On new class of contra continuity in nano topology

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Abstract. The purpose of this paper is to introduce and study the stronger form of nano continuity called nano contra continuity. Further the concept of nano kernel and nano Bi-contra continuity is also discussed. These type of mappings can be used in Biotechnology, to study contra or two way contra effects. Here we applied both contra and Bi-contra mapping between a set of viruses to a set of antiviruses as a treatment for disease causing viruses.

Keywords: nano topology, nano α-open sets, nano rare set, nano contra continuity, nano Bi-contra continuity.

1. Introduction

Ganster and Reily [5] introduced and studied notion of LC-continuous functions. Dontchev [4] presented a new notion of continuous function called contra continuity, a stronger form of LC-continuity. Lellis Thivagar et al. [6] introduced a nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano open sets. But certain nano terms are satisfied simply to mean "very small". It comes from the Greek word 'Nanos' which means 'dwarf', in its modern scientific sense, an order to magnititude-one billionth of something. Nano car is an example. The topology introduced here is named so because of its size, since it has almost five elements in it. Nano

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continuous function [6] has been defined in terms of nano open sets and its characterisations were derived. Certain weak forms of nano α-open sets, nano semi-open sets and nano pre-open sets were also established[6]. In this paper nano contra continuity and nano Bi-contra continuity and their properties are discussed.

2. Preliminaries

The following recalls necessary concepts and preliminaries required in the sequel.

Definition 2.1 ([8]). Let $\mathcal{U}$ be a non-empty finite set of objects called the universe $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, R)$ is said to be the approximation space. Let $X \subseteq \mathcal{U}$.

(i) The Lower appproximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \left\{\bigcup_{x\in\mathcal{U}}\{R(x) : R(x) \subseteq X\}\right\}$, where R(x) denotes the equivalence class determined by x.

(ii) The Upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \left\{\bigcup_{X\in\mathcal{U}}\{R(x) : R(x) \cap X \neq \emptyset\}\right\}$.

(iii) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 ([9]). Let $\mathcal{U}$ be the universe, R be an equivalence relation on $\mathcal{U}$ and $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq \mathcal{U}$ and $\tau_R(X)$satisfies the following axioms.

(i) $\mathcal{U}$ and $\emptyset \in \tau_R(X)$.

(ii) The union of the elements of any subcollection$\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology $\mathcal{U}$ called as the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets. A set A is said to be nano closed if its complement is nano open.

Definition 2.3 ([6]). If $(\mathcal{U}, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq \mathcal{U}$ and if $A \subseteq \mathcal{U}$, then nano interior of A is defined as the union of all nano open subsets contained in A and its denoted by $\mathcal{N}Int(A)$. That is

$\mathcal{N}Int(A)$ is the largest nano open subset contained in A. The nano closure of A is defined as the intersection of all nano closed sets containing A and its denoted by $\mathcal{N}Cl(A)$. That is, $\mathcal{N}Cl(A)$ is the smallest nano closed set containing A.

Definition 2.4 ([6]). Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$. Then A is said to be,

(i) nano semi-open if $A \subseteq \mathcal{N}Cl(\mathcal{N}Int(A))$.

(ii) nano pre-open if $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(A))$.

(iii) nano α-open if $A \subseteq \mathcal{N}Int(\mathcal{N}Cl(\mathcal{N}Int(A))$.

Definition 2.5 ([6]). Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$ is called nano α-closed (respectively,nano semi-closed,nano pre-closed) if its complement is nano α-open (nano semi-open, nano pre-open).

Definition 2.6 ([6]). Let $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ be nano topological spaces. Then a mapping f: $(\mathcal{U}, \tau_R(X)) \to (\mathcal{V}, \tau_{R'}(Y))$ is nano continuous on $\mathcal{U}$ if the inverse image of every nano open set in $\mathcal{V}$ is nano open in $\mathcal{U}$.

Throughout this paper, $\mathcal{U}$ and $\mathcal{V}$ are non empty finite universes, $X \subseteq \mathcal{U}$ and $Y \subseteq \mathcal{V}$, where R and R' are equivalence relations on $\mathcal{U}$ and $\mathcal{V}$ respectively. $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ are the nano topological space with respect to X and Y, respectively.

