ be the projection. Then for all $k \in K$, we have $\pi_1(k) = \pi_1(k - g(k) + g(k)) = \pi_1(g(k)) = g(k)$. Therefore, π_1 extends g and hence M_1 is M_2-full-*LE-Cy*-injective. □

Corollary 2.1. *If $M = M_1 \oplus M_2$ and M_1 is M_2-full-LE-Cy-injective, then $M = M_1 \oplus C$ for every full large extension of cyclic and complement C of M_1 in M.*

Proof. Let C be a full large extension of cyclic and complement of M_1 in M. Since M_1 is M_2-full-*LE-Cy*-injective, there exists a submodule M_3 of M such that $M = M_1 \oplus M_3$ and $C \leq M_3$. Since $C \oplus M_1$ is large submodule of $M_1 \oplus M_3$, C is a large submodule of M_3. Therefore, $C = M_3$. □

3. Modules with closed full *LE-Cy*-submodules summands

In this section, we introduce the concept of modules with the condition that every full *LE-Cy*-submodule is large in a direct summand. This new concept, in turn, generalizes the concept of extending modules.

Definition 3.1. *Consider the following condition on a module M:*

(C_1-LE-Cy): Every full LE-Cy-submodule of M is large in a direct summand of M. A module M, which satisfies the condition (C_1-LE-Cy) is called a (C_1-LE-Cy)-module.

Definition 3.2. *A full LE-Cy-submodule N of M is said to be full LE-Cy-closed in M if N has no proper full LE-Cy-large extension in M.*

Lemma 3.1. *Let M be a module, and K be a full LE-Cy-submodule of M. Then L is a full LE-Cy-submodule, for each Large extension L of K in M.*

Proof. Let K be a full *LE-Cy*-submodule of M, and L be a large extension of K in M. Let D be a submodule of L. It follows that $D \cap K$ is large in D. Since K is a full *LE-Cy*-submodule of M, we have that $D \cap K$ is an *LE-Cy*-module.

Hence D is an LE-Cy-submodule of L. Therefore, L is a full LE-Cy-submodule of M. □

Corollary 3.1. *Every full LE-Cy-closed submodule of a module M is a closed submodule of M.*

Proof. Let K be a full LE-Cy-closed submodule of M and let N be a large extension of K in M. By Lemma 3.1., we have that N is full LE-Cy-submodule of M. Therefore, $K = N$. □

Corollary 3.2. *The following are equivalent for a module M:*

1. *M is a (C_1-LE-Cy)-module.*
2. *Every full LE-Cy-closed submodule of M a direct summand of M.*
3. *For every full LE-Cy-submodule N of M, there exists a decomposition $M = M_1 \oplus M_2$ such that $N \leq M_1$ and $N \oplus M_2$ is large in M.*

Proof. 1) ⇒ 2) Let H be a full LE-Cy-closed submodule of M, then there exists a direct summand submodule D of M such that H is large in D. By Corollary 3.1., we have that $H = D$. Therefore, H is a direct summand of M.
2) ⇒ 3) Let H be a full LE-Cy-submodule of M and let M_1 be a maximal large extension of H in M, then by Lemma 3.1., M_1 is full LE-Cy-closed in M. Therefore, $M = M_1 \oplus M_2$ such that $H \leq M_1$ and $H \oplus M_2$ is large in M.
3) ⇒ 1) It is clear. □

Proposition 3.1. *Let M be an indecomposable module and a full LE-Cy-module. Then M is a (C_1-LE-Cy)-module if and only if M is uniform.*

Proof. Let M be a (C_1-LE-Cy)-module and $0 \neq X$ be submodule of M. Then there exists a decomposition $M = M_1 \oplus M_2$ such that $X \leq M_1$ and $X \oplus M_2$ is large in M. It is clear that $M_2 = 0$. Therefore, M is uniform. □

Lemma 3.2 (Theorem 5, [5])**.** *Let M be a torsion free reduced module over a commutative integral domain R. If M is extending, then M is a finite direct sum of uniform submodules.*

Lemma 3.3 (Theorem 7, [5])**.** *Let M be a torsion free reduced module over an integral domain R with extension field K. Then the following are equivalent:*
1) M is extending.
2) $M = \oplus_{i=1}^{n} M_i$, with all M_i uniform, and for all $q_1, q_2, ..., q_n \in K$ (not all zero) there exist $\alpha_1, \alpha_2, ..., \alpha_n \in K$ such that $\sum_{k=1}^{n} \alpha_k = 1$ and $\alpha_k q_i M_k \subset q_k M_i$ for all k, i.

