Redefined neutrosophic filters in BE-algebras

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Abstract. Neutrosophic set theory is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 2015, neutrosophic set theory is applied to BE-algebra, and the notion of neutrosophic filter is introduced. In this paper, some mistakes and deficiencies of original definition of neutrosophic filter are pointed out by some examples. Moreover, a new definition of neutrosophic filter is established, some basic properties are presented, and the relationships between fuzzy filters and neutrosophic filters are discussed. Finally, the concept of implicative neutrosophic filter in BE-algebra is introduced, and some necessary and sufficient conditions for a neutrosophic filter to be implicative are presented.

Keywords: neutrosophic set, neutrosophic filter, BE-algebra, implicative neutrosophic filter, fuzzy filter.

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1. Introduction

The neutrosophic set is a more general form of mathema- tical concepts that express uncertainty, such as fuzzy sets and intuitionistic fuzzy sets (see [17, and 18]). In the neutrosophic set, truth-membership, indeterminacy-member ship, and falsity-membership are represented independently. In this paper we work with special neutrosophic set (it is called single valued neutrosophic set, see [21]). The neutrosophic set theory is applied to many scientific fields (see [3033]), including algebraic systems (see [4, 10, 16, 28, and 29]), it is similar to the applications of fuzzy set and soft set theory in algebraic structures ([2, 14, 25, 27 and 34-37]).

As a generalization of dual BCK-algebra and related non-classical logic algebras ([23, 24, and 26]), Hee Sik Kim and Young Hee Kim introduced the notion of BE-algebra (see [7]). Since then, many scholars have conducted in-depth research on BE-algebras. For examples, the concept of ideal of BE-algebra is proposed and some characterizations are presented by the notion of upper set in [1]; a procedure which generated a filter by a subset in a transitive BE-algebra is established in [8]; the fuzzy filter (ideal) theory in BE-algebra is investigated in [5, 9, 19]; the theory of pseudo BE-algebra is constructed in [3], and so on (see [6, 11, 12, 13, 15, and 22]).

In this paper, we further study on the applications of neutrosophic sets to BE-algebras. We introduce the new definition of neutrosophic filters in BE-algebras, and investigate some basic properties and present relationships between neutrosophic filters and fuzzy filters. Moreover, we introduce the notion of implicative neutrosophic filters in BE-algebras. The relation between implicative neutrosophic filter and neutrosophic filter is investigated.

2. Basic concepts and properties

Definition 2.1 ([16, 17, 18]). Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is, $T_A(x) : X \to]^-0, 1^+[, I_A(x) : X \to]^-0, 1^+[$, and $F_A(x) : X]^-0, 1^+[$. Thus, there is no restrictionon the sum of $T_A(x), I_A(x)$, and $F_A(x)$, so $^-0 \leq supT_A(x)+supI_A(x)+ supF_A(x) \leq 3+$.

Definition 2.2 ([21]). Let X be a space of points (objects) with generic elements in X denoted by x. A simple valued neutrosophic set A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. Then, a simple valued neutrosophic set A can be denoted by

$$A = \{x, TA(x), IA(x), FA(x)|x \in X\},$$

where $T_A(x), I_A(x), F_A(x) \in [0,1]$ for each point x in X. Therefore, the sum of $T_A(x), I_A(x)$, and $F_A(x)$ satisfies the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3 ([16])**.** A simple valued neutrosophic set A is contained in the other simple valued neutrosophic set B, denote $A \subseteq B$, if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)$ for any x in X.

Remark 2.1. About the inclusion relation of neutrosophic sets, there are two different definitions in the literature. This article adopts the method in [16] (original definition byFlorentin Smarandache). Another way is given in [21], that is, $A \subseteq B$ if and only if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x)$ for any x in X.

Definition 2.4 ([16])**.** Two simple valued neutrosophic sets A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

For convenience, "simple valued neutrosophic set" is abbreviated to "neutrosophic set" later.

Definition 2.5 ([16])**.** The union of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity- membership functions are related to those of A and B by

$$T_C(x) = \max(T_A(x), T_B(x)), I_C(x) = \min(I_A(x), I_B(x)),$$
$$F_C(x) = \min(F_A(x), F_B(x)), \forall x \in X.$$

Definition 2.6 ([16])**.** The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity- membership functions are related to those of A and B by

$$T_C(x) = \min(T_A(x), T_B(x)), I_C(x) = \max(I_A(x), I_B(x)),$$
$$F_C(x) = \max(F_A(x), F_B(x)), \forall x \in X.$$

Definition 2.7 ([7])**.** By a BE-algebra we shall mean an algebraic structure $(X; \to, 1)$ of type (2, 0) satisfying the following axioms:

(BE1) $x \to x = 1$;
(BE2) $x \to 1 = 1$;
(BE3) $1 \to x = x$;
(BE4) $x \to (y \to z) = y \to (x \to z)$, for all$x, y, z \in X$.

For a BE-algebra $(X; \to, 1)$, we can define a relation $\leq$ on X by $x \leq y$ if and only if $x \to y = 1$.

Proposition 2.1[5,8] If $(X; \to, 1)$ is a BE-algebra, then for all $x, y \in X$,

(1) $1 \leq x \Rightarrow x = 1$.
(2) $x \to (y \to x) = 1$,or equivalently, $x \leq y \to x$.
(3)$x \to ((x \to y) \to y) = 1$,or equivalently, $x \leq (x \to y) \to y$.

A BE-algebra $(X; \to, 1)$ is said to be self distributive if $x \to (y \to z) = (x \to y) \to (x \to z)$, for all $x, y, z \in X$.

A BE-algebra $(X; \to, 1)$ is said to be commutative if $(x \to y) \to y = (y \to x) \to x$, for all $x, y \in X$.