3. Nano kernel

In this section, the notion of nano kernel is introduced and its properties are investigated.

Definition 3.1. Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$. The set $\mathcal{N}ker(A) = \bigcap\{$U: $A \subseteq$U, U $\in \tau_R(X)\}$ is called the nano kernel of A and is denoted by $\mathcal{N}ker(A)$.

Example 3.2. Let $\mathcal{U} = \{a, b, c, d, e\}$ with $\mathcal{U}/R = \{\{a, b\}, \{c\}, \{d, e\}\}$. Let $X = \{a\} \subseteq \mathcal{U}$ so that $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$, then the $\mathcal{N}Ker\{a\} = \{a, b\}$, $\mathcal{N}Ker\{a, b\} = \{a, b\}$, $\mathcal{N}Ker\{a, b, c\} = \mathcal{U}$.

Theorem 3.3. *Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A, B \subseteq \mathcal{U}$. Then the following properties hold.*

(i) $x \in \mathcal{N}\ker A$ iff $A \cap F \neq \emptyset$ for any nano closed set containing x.

(ii) If $A \subseteq \mathcal{N}kerA$ and then $A = \mathcal{N}Ker(A)$ if A is nano open in $\mathcal{U}$.

(iii) If $A \subseteq B$, then $\mathcal{N}\ker A \subseteq \mathcal{N}\ker B$.

Proof. (i) $\Rightarrow$. If $x \in \mathcal{N}\ker A$, then $x \in A \subseteq F^c$, where F^c is a nano open set and $A \subseteq F^c, \Rightarrow A \cap F^c \neq \emptyset$. Hence $A \subseteq F^c \subseteq F \Rightarrow A \cap F \neq \emptyset$, F is nano closed. Hence $x \in A \cap F \neq \emptyset$, F is nano closed. Therefore $A \cap F \neq \emptyset$ for any nano closed set containing x.

$\Leftarrow$. Let $A \cap F \neq \emptyset$ for any nano closed set containing x.

Assume that $x \notin \mathcal{N}kerA$, hence their exist an nano open set F^c such that $A \subseteq F^c$ and $x \notin F^c$. Hence $A \subseteq F$ and $x \notin F$, where F is a nano closed which is a contradiction. Hence $x \in \mathcal{N}kerA$.

(ii) Let A be a nano open set. Since A is nano open $\mathcal{N}kerA \subseteq A$ holds.

Let B be any nano open set containing A, then we have $A \subseteq W$. Hence $A \subseteq W \cap A \subseteq A$ and $W \cap A$ is nano open. Now, $A \subseteq \cap\{W.A \subseteq W,\ W \in \tau_R(X)\}$. Therefore $A \subseteq \mathcal{N}kerA$ and hence $A = \mathcal{N}kerA$.

(iii) Let $A \subseteq B$, To prove that. $\mathcal{N}kerA \subseteq \mathcal{N}kerB$. Let $V \in \mathcal{N}kerA \Rightarrow A \subseteq V$ and V is nano open in $\{\mathcal{U}, \tau_R(X)\}$. Since $A \subseteq B$, so $A \subseteq B \subseteq V$, where V is a nano open set in $(\mathcal{U}, \tau_R(X))$. Therefore $V \in \mathcal{N}ker(B)$. Hence $V \in \mathcal{N}ker(A) \Rightarrow V \in \mathcal{N}ker(B)$ and thus $\mathcal{N}kerA \subseteq \mathcal{N}ker(B)$. □

4. Nano kernel

In this section we introduce and define the concept of nano no where dense sets and nano rare sets and some of their properties are investigated.

Definition 4.1. Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and let $A \subseteq \mathcal{U}$, then A is called nano nowhere dense if $\mathcal{N}Int[\mathcal{N}Cl(A)] = \emptyset$.

Example 4.2. Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{a, c\}, \{b, d\}$ and let $X = \{a, c\} \subseteq \mathcal{U}$, $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, c\}\}$, then the nano closed set are $\mathcal{U}, \emptyset, \{b, d\}$. Nano semi-closed sets are $\mathcal{U}, \emptyset, \{b, d\}, \{b\}, \{d\}$. Nano nowhere dense sets are $\mathcal{U}, \emptyset, \{b, d\}, \{b\}, \{d\}$.

