Fuzzy zero suffix algorithm to solve fully fuzzy transportation problems by using element-wise operations

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Abstract. In this paper zero suffix method with element-wise operations of fuzzy numbers is proposed to solve fully fuzzy transportation problem. The proposed method assures the optimality, feasibility and positivity conditions of the fuzzy solution. The proposed method is easy to understand since it follows zero suffix algorithm and easy to compute since it considers the fuzzy numbers as ordered pairs as it uses the element-wise operations.

Keywords: fuzzy number, triangular fuzzy number, trapezoidal fuzzy number, fuzzy arithmetic operations, fuzzy transportation problems, fuzzy optimal solution.

1. Introduction

Transportation problem (TP) plays predominant role in supply chain management for reducing the transporting cost. The primary task of the algorithm is to optimize the transportation cost of commodity while transporting the commodity from sources to sinks. Hitchcock [20] constructed the transportation problem. Dantzig and Thapa [22] applied the simplex method to solve transportation problem. Charnes and Cooper [21] developed the stepping stone method as an alternative to the simplex method. The decision variables in the transporta-

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tion problem such as availability, requirement and the transportation cost per unit should be crisp to get a solution. Due to some unmanageable environment, these decision variables may not be precise. The unpredictability in determining the data may be designed using fuzzy variables which was introduced by Zadeh [1, 2] in the year 1965. If the variables are represented by fuzzy numbers in a TP, then the TP is called as a Fully Fuzzy TP or TP with fuzzy environment. Several authors proposed several approaches to solve a fuzzy TP. Chanas et al., [4, 5] applied parametric programming technique to solve fuzzy TP and also solved the given problem by converting the given problem into a bi-criterial TP with a crisp objective function. Liu Kao [3] used the extension principle for solving fuzzy TP. Verma et al., [10] solved the fuzzy TP with hyperbolic and exponential membership function by appling the fuzzy programming technique. Liang et al., [23], [11] proposed possibilistic linear programming technique for fuzzy TP and solved interactive multi objective transportation planning decision problems by using fuzzy linear programming. Nagoorgani et al., [6] approached a two stage cost minimizing fuzzy transportation problem by parametric technique. Pandian et al., [7] proposed fuzzy zero point method to solve fuzzy TP. Amit Kumar et al., [14] fuzzified least cost method, north west corner rule and VAM to solve fuzzy TP with generalized fuzzy numbers. Many authors [24], [25], [26], [27] used zero suffix method for solving transportation problem with crisp values and fuzzy values. All the existing methods transform the given problem in to crisp problem then implemented the zero suffix method. In this paper fuzzified version of zero suffix method and to order the fuzzy numbers Yager's ranking technique [8] is used. In the proposed method, the fuzzy zero suffix method is applied with element-wise addition, subtraction [9], [17], [18] and element-wise division to get the solution. In this paper, Section 2 deals with fuzzy preliminaries followed by Section 3 in which the proposed algorithm is given in detail. In Section 4, the implementation of the algorithm through example is explained. Finally,the conclusion is given in Section 5.

2. Preliminaries

Definition 1. *A fuzzy set can be obtained by mapping each possible individual in the universe of discourse to a value represented by its grade of membership.*

Definition 2. *A fuzzy number $\tilde{A}$ is a fuzzy set whose membership function is piecewise continuous, convex and normal.*

Definition 3. *A fuzzy number $\tilde{A} = (a, b, c)$ with membership function of the form*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \leq x \leq b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ 0, & otherwise \end{cases}$$

is called a triangular fuzzy number and a fuzzy number $\tilde{A} = (a, b, c, d)$ *with membership function of the form*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & otherwise \end{cases}$$

is called a trapezoidal fuzzy number.