A BE-algebra $(X; \to, 1)$ is said to be transitive if for all $x, y, z \in X, y \to z \leq (x \to y) \to (x \to z)$.

Proposition 2.2 ([8])**.** If a BE-algebra $(X; \to, 1)$ is transitive, then for all $x, y, z \in X$,
(1) $y \leq z$ implies $x \to y \leq x \to z$.
(2) $y \leq z$ implies $z \to x \leq y \to x$.
(3) $x \leq y$ and $y \leq z$ imply $x \leq z$.

Definition 2.8 ([7,8])**.** A subset F of BE-algebra $(X; \to, 1)$ is called a filter of X if it satisfies:
(F1) $1 \in F$;
(F2) $x \in F$ and $x \to y \in F \Rightarrow y \in F$.

Definition 2.9 ([7])**.** Let $(X; \to, 1)$ be a BE-algebra and let $x, y, z \in X$. Define $A(x, y) = \{z \in X | x \to (y \to z) = 1\}$. We call $A(x, y)$ an upper set of x and y. It is easy to see that $1, x, y \in A(x, y)$, for any $x, y \in X$.

Proposition 2.3 ([7])**.** *Let F be a non-empty subset of a BE- algebra* $(X; \to, 1)$*. Then F is a filter of X if and only if* $A(x, y) \subseteq F$*, for all* $x, y \in F$*.*

Definition 2.10 ([15])**.** A non-empty subset F of BE-algebra $(X; \to, 1)$ is called an implicative filter if satisfies the following conditions:
(IF1) $1 \in F$;
(IF2) $x \to (y \to z) \in F$ and$x \to y \in F$ imply that $x \to z \in F$, for all $x, y, z \in X$.

Definition 2.11 ([5,6,9])**.** A fuzzy set μ in BE-algebra $(X; \to, 1)$ is called a fuzzy filter of X if it satisfies:
(FF1) $\mu(1) \geq \mu(x)$;
(FF2)$\mu(y) \geq \min\{\mu(x), \mu(x \to y)\}$ for all $x, y \in X$.

Proposition 2.4 ([5,9])**.** *Let* μ *be a fuzzy filter of a BE-algebra* $(X; \to, 1)$*. Then, for any* $x, y \in X$*, if* $x \leq y$*, then* $\mu(x) \leq \mu(y)$*.*

Proposition 2.5 ([5,9])**.** *Let* μ *be a fuzzy set of a BE-algebra* $(X; \to, 1)$*.. Then the following conditions are equivalent.*
(1) μ *is a fuzzy filter in* X*;*

(2) *for all* $x \in X, \mu(1) \geq \mu(x)$; *and, for all* $x, y, z \in X, x \to (y \to z) = 1$ *implies* $\mu(z) \geq \min\{\mu(x), \mu(y)\}$;

(3) *for each* $\alpha \in [0, 1]$, *the level subset* $U(\mu; \alpha) = \{x \in X : \mu(x) \geq \alpha\}$ *is a filter of* X, *when* $U(\mu; \alpha) \neq \emptyset$.

3. Deficiencies of original definition of neutrosophic filter

In 2015, A. Rezaei, A. B. Saeid, and F. Smarandache [10] introduced the notion of neutrosophic filter in BE-algebras, and discussed some properties of neutrosophic filters.

Definition 3.1 (Definition 3.1 in [10])**.** A neutrosophic set A in a BE-algebra X is called a neutrosophic filter in X if satisfies the following conditions:

(NF1) $T_A(x) \leq T_A(1), I_A(x) \geq I_A(1)$ and $F_A(x) \geq F_A(1)$;

(NF2) $\min\{T_A(x), T_A(x \to y)\} \leq T_A(y), \min\{I_A(x), I_A(x \to y)\} \geq I_A(y)$ and $\min\{F_A(x), F_A(x \to y)\} \geq F_A(y)$,for all $x, y \in X$.

Proposition 3.1 (Theorem 3.4 in [10])**.** *Let* A *be a neutrosophic set of* X. *Then the following are equivalent:*

(i) A *is a neutrosophic filter in* X;

(ii) $(\forall t \in [0, 1])U(A; t) = \{x \in X : t \leq T_A(x), I_A(x) \leq t, F_A(x) \leq t\} \neq \emptyset$ *imply* $U(A; t)$ *is a filter of* X.

Now, we give some counterexamples to show that Theorem 3.4 in [10] is not true and the original definition of neutrosophic filter in BE-algebras (Definition 3.1 in [10]) is not well-defined.

Example 3.1. Let $X = \{1, a, b, c, d\}$ be a set with the following operation table:

$\to$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	b
b	1	a	1	b	a
c	1	a	1	1	a
d	1	1	1	b	1

Then $(X; , 1)$ is a BE-algebra. Define a neutrosophic set A in X as follows:

$$T_A(x) = 0.79, for\ all\ x \in X;$$

$$I_A(x) = \begin{cases} 0.17, & if x = 1, a \\ 0.79, & \text{otherwise.} \end{cases}, F_A(x) = \begin{cases} 0.17, & if x = 1, a \\ 0.79, & \text{otherwise.} \end{cases}$$

Then we can verify that $(\forall t \in [0, 1])U(A; t) = \{x \in X : t \leq T_A(x), I_A(x) \leq t, F_A(x) \leq t\} \neq \emptyset$ imply $U(A; t)$ is a filter of X. But A is not a neutrosophic filter in X, since

$$\min\{I_A(a), I_A(a \to d)\} = \min\{0.17, 0.79\} = 0.17 \ngeq I_A(d) = 0.79,$$
$$\min\{F_A(a), F_A(a \to d)\} = \min\{0.17, 0.79\} = 0.17 \ngeq F_A(d) = 0.79.$$