Remark 4.3. The following theorem is the consequence of the above example.

Theorem 4.4. *Every nano nowhere dense set is nano semi-closed.*

Proof. Let A$\subseteq \mathcal{U}$ be a nano nowhere dense set,then $\mathcal{N}Int[\mathcal{N}Cl(A)] = \emptyset$. Clearly, $\mathcal{N}Int[\mathcal{N}Cl(A)] = \emptyset \subseteq A \Rightarrow \mathcal{N}Int[\mathcal{N}Cl(A)] \subseteq A$. Hence A is nano semi-closed. □

Remark 4.5. The converse of Theorem 4.4 is not true which can be shown by the following example.

Example 4.6. Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a\}, \{c\}, \{b, d\}\}$, and let $X = \{a, b\} \subseteq \mathcal{U}$, $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$. The set $\{b, c, d\}$ is a nano semi-closed set in $\mathcal{U}$ but not nano nowhere dense.

Definition 4.7. Let $(\mathcal{U}, \tau_R(X))$ be a nano topological space and $A \subseteq \mathcal{U}$,then A is said to be a nano rare set if $\mathcal{N}Int(A) = \emptyset$, otherwise it is known as nano non-rare set in $\mathcal{U}$.

Example 4.8. Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\} \subseteq \mathcal{U}$. Then the nano topology $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$. Then the nano rare sets are $\emptyset$, $\{b\}, \{c\}, \{d\}, \{c, d\}, \{b, c\}$. Nano non-rare sets are $\mathcal{U}$, $\{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$.

Theorem 4.9. *In a nano topological space $[\mathcal{U}, \tau_R(X)]$ if $L_R(X) = U_R(X)$, then $\mathcal{U}, L_R(X)$ and any set $A \supseteq L_R(X)$ are the only nano non-rare sets in $\mathcal{U}$.*

Proof. Since $L_R(X) = U_R(X) = X$, the nano topological space, $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X)\}$. If $A \neq \emptyset$ and $A \subset L_R(X)$, then $\mathcal{N}Int(A) = \emptyset$, since $\emptyset$ is the only nano open subset of A. Hence A is a nano non-rare subset of A. If $A \supset L_R(X)$, then $L_R(X)$ is the largest nano open subset of A and hence $\mathcal{N}Int(A) = L_R(X) \neq \emptyset$. Therefore, A is a nano non-rare set. Thus $\mathcal{U}, L_R(X)$ and any set $A \supset L_R(X)$ are the only nano non-rare sets in $\mathcal{U}$, if $L_R(X) = U_R(X)$. □

Theorem 4.10. *In a nano topological space if $L_R(X) = \emptyset$ and $U_R(X) \neq \mathcal{U}$, then $\mathcal{U}, U_R(X)$ and any set $A \supseteq U_R(X)$ are the only nano non-rare sets.*

Proof. Since $L_R(X) = \emptyset$ and $U_R(X) = X$, the nano topological space, $\tau_R(X) = \{\mathcal{U}, \emptyset, U_R(X)\}$ and the members of $\tau_R(X)$ are nano open in $\mathcal{U}$. Let $A \subset U_R(X)$, then $\mathcal{N}Int(A) = \emptyset$. Therefore A is a nano rare set in $\mathcal{U}$. If $A \supseteq U_R(X)$, then $U_R(X)$ is the largest nano open subset of A. Therefore $\mathcal{N}Int(A) = U_R(X)$ and hence A is a nano non-rare set in $\mathcal{U}$. Thus $\mathcal{U}$, and any set $A \supseteq U_R(X)$ are the only nano non-rare sets in $\mathcal{U}$. □

Theorem 4.11. *In $[\mathcal{U}, \tau_R(X)]$ if $U_R(X) = \mathcal{U}$ and $L_R(X) \neq \emptyset$ then $\mathcal{U}$ and any set $A \supseteq L_R(X), B_R(X)$ are the only nano non-rare sets in $\mathcal{U}$.*