Definition 4. *Fuzzy Addition:*

$$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

$$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

Fuzzy Subtraction:

$$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

$$(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$

Definition 5. *Element-wise Addition:*
$(a_1, b_1, c_1, d_1) + (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$
Element-wise Subtraction:
$(a_1, b_1, c_1, d_1) - (a_2, b_2, c_2, d_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2)$
$(a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2)$
Element-wise Multiplication:
$(a_1, b_1, c_1, d_1) * (a_2, b_2, c_2, d_2) = (a_1 * a_2, b_1 * b_2, c_1 * c_2, d_1 * d_2)$
$(a_1, b_1, c_1) * (a_2, b_2, c_2) = (a_1 * a_2, b_1 * b_2, c_1 * c_2)$
Element-wise Division:
$(a_1, b_1, c_1, d_1)/(a_2, b_2, c_2, d_2) = (a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2)$
$(a_1, b_1, c_1)/(a_2, b_2, c_2) = (a_1/a_2, b_1/b_2, c_1/c_2)$

Definition 6. *The Yager's ranking of a fuzzy number* $\tilde{a}$ *is given by*

$$Y(\tilde{a}) = \int_0^1 (0.5)(a_U^\alpha + a_L^\alpha) d\alpha,$$

where a_L^α = *Lower* α*- level cut and* a_U^α = *Upper* α*- level cut.If* $Y(\tilde{s}) \le Y(\tilde{i})$ *then* $\tilde{s} \le \tilde{i}$.

Definition 7. *A Fully Fuzzy transportation problem is defined by*

$$\min \tilde{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij}$$

subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{s}_i \text{ for } i = 1,2,3\ldots m,$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{d}_j \text{ for } j = 1,2,3\ldots n.$$

for all $\tilde{x}_{ij} \succ \tilde{0}$, *where* $i = 1,2,3\ldots m$ *and* $j = 1,2,3\ldots n$.

Here $\tilde{x}_{ij}$ *is the number of units to be transported from* i^{th} *source to* j^{th} *destination,* $\tilde{c}_{ij}$ *is the cost of transporting one unit from* i^{th} *source to* j^{th} *destination,* $\tilde{s}_i$ *is the number of units available in the* i^{th} *source and* $\tilde{d}_j$ *is the number of units required in the* j^{th} *destination.*

The matrix form of fuzzy transportation problem is given as follows

	A	B	C	...	E	Supply
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	$\tilde{c}_{13}$	...	$\tilde{c}_{1n}$	$\tilde{s}_1$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	$\tilde{c}_{23}$	...	$\tilde{c}_{2n}$	$\tilde{s}_2$
3	$\tilde{c}_{31}$	$\tilde{c}_{32}$	$\tilde{c}_{33}$	...	$\tilde{c}_{3n}$	$\tilde{s}_3$
...	...	...	...	...	...	...
m	$\tilde{c}_{m1}$	$\tilde{c}_{m2}$	$\tilde{c}_{m3}$	...	$\tilde{c}_{mn}$	$\tilde{s}_m$
Demand	$\tilde{d}_1$	$\tilde{d}_2$	$\tilde{d}_3$	...	$\tilde{d}_n$	

3. Fuzzy zero suffix method

Step 1. Construct the fuzzy transportation table for the given fuzzy transportation problem and then,convert it into a balanced one, if it is not.Subtract each row entries of the fuzzy transportation table from the row minimum. Do the same for columns also.

Step 2. In the reduced cost matrix there will be at least one fuzzy zero in each row and column. Find fuzzy suffix value $\tilde{S}$ of all the fuzzy zeros in the reduced cost matrix by the ratio of addition fuzzy costs of nearest adjacent sides of fuzzy zeros which are greater than fuzzy zero to the number of fuzzy values added. Here we should take the denominator as fuzzy values. i.e., if the number of values is 3, we should take that as fuzzy number $(3, 3, 3)$. $\tilde{S}$ = Addition of the fuzzy costs of adjacent sides of fuzzy zero which are greater than fuzzy zero/number of fuzzy values added.

Step 3. Select the maximum of $\tilde{S}$, and supply to that fuzzy demand corresponding to that cell. If it has more equal fuzzy values then select any one and supply to that fuzzy demand maximum possible.