Example 3.2. Let $X = \{1, a, b\}$ be a set with the following operation table:

$\to$	1	a	b
1	1	a	b
a	1	1	a
b	1	a	1

Then $(X; , 1)$ is a BE-algebra. Define a neutrosophic set A in X as follows: $T_A(1) = 0.9, T_A(a) = T_A(b) = 0.5; I_A(1) = 0.2, I_A(a) = I_A(b) = 0.35; F_A(1) = 0.1, F_A(a) = F_A(b) = 0.$

In [10], the authors said that A is a neutrosophic filter in X (see Example 3.1 in [10]). This is a mistake, since $\min\{I_A(a), I_A(a \to a)\} = \min\{0.35, 0.2\} = 0.2 \ngeq I_A(a) = 0.35, \min\{F_A(b), F_A(b \to b)\} = \min\{0, 0.1\} = 0 \ngeq F_A(1) = 0.1.$

Proposition 3.2. *Assume that the concept of neutrosophic filters in a BE-algebra X is defined by Definition 3.1. Let A be a neutrosophic filter in a BE-algebra X. Then:*
(i) $I_A(x) = I_A(1)$ *for all* $x \in X$*;*
(ii) $F_A(x) = F_A(1)$ *for all* $x \in X$*.*

Proof. For all $x \in X$, by Definition 3.1 (NF2) we have

$$I_A(1) \geq \min\{I_A(1), I_A(x)\} = \min\{I_A(1), I_A(1 \to x)\} \geq I_A(x).$$

That is, $I_A(x) \leq I_A(1)$. On the other hand, using (NF1) we have $I_A(x) \geq I_A(1)$. Hence $I_A(x) = I_A(1)$.

Similarly, we can get that $F_A(x) = F_A(1)$ for all $x \in X$.

By Proposition 3.2 we know that $I_A(x)$ and $F_A(x)$ are two constant valued functions for neutrosophic filter (it is defined by Definition 3.1 in [10]) in BE-algebras. Moreover, by Example 3.1 we know that a neutrosophic filter cannot be completely determined by its level subsets. All of these are inconsistent with the properties of the traditional fuzzy filter, so it is necessary to redefine the concept of neutrosophic filter in BE-algebras.

4. New definition of neutrosophic filters in BE-algebras

Definition 4.1. A neutrosophic set A in a BE-algebra X is called a neutrosophic filter in X if it satisfies:

(NSF1)$\forall x \in X, T_A(x) \leq T_A(1), I_A(x) \geq I_A(1)$ and $F_A(x) \geq F_A(1)$;

(NSF2) $\forall x, y \in X, \min\{T_A(x), T_A(x \to y)\} \le T_A(y), \max\{I_A(x), I_A(x \to y)\} \ge I_A(y)$ and $\max\{F_A(x), F_A(x \to y)\} \ge F_A(y)$;

Proposition 4.1 *Let A be a neutrosophic filter in BE- algebra X. Then:*

$(NSF3)\ \forall x, y \in X, x \le y \Rightarrow T_A(x) \le T_A(y), I_A(x) \ge I_A(y)$ *and* $\max F_A(x) \ge F_A(y)$.

Proof. If $x \le y$, then $x \to y = 1$. It follows that $T_A(x \to y) = T_A(1)$. From this, using Definition 4.1 (NSF1) and (NSF2) we get

$$T_A(x) = \min\{T_A(x), T_A(1)\} = \min\{T_A(x), T_A(x \to y)\} \le T_A(y).$$

That is,$x \le y \Rightarrow T_A(x) \le T_A(y)$.

Similarly, we can get that $x \le y \Rightarrow I_A(x) \ge I_A(y)$ and $x \le y \Rightarrow F_A(x) \ge F_A(y)$.

It is easy to verify that the following proposition is true.

Proposition 4.2. *If A and B are two neutrosophic filters in a BE-algebra X, then $A \cap B$ is also a neutrosophic filter in X.*

Example 4.1. Let $(X; \to, 1)$ be the BE-algebra in Example 3.1. Define a neutrosophic set A in X as follows:

$$T_A(x) = \begin{cases} 0.86, & if x = 1, a \\ 0.13, & \text{otherwise} \end{cases}, I_A(x) = \begin{cases} 0.15, & if x = 1, a \\ 0.82, & \text{otherwise} \end{cases},$$

$$F_A(x) = \begin{cases} 0.15, & if x = 1, a \\ 0.82, & \text{otherwise.} \end{cases}$$

Then A is a neutrosophic filter in X.

Moreover, we can verify that $(\forall t \in [0,1])U(A;t) = \{x \in X : t \le T_A(x), I_A(x) \le t, F_A(x) \le t\} \ne \emptyset$ imply $U(A;t)$ is a filter of X. This is a general result for every neutrosophic filter, it is proved as follows.

Proposition 4.3. *Let X be a BE-algebra, A be a neutrosophic filter in X. Then $(\forall t \in [0,1])U(A;t) = \{x \in X : t \le T_A(x), I_A(x) \le t, F_A(x) \le t\} \ne \emptyset$ imply $U(A;t)$ is a filter of X.*

Proof. Assume that A is neutrosophic filter in X and let $t \in [0,1]$ such that $U(A;t) \ne \emptyset$. Then there exists $x_0 \in X$ such that $t \le T_A(x_0), I_A(x_0) \le t$, and $F_A(x_0) \le t$. By applying Definition 4.1 (NSF1) we have

$$t \le T_A(x_0) \le T_A(1), I_A(1) \le I_A(x_0) \le t \text{ and } F_A(1) \le F_A(x_0) \le t.$$

This means that $1 \in U(A;t)$. Let $x, y \in X, x \to y \in U(A;t)$ and $x \in U(A;t)$. Then $t \le T_A(x \to y), I_A(x \to y) \le t$ and $F_A(x \to y) \le t$, $t \le T_A(x), I_A(x) \le t, and\ F_A(x) \le t$. From these, using Definition 4.1 (NSF2) we have

$$t \le \min\{T_A(x), T_A(x \to y)\} \le T_A(y), I_A(y) \le \max\{I_A(x), I_A(x \to y)\} \le t,$$

$$F_A(y) \leq \max\{F_A(x), F_A(x \to y)\} \leq t.$$

This means that $y \in U(A;t)$. By Definition 2.8 we know that $U(A;t)$ is a filter of X.