Proof. Since $U_R(X) = \mathcal{U}$ and $L_R(X) \neq \emptyset$ the nano open sets in $\mathcal{U}$ are $\mathcal{U}, \emptyset$, $L_R(X)$ and $B_R(X)$. Let A $\subseteq \mathcal{U}$ such that $A \neq \emptyset$. If $A = \emptyset$, then A is nano rare open. Therefore, let $A \neq \emptyset$, when $A \subset L_R(X)$, then $\mathcal{N}Int(A) = \emptyset$, since the largest nano open subset of A is $\emptyset$. That is, A is nano rare open in $\mathcal{U}$. When $L_R(X) \subset A$, $\mathcal{N}Int(A) = L_R(X)$ and therefore A is nano non-rare open in $\mathcal{U}$. Similarly it can be shown that any set $A \subset B_R(X)$ is nano rare set in $\mathcal{U}$ and $A \supseteq B_R(X)$ is a nano non-rare set in $\mathcal{U}$. Thus $\mathcal{U}$ and any set $A \supseteq L_R(X), B_R(X)$ are the only nano non-rare sets in $\mathcal{U}$. □

Theorem 4.12. *If $L_R(X) \neq U_R(X)$ where $L_R(X) \neq \emptyset$ and $U_R(X) \neq \mathcal{U}$ in a nano topological space then any set $A \supseteq L_R(X), U_R(X)$ and $B_R(X)$ are the only nano non-rare sets in $\mathcal{U}$.*

Proof. The nano topology on $\mathcal{U}$ is given by $\tau_R(X) = \{\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)\}$. Then $A \supseteq \mathcal{U}$ such that $A \supseteq U_R(X)$, then $\mathcal{N}Int(A) = U_R(X)$ and therefore any $A \supseteq U_R(X)$ is nano non-rare in $\mathcal{U}$. Similarly it can be shown that $A \supseteq B_R(X)$ and $A \subseteq L_R(X)$ are nano non-rare in $\mathcal{U}$. When $A \subset B_R(X)$, $\mathcal{N}Int(A) = \emptyset$ and hence A is nano rare in $\mathcal{U}$. When $A \subset U_R(X)$ such that A is neither a subset of $L_R(X)$ nor a subset of $B_R(X)$, $\mathcal{N}Int(A) = \emptyset$ and hence A is nano rare in $\mathcal{U}$. Thus any set $A \supseteq L_R(X), U_R(X)$ *and* $B_R(X)$ are the only nano non-rare sets in $\mathcal{U}$. □

Theorem 4.13. *Every nano nowhere dense set is nano rare.*

Proof. Let $A \subseteq \mathcal{U}$ be a nano nowhere dense set in $\mathcal{U}$, then by definition $\mathcal{N}Int[\mathcal{N}Cl(A)] = \emptyset$. We know that $A \subseteq \mathcal{N}Cl(A) \Longrightarrow \mathcal{N}Int(A) \subseteq \mathcal{N}Int[\mathcal{N}Cl(A)] = \emptyset \Longrightarrow \mathcal{N}Int(A) = \emptyset$. That is, A is a nano rare set in $\mathcal{U}$. □

Remark 4.14. The converse of Theorem 4.13 is not true which can be seen from the following example.

Example 4.15. Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a\}, \{b, c, d\}\}$ and $X = \{a, d\} \subseteq \mathcal{U}$. Then the nano topology $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a\}, \{b, c, d\}\}$. Then the nano rare sets in $\mathcal{U}$ are $\{b\}, \{c\}, \{d\}, \{b, d\}, \{c, d\}$ but are not nano nowhere dense in $\mathcal{U}$.