Step 4. After the above step, the exhausted fuzzy demands or fuzzy supplies to be trimmed. The resultant fuzzy matrix posses at least one fuzzy zero in each row and column else repeat Step 1.

Step 5. Repeat Step 3 to Step 4 until the optimal solution is obtained.

4. Numerical examples

4.1 Example

Consider the fully fuzzy transportation problem with triangular numbers given as follows

Solution:
Since the total supply $(4, 15, 27)$ is equal to the total demand $(4, 15, 27)$, this is a balanced fuzzy TP.

	A	B	C	D	Supply
1	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	(0,3,6)
2	(4 9 16)	(4 8 12)	(2,5,8)	(1,4,7)	(2,7,13)
3	(2,7,13)	(0,5,10)	(0,5,10)	(4,8,12)	(2,5,8)
Demand	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

The given transportation problem rewritten as assignment problem.

	A	B	C	D
1	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)
2	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)
3	(2,7,13)	(0,5,10)	(0,5,10)	(4,8,12)

The fuzzy costs and fuzzy units of fuzzy transportation table are given with their crisp values in the following table.

	A	B	C	D
1	(-2,3,8)(3)	(-2,3,8)(3)	(-2,3,8)(3)	(-1,1,4)(1.33)
2	(4,9,16)(9.667)	(4,8,12)(8)	(2,5,8)(5)	(1,4,7)(4)
3	(2,7,13)(7.33)	(0,5,10)(5)	(0,5,10)(5)	(4,8,12)(8)

Choose the smallest fuzzy number in each and every row and subtract it with the other elements in the corresponding row. Repeat the same for the columns also. It is noted that the reduced matrix has at least one fuzzy zero in each row and column. The reduced matrix is given in

	A	B	C	D
1	(0,0,0)	(-1,2,4)	(-1,2,4)	(0,0,0)
2	(4,3,5)	(3,4,5)	(1,1,1)	(0,0,0)
3	(3,0,-1)	(0,0,0)	(0,0,0	(4,3,2)

The fuzzy zeros are in the position $(1,1),(1,4),(2,4),(3,2),(3,3)$ of the reduced matrix. If we take the fuzzy zero in the $(1,1)$, the adjacent values $(-1,2,4)$ and $(4,3,5)$ which are greater than fuzzy zero. So the fuzzy suffix value for that position $(1,1)$ is given by $\frac{((-1,2,4)+(4,3,5))}{(2,2,2)} = (1.5,2.5,4.5)$, where the fuzzy number $(2,2,2)$ is the fuzzy value of the number of adjacent values which are greater than fuzzy zero added. Similarly find the fuzzy suffix value for all other fuzzy zeros. The values are given below : for the position $(1,4)$ is $(-1,2,4)$, for the position $(2,4)$ is $(2.5,2,1.5)$, for the position $(3,2)$ is $(3,2,2)$ and for the position $(3,3)$ is $(2.5,2,1.5)$. Out of all these fuzzy suffix value, the fuzzy suffix value of fuzzy zero in the position $(1,1)$ is maximum. Therefore allocate the corresponding fuzzy supply or fuzzy demand whichever is less to that $(1,1)$ position. From the problem it is noted that in that position the corresponding fuzzy supply $(0,3,6)$ is minimum. So allocate the corresponding fuzzy supply $(0,3,6)$ to that position and delete the corresponding row. This is given as follows.

	A	B	C	D	Supply
1	(0,3,6) (-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	
2	(4 9 16)	(4 8 12)	(2,5,8)	(1,4,7)	(2,7,13)
3	(2,7,13)	(0,5,10)	(0,5,10)	(4,8,12)	(2,5,8)
Demand	(1,1,1)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

After deleting the first row the reduced matrix is given as follows.

	A	B	C	D
2	(4,3,5)	(3,4,5)	(1,1,1)	(0,0,0)
3	(3,0,-1)	(0,0,0)	(0,0,0)	(4,3,2)

Again apply the first step the resultant reduced matrix is given as follows.