The following example shows that the inverse of Proposition 4.3 is not true in general.

Example 4.2. Let $(X; \to, 1)$ be the BE-algebra in Example 3.1. Define a neutrosophic set B in X as follows:

$$T_B(x) = \begin{cases} 0.82, & if x = 1, a \\ 0.16, & \text{otherwise} \end{cases}, I_B(x) = \begin{cases} 0.16, & if x = 1, a \\ 0.82, & \text{otherwise} \end{cases},$$

$$F_B(x) = \begin{cases} 0.11, & if x = 1, a, b \\ 0.82, & \text{otherwise.} \end{cases}$$

Then we can get that:

If $t > 0.82, U(B;t) = \emptyset$;

If $0.82 \geq t > 0.16, U(B;t) = \{1, a\}$;

If $0.16 \geq t > 0.11, U(B;t) = \emptyset$;

If $0.11 \geq t, U(B;t) = \emptyset$.

This means that $(\forall t \in [0,1])U(B;t) = \{x \in X : t \leq T_B(x), I_B(x) \leq t, F_B(x) \leq t\} \neq \emptyset$ imply $U(B;t)$ is a filter of X. But B is not a neutrosophic filter in X, since $\max\{F_B(b), F_B(b \to d)\} = 0.11 \ngeq 0.82 = F_B(d)$.

Theorem 4.1. *Let A be a neutrosophic set in a BE-algebra X. Then A is a neutrosophic filter in X if and only if A satisfies:*

(i) T_A is a fuzzy filter of X;

(ii) $1 - I_A$ is a fuzzy filter of X, where $(1 - I_A)(x) = 1 - I_A(x), x \in X$;

(iii) $1 - F_A$ is a fuzzy filter of$X$, where $(1 - F_A)(x) = 1 - F_A(x), x \in X$.

Proof. Suppose that A is a neutrosophic filter in X. Then T_A is a fuzzy set on X; and using Definition 4.1 we have $\forall x, y \in X, T_A(x) \leq T_A(1), \min\{T_A(x), T_A(x \to y)\} \leq T_A(y)$. By Definition 2.11 we know that T_A is a fuzzy filter of X.

Moreover, it is easy to verify that $1 - I_A$ is a fuzzy set on X; and using Definition 4.1 we have: $\forall x, y \in X$,

$$(1 - I_A)(x) = 1 - I_A(x) \leq 1 - I_A(1) = (1 - I_A)(1);$$

$\min\{(1 - I_A)(x), (1 - I_A)(x \to y)\} = \min\{1 - I_A(x), 1 - I_A(x \to y)\} = 1 - \max\{I_A(x), I_A(x \to y)\} \leq 1 - I_A(y) = (1 - I_A)(y)$.

By Definition 2.11 we know that $1 - I_A$ is a fuzzy filter of X. Similarly, we can get that $1 - F_A$ is a fuzzy filter of X.

Conversely, suppose that neutrosophic set A satisfies the conditions (i), (ii) and (iii).Then by Definition 2.11 we have $(\forall x, y \in X)$,

$$T_A(x) \leq T_A(1), \min\{T_A(x), T_A(x \to y)\} \leq T_A(y);$$

$$(1-I_A)(x) \leq (1-I_A)(1), \min\{(1-I_A)(x), (1-I_A)(x \to y)\} \leq (1-I_A)(y);$$
$$(1-F_A)(x) \leq (1-F_A)(1), \min\{(1-F_A)(x), (1-F_A)(x \to y)\} \leq (1-F_A)(y).$$

It follows that, $\forall x, y \in X$,

$$\begin{aligned}
&T_A(x) \leq T_A(1),\\
&I_A(x) = 1-(1-I_A)(x) \geq 1-(1-I_A)(1) = I_A(1),\\
&F_A(x) = 1-(1-F_A)(x) \geq 1-(1-F_A)(1) = F_A(1),\\
&\min\{T_A(x), T_A(x \to y)\} \leq T_A(y),\\
&\max\{I_A(x), I_A(x \to y)\} = 1-\min\{(1-I_A)(x), (1-I_A)(x \to y)\}\\
&\geq 1-(1-I_A)(y) = I_A(y),\\
&\max\{F_A(x), F_A(x \to y)\} = 1-\min\{(1-F_A)(x), (1-F_A)(x \to y)\}\\
&\geq 1-(1-F_A)(y) = F_A(y).
\end{aligned}$$

From this, by Definition 4.1 we get that A is a neutrosophic filter in X.