Example 3.1. 1) Every extending module is a (C_1-LE-Cy)-module, while there exists (C_1-LE-Cy)-modules, which are not extending, for example the $\mathbb{Z}$-module $M = \bigoplus_{i=1}^{\infty} M_i$, where $M_i = \mathbb{Z}$ for all $i \in \mathbb{N}$. It is clear, from Lemma 3.2., that M

is not extending, and from Lemma 3.3, that each finite subsum of $M = \bigoplus_{i=1}^{\infty} M$ is extending. Since every full LE-Cy-submodule of M is contained in a finite subsum of M, we have that M is a $(C_1$-LE-$Cy)$-module.

2) A direct sum of two $(C_1$-LE-$Cy)$-modules need not be a $(C_1$-LE-$Cy)$-module, for example the $\mathbb{Z}$-module $M = \mathbb{Z} \oplus (\mathbb{Z}/2\mathbb{Z})$ is not a $(C_1$-LE-$Cy)$-module. In fact the submodule $(2, \overline{1})\mathbb{Z}$ is full-LE-Cy-closed in M, while it is not a direct summand of M.

3) Let $\mathbb{F}$ be a field, then R_R is not a full cyclic large extending module where,

$$R = \begin{pmatrix} \mathbb{F} & \mathbb{F} & \mathbb{F} \\ 0 & \mathbb{F} & 0 \\ 0 & 0 & \mathbb{F} \end{pmatrix}$$

In fact, R_R contains a simple and closed submodule which is not a direct summand.

Lemma 3.4 ([8], 1.10 (4))**.** *If K is closed in L and L is closed in M then K is closed in M.*

Lemma 3.5. *Let M be a $(C_1$-LE-$Cy)$-module and N be a direct summand submodule of M. Then N is a $(C_1$-LE-$Cy)$-module.*

Proof. Let C be full LE-Cy-closed in N. Since $N \leq^{\oplus} M$, we have, by Lemma 3.4., that C is full LE-Cy-closed in M. As M is a $(C_1$-LE-$Cy)$-module, we have that $C \leq^{\oplus} M$; and hence $C \leq^{\oplus} N$. Therefore, N is $(C_1$-LE-$Cy)$-module. □

Proposition 3.2. *Let M_1 and M_2 be R-modules and let $M = M_1 \oplus M_2$. Then the following are equivalent:*

1) M is a $(C_1$-LE-$Cy)$-module.

2) Every full LE-Cy-closed submodule K of M, with $K \cap M_1 = 0$ or $K \cap M_2 = 0$ is a direct summand of M.

Proof. 1) ⇒ 2). It is clear by Lemma 3.5.,.

2) ⇒ 1). Let L be a full LE-Cy-closed in M. Let X be a maximal large extension of $L \cap M_2$ in L. Since, by Lemma 3.1., X is full LE-Cy-closed in M, with $X \cap M_1 = 0$. By 2), $M = X \oplus Y$ for some submodule Y of M. As $L = X \oplus (Y \cap L)$, by Lemma 3.4., we have that $(Y \cap L)$ is a full LE-Cy-closed submodule of M. $(L \cap M_2) \leq X$, then $(Y \cap L) \cap M_2 = 0$. Again by 2), $Y \cap L$ is a direct summand of M, and hence it is a direct summand of Y. Therefore, $M = X \oplus (Y \cap L) \oplus K = L \oplus K$ for some submodule K of M. □

Lemma 3.6. *Let M and N be isomorphic R-modules. If M is a $(C_1$-LE-$Cy)$-module, then N is a $(C_1$-LE-$Cy)$-module.*