	A	B	C	D
2	(1,3,6)	(3,4,5)	(1,1,1)	(0,0,0)
3	(0,0,0)	(0,0,0)	(0,0,0)	(4,3,2)

The fuzzy zeros are in the position $(2,4),(3,1),(3,2),(3,3)$ of the reduced matrix. If we take the fuzzy zero in the $(2,4)$, the adjacent values $(1,1,1)$ and $(4,3,2)$ which are greater than fuzzy zero. So the fuzzy suffix value for that position $(2,4)$ is given by $\frac{((-1,1,1)+(4,3,2))}{(2,2,2)} = (2.5,2,1.5)$, where the fuzzy number $(2,2,2)$ is the fuzzy value of the number of adjacent values which are greater than fuzzy zero added. Similarly find the fuzzy suffix value for all other fuzzy zeros. The values are given below : for the position $(3,1)$ is $(1,3,6)$, for the position $(3,2)$ is $(3,4,5)$ and for the position $(3,3)$ is $(2.5,2,1.5)$. Out of all these fuzzy suffix value, the fuzzy suffix value of fuzzy zero in the position $(3,2)$ is maximum. Therefore allocate the corresponding fuzzy supply or demand whichever is less to that $(3,2)$ position. From the problem it is noted that in that position the corresponding fuzzy demand $(0,3,5)$ is minimum. So allocate the corresponding fuzzy demand $(0,3,5)$ to that position and delete the corresponding column. The matrix is given as follows.

	A	B	C	D	Supply
1	(0,3,6) (-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	
2	(4 9 16)	(4 8 12)	(2,5,8)	(1,4,7)	(2,7,13)
3	(2,7,13)	(0,3,5) (0,5,10)	(0,5,10)	(4,8,12)	(2,2,3)
Demand	(1,1,1)		(1,4,7)	(2,4,8)	(4,15,27)

After deleting the second column, the reduced matrix is given as follows

	A	C	D
2	(1,3,6)	(1,1,1)	(0,0,0)
3	(0,0,0)	(0,0,0)	(4,3,2)

The fuzzy zeros are in the position $(2,4),(3,1),(3,3)$ of the reduced matrix. If we take the fuzzy zero in the $(2,4)$, the adjacent values $(1,1,1)$ and $(4,3,2)$ which are greater than fuzzy zero. So the fuzzy suffix value for that position $(2,4)$ is given by $\frac{((-1,1,1)+(4,3,2))}{(2,2,2)} = (2.5,2,1.5)$, where the fuzzy number $(2,2,2)$ is the fuzzy value of the number of adjacent values which are greater than fuzzy zero added. Similarly find the suffix value for all other fuzzy zeros. The values are given below : for the position $(3,1)$ is $(1,3,6)$, and for the position $(3,3)$ is $(2.5,2,1.5)$. Out of all these fuzzy suffix value, the fuzzy suffix value of fuzzy zero in the position $(3,1)$ is maximum. Therefore allocate the corresponding fuzzy supply or demand whichever is less to that $(3,1)$ position. From the problem it is noted that in that position the corresponding fuzzy demand $(1,1,1)$ is minimum. So allocate the corresponding fuzzy demand $(1,1,1)$ to that position and delete the corresponding column. This is given in the following table

	A	B	C	D	Supply
1	(0,3,6) (-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	
2	(4,9,16)	(4 8 12)	(2,5,8)	(1,4,7)	(2,7,13)
3	(1,1,1) (2,7,13)	(0,3,5) (0,5,10)	(0,5,10)	(4,8,12)	(1,1,2)
Demand			(1,4,7)	(2,4,8)	(4,15,27)

After deleting the first column , the reduced matrix is given in the following table.