Applying Theorem 4.1 and Proposition 2.5 (3) we can get:

Corollary 4.1. *Let A be a neutrosophic filter in a BE-algebra X. Then:*

(1) *for any $t \in [0,1], U(T_A; t) = \{x \in X | T_A(x) \geq t\}$ is a filter of X when $U(T_A; t) \neq \emptyset$;*

(2) *for any $t \in [0,1], U(1-I_A; t) = \{x \in X | 1-I_A(x) \geq t\}$ is a filter of X when $U(1-I_A; t) \neq \emptyset$;*

(3) *for any $t \in [0,1], U(1-F_A; t) = \{x \in X | 1-F_A(x) \geq t\}$ is a filter of X when $U(1-F_A; t) \neq \emptyset$.*

Definition 4.2 ([20])**.** Let A be a neutrosophic set in X and $\alpha, \beta, \gamma \in [0,1]$ with $0 \leq \alpha+\beta+\gamma \leq 3$ and (α, β, γ)-level set of A denoted by $A^{(\alpha,\beta,\gamma)}$ is defined as:

$$A^{(\alpha,\beta,\gamma)} = \{x \in X | T_A(x) \geq \alpha, I_A(x) \leq \beta, F_A(x) \leq \gamma\}.$$

Remark 4.1. In fact, the original definition of (α, β, γ)-level set in [20] is as follows:

$$A^{(\alpha,\beta,\gamma)} = \{x \in X | T_A(x) \geq \alpha, I_A(x) \geq \beta, F_A(x) \leq \gamma\}.$$

In this paper, level set is defined as above in order to match the order relations of the neutrosophic set. Since this paper uses another ordering relationship (see Remark 2.1), the corresponding (α, β, γ)-level set uses the above Definition 4.2.

Theorem 4.2. *Let X be a BE-algebra, A be a neutrosophic set in X. Then A is a neutrosophic filter in X if and only if all of (α, β, γ)-level set $A^{(\alpha,\beta,\gamma)}$ are filters of X when $\alpha, \beta, \gamma \in [0,1]$ such that $A^{(\alpha,\beta,\gamma)} \neq \emptyset$.*

Proof. Assume that A is a neutrosophic filter in X and let $\alpha, \beta, \gamma \in [0,1]$ such that$A^{(\alpha,\beta,\gamma)} \neq \emptyset$. By Definition 4.2, we have $U(T_A; \alpha) \neq \emptyset, U(1-I_A; 1-\beta) \neq \emptyset$, and $U(1-F_A;1-\gamma)\neq\emptyset$. Applying Theorem 4.1 and Proposition 2.5 (3), we get that $U(T_A;\alpha), U(1-I_A;1-\beta)$, and $U(1-F_A;1-\gamma)$ are filters of X. Thus $U(T_A;\alpha)\cap U(1-I_A;1-\beta)\cap U(1-F_A;1-\gamma)$ is also filter of X. Moreover, by Definition 4.2, it is easy to verify that

$$A^{(\alpha,\beta,\gamma)} = U(T_A;\alpha)\cap U(1-I_A;1-\beta)\cap U(1-F_A;1-\gamma).$$

Hence,(α,β,γ)-level set $A^{(\alpha,\beta,\gamma)}$ is a filter of X.

Conversely, assume that all of (α,β,γ)-level sets $A^{(\alpha,\beta,\gamma)}$ are filters of X when $\alpha,\beta,\gamma\in[0,1]$such that $A^{(\alpha,\beta,\gamma)}\neq\emptyset$. For any $t\in[0,1]$, if $U(T_A;t)=\{x\in X|T_A(x)\geq t\}\neq\emptyset$, then there exists $x_0\in X$ such that $T_A(x_0)\geq t$. Obviously, $I_A(x_0)\leq 1, F_A(x_0)\leq 1$. It follows that $x_0\in A^{(t,1,1)}$, that is, $A^{(t,1,1)}=\{x\in X : t\leq T_A(x), I_A(x)\leq 1, F_A(x)\leq 1\}=\{x\in X|T_A(x)\geq t\}=U(T_A;t\neq\emptyset)$.

Hence, by the assumption $U(T_A;t)=A^{(t,1,1)}$ is a filter of X. Applying Proposition 2.5 we know thatT_A is a fuzzy filter of X. Moreover, for any $t\in[0,1]$, if $U(1-I_A;t)=\{x\in X|1-I_A(x)\geq t\}\neq\emptyset$, then there exists $x_0\in X$ such that $1-I_A(x_0)\geq t$, that is, $I_A(x_0)\leq 1-t$. Obviously, $T_A(x_0)\geq 0, F_A(x_0)\leq 1$. It follows that $x_0\in A^{(0,1-t,1)}$, that is, $A^{(0,1-t,1)}=\{x\in X : 0\leq T_A(x), I_A(x)\leq 1-t, F_A(x)\leq 1\}=\{x\in X|I_A(x)\leq 1-t\}=U(1-I_A;t)\neq\emptyset$.

Hence, by the assumption $U(1-I_A;t)=A^{(0,1-t,1)}$ is a filter of X. Applying Proposition 2.5 we know that$1-T_A$ is a fuzzy filter of X.

Similarly, we can get that $1-F_A$ is a fuzzy filter of X. Combining the above results, using Theorem 4.1, we know that A is a neutrosophic filter in X.

Applying Theorem 4.2 we can get

Corollary 4.2. *Let A be a neutrosophic filter in a BE-algebra X, we denote that:*

(1) $A_T=\{x\in X|T_A(x)=T_A(1)\}$;
(2) $A_I=\{x\in X|I_A(x)=I_A(1)\}$;
(3) $A_F=\{x\in X|F_A(x)=F_A(1)\}$.

Then A_T, I_T and F_T are are filters of X.

Corollary 4.3. *Let A be a neutrosophic filter in a BE-algebra X, we denote*

$$A_b=\{x\in X|T_A(x)\geq T_A(b), I_A(x)\leq I_A(b), F_A(x)\leq F_A(b)\}, b\in X.$$

Then A_b is a filter of X for every $b\in X$.

Theorem 4.3. *Let X be a BE-algebra, A be a neutrosophic set in X. Then A is a neutrosophic filter in X if and only if it satisfies (NSF1) and*

$(NSF4)$ $\forall x,y,z\in X$, *if* $x\to(y\to z)=1$, *then* $\min\{T_A(x),T_A(y)\}\leq T_A(z)$, $\max\{I_A(x),I_A(y)\}\geq I_A(z)$, *and* $\max\{F_A(x),F_A(y)\}\geq F_A(z)$.