Proof. Let $f : M \longrightarrow N$ be an R-isomorphism and let C be a full LE-Cy-closed submodule of N. It is clear, by Lemma 1, that $f^{-1}(C)$ is a full LE-Cy-closed submodule of M. By the condition $(C_1$-LE-$Cy)$ for M, $f^{-1}(C)$ is a direct summand of M. Therefore, C is a direct summand of N, i.e. N is a $(C_1$-LE-$Cy)$-module. □

Proposition 3.3. *Let $M = M_1 \oplus M_2$ be a module, where M_i is M_j-full-LE-Cy-injective $(i \neq j = 1, 2)$. Then the following are equivalent :*

1) M is a (C_1-LE-Cy)-module.

2) M_i is a (C_1-LE-Cy)-module, $(i = 1, 2)$.

Proof. 1) $\Rightarrow$ 2) It is clear from Lemma 3.5.,.

2) $\Rightarrow$ 1) Let K be a full LE-Cy-closed submodule of M, with $K \cap M_1 = 0$. By Theorem 2.1., there exists a submodule M_3 of M such that $M = M_1 \oplus M_3$ and $K \leq M_3$. As $M_3 \cong M_2$ and M_2 is a (C_1-LE-Cy)-module, we have, by Lemma 3.6., that M_3 is a (C_1-LE-Cy)-module. Therefore, K is a direct summand submodule of M_3, and hence K is a direct summand submodule of M. By proposition 3.2., we have that M is a (C_1-LE-Cy)-module. □

Corollary 3.3. *Let $M = M_1 \oplus ... \oplus M_n$, where M_i is M_j-full-LE-Cy-injective, for all $i \neq j, (i, j = 1, 2, ..., n)$ for some positive integer n. Then the following are equivalent :*

1) M is a (C_1-LE-Cy)-module.

2) M_i is a (C_1-LE-Cy)-module, $(i = 1, .., n)$.

Proof. 1) $\Rightarrow$ 2) It is clear from Lemma 3.5.,.

2) $\Rightarrow$ 1) By induction on n, it is enough to prove that M is a (C_1-LE-Cy)-module by consider in the case, when $n = 2$, which is shown in Proposition 3.3.,. □

Corollary 3.4. *Let $M = Z_2(M) \oplus N$ be a module, where $Z_2(M)$ is the second singular submodule of M. If $Z_2(M)$ and N are both (C_1-LE-Cy)-modules, and $Z_2(M)$ is N-full-LE-Cy-injective, then M is a (C_1-LE-Cy)-module.*

Proof. It is clear that $Hom(Z_2(M), N) = 0$, (due to N is non-singular), and hence N is $Z_2(M)$-injective. By Proposition 3.3., we have that M is a (C_1-LE-Cy)-module. □

Proposition 3.4. *Let M be a (C_1-LE-Cy)-module and $Z_2(M)$ be a full LE-Cy-module. Then we have the following :*

1) $M = Z_2(M) \oplus N$, for some submodule N of M, and both $Z_2(M)$, N are (C_1-LE-Cy)-modules.

2) $Z_2(M)$ is N-full-LE-Cy-injective.

Proof. 1) As $Z_2(M)$ is full LE-Cy-closed submodule of M, we have that $M = Z_2(M) \oplus N$. By Lemma 3.5., we have $Z_2(M)$ and N are (C_1-LE-Cy)-modules.
2) Let L be a full LE-Cy-submodule of M with $L \cap Z_2(M) = 0$. Let C be a maximal large extension of L in M. By Lemma 3.1., we have that C is full LE-Cy-closed submodule of M. By hypothesis, we have $M = C \oplus C'$ for some submodule C' of M. As $C \cap Z_2(M) = 0$, we have that $Z_2(M) \leq C'$. Thus $M = C' \oplus C = Z_2(M) \oplus (N \cap C') \oplus C$ and $L \leq (N \cap C') \oplus C$. Therefore, by Theorem 2.1., $Z_2(M)$ is N-full-LE-Cy-injective. □

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