	C	D
2	(1,1,1)	(0,0,0)
3	(0,0,0)	(4,3,2)

The fuzzy zeros are in the position $(2,4),(3,3)$ of the reduced matrix. If we take the fuzzy zero in the $(2,4)$, the adjacent values $(1,1,1)$ and $(4,3,2)$ which are greater than fuzzy zero. So the fuzzy suffix value for that position $(2,4)$ is given by $\frac{((-1,1,1)+(4,3,2))}{(2,2,2)} = (2.5,2,1.5)$, where the fuzzy number $(2,2,2)$ is the fuzzy value of the number of adjacent values which are greater than fuzzy zero added. Similarly find the fuzzy suffix value for all other fuzzy zeros. The values are given below : for the position $(3,3)$ is $(2.5,2,1.5)$. Both the fuzzy suffix values are same, So we can take any position for allocation. Here we choose $(2,4)$ for allocation. Therefore allocate the corresponding fuzzy supply or demand whichever is less to that $(2,4)$ position. From the problem it is noted that in that position the corresponding fuzzy demand $(2,4,8)$ is minimum. So allocate the corresponding fuzzy demand $(2,4,8)$ to that position and delete the corresponding column. This is given as follows

	A	B	C	D	Supply
1	(0,3,6) (-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	
2	(4,9,16)	(4 8 12)	(2,5,8)	(2,4,8) (1,4,7)	(0,3,5)
3	(1,1,1) (2,7,13)	(0,3,5) (0,5,10)	(0,5,10)	(4,8,12)	(1,1,2)
Demand			(1,4,7)		(4,15,27)

After deleting the fourth column , the reduced matrix is given as follows

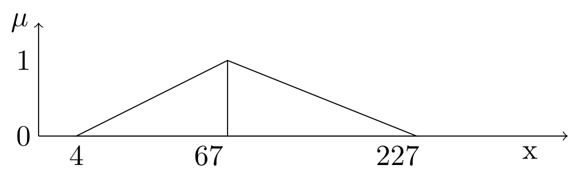
	C	
2	(1,1,1)	
3	(0,0,0)	

Applying the above mentioned procedure we get the optimal table which is given as follows.

	A	B	C	D	Supply
1	(-2,3,8)	(-2,3,8)	(-2,3,8)	[(0,3,6)] (-1,1,4)	(0,3,6)
2	[(-1,2,4)] (4,9,16)	(4,8,12)	[(1,4,7)] (2,5,8)	[(2,1,2)] (1,4,7)	(2,7,13)
3	[(2,2,3)] (2,7,13)	[(0,3,5)] (0,5,10)	(0,5,10)	(4,8,12)	(2,5 ,8)
Demand	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

Fuzzy T.C ≈ (-2,3,8) * (0,3,6) + (2,5,8) * (0,3,5) + (0,5,10) * (0,3,5) + (0,5,10) * (1,1,2) + (1,4,7) * (2,4,8) + (2,7,13) * (1,1,1)≈ (4,67,227)

The membership function for the obtained result is



- According to the decision maker the minimum transportation cost will lie between 4 and 227

- The overall level of satisfaction of the decision maker about the statement that the minimum transportation cost will be 67 dollars is 100 percent.

- The overall level of satisfaction of the decision maker for the remaining values of minimum transportation cost can be obtained as follows:Let x_0 represents the minimum transportation cost then the overall level of satisfaction of the decision maker for x_0 is $\mu_{\tilde{A}}(x_0) \times 100$
  where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-4}{63}, & 4 \leq x \leq 67 \\ 1, & x = 67 \\ \dfrac{227-x}{160}, & 67 \leq x \leq 227 \\ 0, & otherwise. \end{cases}$$

5. Conclusions

In this paper, an efficient method called Fuzzy zero suffix algorithm is proposed to solve fully fuzzy transportation problem with element-wise subtraction and element-wise division. The solution obtained by using the fuzzy zero suffix method satisfies the feasibility, optimality conditions and the positive values in all the allocated cells. An advantage of the proposed method is that it follows the zero suffix method which is easy to understand and to apply. It can also be used to solve special type of fuzzy transportation problem like unbalanced transportation problems and transportation problem with Degeneracy.

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