Proof. Let A be a neutrosophic filter in X and let $x, y, z \in X$. Suppose that $x \to (y \to z) = 1$. Applying Definition 4.1 we have

$$T_A(y \to z) \geq \min\{T_A(x), T_A(x \to (y \to z))\} = \min\{T_A(x), T_A(1)\} = T_A(x),$$
$$T_A(z) \geq \min\{T_A(y \to z), T_A(y)\} \geq \min\{T_A(x), T_A(y)\};$$
$$I_A(y \to z) \leq \max\{I_A(x), I_A(x \to (y \to z))\} = \max\{I_A(x), I_A(1)\} = I_A(x),$$
$$I_A(z) \leq \max\{I_A(y \to z), I_A(y)\} \leq \max\{I_A(x), I_A(y)\};$$
$$F_A(y \to z) \leq \max\{F_A(x), F_A(x \to (y \to z))\} = \max\{F_A(x), F_A(1)\} = F_A(x),$$
$$F_A(z) \leq \max\{F_A(y \to z), F_A(y)\} \leq \max\{F_A(x), F_A(y)\}.$$

That is, (NSF4) holds.

Conversely, let A satisfies (NSF1) and (NSF4). From Definition 2.7 (BE1) we have $(x \to y) \to (x \to y) = 1$. By (NSF4),

$\min\{T_A(x \to y), T_A(x)\} \leq T_A(y), \max\{I_A(x \to y), I_A(x)\} \geq I_A(y)$, and $\max\{F_A(x \to y), F_A(x)\} \geq F_A(y)$.

This means that A satisfy (NSF2). Using Definition 4.1 we get that Ais a neutrosophic filter in X.

5. Implicative neutrosophic filters

Definition 5.1. A neutrosophic set A in a BE-algebra X is called an implicative neutrosophic filter if it satisfies:$\forall x, y, z \in X$,

(1) $T_A(x) \leq T_A(1), I_A(x) \geq I_A(1)$ and $F_A(x) \geq F_A(1)$;

(2) $\min\{T_A(x \to (y \to z)), T_A(x \to y)\} \leq T_A(x \to z), \max\{I_A(x \to (y \to z)), I_A(x \to y)\} \geq I_A(x \to z)$, and $\max\{F_A(x \to (y \to z)), F_A(x \to y)\} \geq F_A(x \to z)$.

Example 5.1. Let $X = \{a, b, c, 1\}$ be a BE-algebra with a binary operation given by the following table

→	a	b	c	1
a	a	b	c	1
b	1	b	b	1
c	a	1	a	1
1	a	b	c	1

Define neutrosophic set A in X as following:

$$T_A(x) = \begin{cases} 0.87, & if x = 1 \\ 0.69, & if x = b \\ 0.11, & \text{otherwise} \end{cases}, I_A(x) = \begin{cases} 0.09, & if x = 1 \\ 0.15, & if x = b \\ 0.84, & \text{otherwise} \end{cases},$$

$$F_A(x) = \begin{cases} 0.05, & if x = 1 \\ 0.14, & if x = b \\ 0.79, & \text{otherwise.} \end{cases}$$

We can verify that A is an implicative neutrosophic filter.

When $x = 1$ in Definition 5.1 (2), we can get (NSF2) in Definition 4.1, this means that the following proposition is true.

Proposition 5.1. *Let A be an implicative neutrosophic filter in a BE-algebra X. Then A is a neutrosophic filter in X.*

The following example shows that the converse of Proposition 5.1 is not true in general.

Example 5.2. Let $(X; \to, 1)$ be the BE-algebra in Example 3.1, and A be the neutrosophic filter in Example 4.1. Then we can verify that A is not an implicative neutrosophic filter in$X$, since

$\min\{T_A(b \to (d \to c)), T_A(b \to d)\} = \min\{T_A(1), T_A(a)\} = \min\{0.86, 0.86\} = 0.86 \not\leq 0.13 = T_A(b) = T_A(b \to c)$.

Proposition 5.2. *Let A be an implicative neutrosophic filter in a BE-algebra X. Then A satisfies the following conditions:*

(i) $\forall x, y \in X, T_A(x \to y) = T_A(x \to (x \to y))$;

(ii) $\forall x, y \in X, I_A(x \to y) = I_A(x \to (x \to y))$;

(iii) $\forall x, y \in X, F_A(x \to y) = F_A(x \to (x \to y))$.

Proof. Putting $y = x$ and $z = y$ in Definition 5.1 (2), we can get that

$\min\{T_A(x \to (x \to y)), T_A(x \to x)\} \leq T_A(x \to y), \max\{I_A(x \to (x \to y)), I_A(x \to x)\} \geq I_A(x \to y)$, and $\max\{F_A(x \to (x \to y)), F_A(x \to x)\} \geq F_A(x \to y)$.

Applying Definition 2.7 (BE1) and Definition 5.1 (1) we have $T_A(x \to (x \to y)) = \min\{T_A(x \to (x \to y)), T_A(1)\} = \min\{T_A(x \to (x \to y)), T_A(x \to x)\} \leq T_A(x \to y)$, $I_A(x \to (x \to y)) = \max\{I_A(x \to (x \to y)), I_A(1)\} = \max\{I_A(x \to (x \to y)), I_A(x \to x)\} \geq I_A(x \to y)$, and $F_A(x \to (x \to y)) = \min\{F_A(x \to (x \to y)), F_A(1)\} = \min\{F_A(x \to (x \to y)), F_A(x \to x)\} \geq F_A(x \to y)$.

On the other hand, by Proposition 2.1 (2),we have $x \to y \leq x \to (x \to y)$. Using Proposition 4.1, $T_A(x \to y) \leq T_A(x \to (x \to y)), I_A(x \to y) \geq I_A(x \to (x \to y)), F_A(x \to y) \geq F_A(x \to (x \to y))$.

Combining the above two hands, we get that $T_A(x \to y) = T_A(x \to (x \to y)), I_A(x \to y) = I_A(x \to (x \to y)), F_A(x \to y) = F_A(x \to (x \to y))$.

Theorem 5.1. *Let A be a neutrosophic filter in a transitive BE-algebra X. Then A is an implicative neutrosophic filter in X if and only if it satisfies:*

(i) $\forall x, y \in X, T_A(x \to y) = T_A(x \to (x \to y))$;

(ii) $\forall x, y \in X, I_A(x \to y) = I_A(x \to (x \to y))$;

(*iii*) $\forall x, y \in X, F_A(x \to y) = F_A(x \to (x \to y))$.

Proof. If A is an implicative neutrosophic filter in X, then by Proposition 5.2 we know that the conditions (i), (ii) and (iii) hold.

Conversely, suppose that A satisfies the conditions (i), (ii) and (iii). For any $x, y, z \in X$, by the definition of a transitive BE-algebra and Definition 2.7 we have

$$x \to y \le (y \to z) \to (x \to z) \le (x \to (y \to z)) \to (x \to (x \to z)),$$

$$(x \to y) \to ((x \to (y \to z)) \to (x \to (x \to z))) = 1,$$

$$(x \to (y \to z)) \to ((x \to y) \to (x \to (x \to z))) = 1.$$

Applying Theorem 4.3 (NSF4) we have

$$\min\{T_A(x \to (y \to z)), T_A(x \to y)\} \le T_A(x \to (x \to z)),$$

$$\min\{I_A(x \to (y \to z)), I_A(x \to y)\} \ge I_A(x \to (x \to z)),$$

$$\min\{F_A(x \to (y \to z)), F_A(x \to y)\} \ge F_A(x \to (x \to z)).$$

From these, by (i), (ii) and (iii) we get

$$\min\{T_A(x \to (y \to z)), T_A(x \to y)\} \le T_A(x \to (x \to z)) = T_A(x \to z),$$

$$\max\{I_A(x \to (y \to z)), I_A(x \to y)\} \ge I_A(x \to (x \to z)) = I_A(x \to z),$$

$$\max\{F_A(x \to (y \to z)), F_A(x \to y)\} \ge F_A(x \to (x \to z)) = F_A(x \to z),$$

Hence, by Definition 5.1 we know that A is an implicative neutrosophic filter in X.

Theorem 5.2. *Let X be a self distributive BE-algebra. Then every neutrosophic filter in X is an implicative neutrosophic filter in X.*

Proof. Let A be a neutrosophic filter in X. Then

$$\forall x \in X, T_A(x) \le T_A(1), I_A(x) \ge I_A(1) and F_A(x) \ge F_A(1).$$

For any $x, y, z \in X$, by the definition of a self distributive BE-algebra, $x \to (y \to z) = (x \to y) \to (x \to z)$. By applying (NSF2) in Definition 4.1 we have:

$$\begin{aligned}
&\min\{T_A(x \to (y \to z)), T_A(x \to y)\}\\
&= \min\{T_A((x \to y) \to (x \to z)), T_A(x \to y)\} \le T_A(x \to z),\\
&\max\{I_A(x \to (y \to z)), I_A(x \to y)\}\\
&= \max\{I_A((x \to y) \to (x \to z)), I_A(x \to y)\} \ge I_A(x \to z),
\end{aligned}$$

$$\max\{F_A(x \to (y \to z)), F_A(x \to y)\}$$
$$= \max\{F_A((x \to y) \to (x \to z)), F_A(x \to y)\} \geq F_A(x \to z).$$

By Definition 5.1 we know that A is an implicative neutrosophic filter in X.

Theorem 5.3. *Let X be a self distributive BE-algebra and A be a neutrosophic filter in X. Then the following conditions are equivalent:*

(1) *A is an implicative neutrosophic filter in X;*

(2) $\forall x, y \in X, T_A(x \to (x \to y)) \leq T_A(x \to y), I_A(x \to (x \to y)) \geq I_A(x \to y), F_A(x \to (x \to y)) \geq F_A(x \to y)$;

(3) $\forall x, y, z \in X, \min\{T_A(z \to (x \to (x \to y))), T_A(z)\} \leq T_A(x \to y)$, $\max\{I_A(z \to (x \to (x \to y))), I_A(z)\} \geq I_A(x \to y)$, and $\max\{F_A(z \to (x \to (x \to y))), F_A(z)\} \geq F_A(x \to y)$.

Proof. (1) $\Rightarrow$ (2).$\forall x, y \in X$, by Definition 5.1 (2), we have

$$\min\{T_A(x \to (x \to y)), T_A(x \to x)\} \leq T_A(x \to y),$$

$$\max\{I_A(x \to (x \to y)), I_A(x \to x)\} \geq T_A(x \to y),$$

$$\max\{F_A(x \to (x \to y)), F_A(x \to x)\} \geq F_A(x \to y).$$

Applying Definition 2.7 (BE1) and Definition 5.1 (1) we have $T_A(x \to (x \to y)) = \min\{T_A(x \to (x \to y)), T_A(1)\} = \min\{T_A(x \to (x \to y)), T_A(x \to x)\} \leq T_A(x \to y)$, $I_A(x \to (x \to y)) = \max\{I_A(x \to (x \to y)), I_A(1)\} = \max\{T_A(x \to (x \to y)), I_A(x \to x)\} \leq I_A(x \to y), and\ F_A(x \to (x \to y)) = \max\{F_A(x \to (x \to y)), F_A(1)\} = \max\{T_A(x \to (x \to y)), F_A(x \to x)\} \leq F_A(x \to y)$.

Hence, (2) holds.

(2) $\Rightarrow$ (3). $\forall x, y, z \in X$, by Definition 4.1 (NSF2), we have:

$$\min\{T_A(z \to (x \to (x \to y))), T_A(z)\} \leq T_A(x \to (x \to y)),$$

$$\max\{I_A(z \to (x \to (x \to y))), I_A(z)\} \geq I_A(x \to (x \to y)),$$

$$\max\{F_A(z \to (x \to (x \to y))), F_A(z)\} \geq F_A(x \to (x \to y)).$$

From these, using (2) we get

$$\min\{T_A(z \to (x \to (x \to y))), T_A(z)\} \leq T_A(x \to y),$$

$$\max\{I_A(z \to (x \to (x \to y))), I_A(z)\} \geq I_A(x \to y),$$

$$\max\{F_A(z \to (x \to (x \to y))), F_A(z)\} \geq F_A(x \to y).$$

Therefore, (3) holds.

(3) $\Rightarrow$ (1).$\forall x, y, z \in X$, by the definition of a self distributive BE-algebra and Definition 2.7, we have

$$x \to (y \to z) = y \to (x \to z) \leq (x \to y) \to (x \to (x \to z)).$$

By applying Proposition 4.1,

$$T_A(x \to (y \to z)) \le T_A((x \to y) \to (x \to (x \to z))),$$

$$I_A(x \to (y \to z)) \ge I_A((x \to y) \to (x \to (x \to z))),$$

$$F_A(x \to (y \to z)) \ge F_A((x \to y) \to (x \to (x \to z))).$$

From these, using (3) we get $\min\{T_A(x \to (y \to z)), T_A(x \to y)\} \le \min\{T_A((x \to y) \to (x \to (x \to z))), T_A(x \to y)\} \le T_A(x \to z)$, $\max\{I_A(x \to (y \to z)), I_A(x \to y)\} \ge \max\{I_A((x \to y) \to (x \to (x \to z))), I_A(x \to y)\} \ge I_A(x \to z)$, $\max\{F_A(x \to (y \to z)), F_A(x \to y)\} \ge \max\{F_A((x \to y) \to (x \to (x \to z))), F_A(x \to y)\} \ge T_A(x \to z)$.

Hence, by Definition 5.1 we know that A is an implicative neutrosophic filter in X.

Definition 5.2. A fuzzy set μ in a BE-algebra X is called an implicative fuzzy filter if it satisfies:$x, y, z \in X$,

(1) $\mu(x) \le \mu(1)$;

(2) $\min\{\mu(x \to (y \to z)), \mu(x \to y)\} \le \mu(x \to z)$.

It is similar to Theorem 3.6 in [5] we can get the following theorem (the proof is omitted).

Theorem 5.4. *Let μ be a fuzzy set of a BE-algebra X. Then the following conditions are equivalent:*

(1) *μ is an implicative fuzzy filter in X;*

(2) *for each $\alpha \in [0, 1]$, the level subset $U(\mu; \alpha) = \{x \in X : \mu(x) \ge \alpha\}$ is an implicative filter of X, when$U(\mu; \alpha) \ne \emptyset$.*

It is similar to Theorem 4.1 we can get the following theorem (the proof is omitted).

Theorem 5.5. *Let A be a neutrosophic set in a BE-algebra X. Then A is an implicative neutrosophic filter in X if and only if A satisfies:*

(*i*) *T_A is a fuzzy implicative filter of X;*

(*ii*) $1-I_A$ *is a fuzzy implicative filter of X, where* $(1-I_A)(x) = 1-I_A(x), x \in X$;

(*iii*) $1 - F_A$ *is a fuzzy implicative filter of X, where* $(1 - F_A)(x) = 1 - F_A(x), x \in X$.

It is similar to Theorem 4.2 we can get the following theorem (the proof is omitted).

Theorem 5.6. *Let X be a BE-algebra, A be a neutrosophic set in X. Then A is an implicative neutrosophic filter in X if and only if all of (α, β, γ)-level set $A^{(\alpha,\beta,\gamma)}$ are implicative filters of X when $\alpha, \beta, \gamma \in [0, 1]$such that $A^{(\alpha,\beta,\gamma)} \ne \emptyset$.*

6. Conclusion

This paper further studied the application of neutrosophic set theory to BE-algebras. First of all, we analyzed the defects of the original definition of neutrosophic filter in a BE-algebra, by using some examples we pointed out the following facts:

(1) An example of neutrosophic filter (Example 3.1 in [10]) is wrong;

(2) A theorem on neutrosophic filters (Theorem 3.4 in [10]) is wrong;

(3) The original definition of neutrosophic filter in BE- algebra is not normal, since the indeterminacy-membership function and falsity-membership function are constants for any neutrosophic filter (see Proposition 3.2).

In order to solve the above problems, we given a reasonable new definition of neutrosophic filter in BE- algebras, and through in-depth study its properties, we know that the new definition is good and overcomes the shortcomings of the original definition. Especially, some necessary and sufficient conditions are given, and an important fact is shown: a neutrosophic filter in BE-algebra can be completely determined by its (alpha, beta, gamma)- level sets. Moreover, the relationships between fuzzy filters and neutrosophic filters are investigated. Finally, the new concept of implicative neutrosophic filter in BE-algebra is introduced, and some necessary and sufficient conditions for a neutrosophic filter to be implicative neutrosophic filter are given. All these results are new and important, which can be used for reference to other research on non-classical logic algebra systems.

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