Theorem 4.16. *If a set $A \subseteq \mathcal{U}$ in a nano topological space is both nano pre-open and nano semi-closed then it is a nano non-rare set.*

Proof. By definition of nano pre-open and nano semi-closed, $A \subseteq \mathcal{N}Int[\mathcal{N}Cl(A)]$ and $\mathcal{N}Int[\mathcal{N}Cl(A)] \subseteq A$ which implies $A = \mathcal{N}Int[\mathcal{N}Cl(A)]$. Hence A becomes nano regular open set in $\mathcal{U}$. Since every nano regular open set is nano open, then $\mathcal{N}Int(A) \neq \emptyset$. Hence A is not a nano rare set in $\mathcal{U}$ if it is both nano pre-open and nano semi-closed in $\mathcal{U}$. □

5. Nano contra continuous function

In this section, the notion of nano form of contra continuity is introduced and its properties are investigated. This nano contra continuity is also compared with other nano continuous functions.

Definition 5.1. Let $(\mathcal{U}, \tau_R(X))$ and $(\mathcal{V}, \tau_{R'}(Y))$ be nano topological spaces, then the mapping $f : (\mathcal{U}, \tau_R(X)) \to (\mathcal{V}, \tau_{R'}(Y))$ is nano contra continuous, if the inverse image of every nano open set in $\mathcal{V}$ is nano closed in $\mathcal{U}$. That is, if $f^{-1}(B)$ is nano closed in $\mathcal{U}$ for every nano open set B of $\mathcal{V}$.

Example 5.2. Let $\mathcal{U} = \{a, b, c, d, e\}$ with $\mathcal{U}/R = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$ and $X = \{d, e\} \subseteq \mathcal{U}$. Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{d, e\}\}$. Then nano closed sets are $\mathcal{U}, \emptyset, \{a, b, c\}$. Let $\mathcal{V} = \{x, y, z, u, v\}$ with $\mathcal{V}/R' = \{\{x, y\}, \{z, v\}, \{u\}\}$. Let $Y = \{z, u, v\} \subseteq \mathcal{V}$. Then $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{z, u, v\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ by $f(a) = z, f(b) = u, f(c) = v, f(d) = v, f(d) = x, f(e) = y$. Then $f^{-1}(\{z, u, v\}) = \{a, b, c\}$ which is a nano closed set in $\mathcal{U}$. That is, the inverse image of every nano open set in $\mathcal{V}$ is nano closed in $\mathcal{U}$. Therefore f is nano contra continuous.

Definition 5.3. A function $f : \mathcal{U} \to \mathcal{V}$ is said to be

(i) Nano perfectly continuous. if $f^{-1}(V)$ is nano clopen in $\mathcal{U}$ for every nano open set V in $\mathcal{V}$.

(ii) Nano strongly continuous. if $f^{-1}(V)$ is nano clopen in $\mathcal{U}$ for every subset V in $\mathcal{V}$.

(iii) Nano α-continuous. if $f^{-1}(V)$ is nano α-open in $\mathcal{U}$ for every nano α- open set V in $\mathcal{V}$.

(iv) Nano pre-continuous. if $f^{-1}(V)$ is nano pre-open in $\mathcal{U}$ for every nano open set V in $\mathcal{V}$.

Theorem 5.4. *For a function $f : (\mathcal{U}, \tau_R(X)) \to (\mathcal{V}, \tau_R(Y))$ the following conditions are equivalent.*

(i) f is nano contra continuous.

(ii) The inverse image of each nano closed set in $\mathcal{V}$ is nano open in $\mathcal{U}$.

(iii) For each $x \in \mathcal{U}$ and each nano closed set B in $\mathcal{V}$ with $f(x) \in B$, there exists a nano open set A in $\mathcal{U}$ such that $f(A) \subseteq B$.

(iv) $f(\mathcal{N}(cl(A)) \subseteq \mathcal{N}Kerf(A)$ for every subset A of $\mathcal{U}$.

(v) $\mathcal{N}cl[f^{-1}(B)] \subseteq f^{-1}[\mathcal{N}Ker(B)]$.

Proof. $(i) \Rightarrow (ii)$. Let f be nano contra continuous. Let B be a nano closed set in $\mathcal{V}$ and therefore B^C is nano open in $\mathcal{V}$. By (i) $f^{-1}(B^C)$ is nano closed in $\mathcal{U}$. But, $f^{-1}(B^C) = \{f^{-1}(B)\}^C$. Hence $f^{-1}(B)$ is nano open in $\mathcal{U}$.

$(ii) \Rightarrow (i)$. Let B be a nano open set in $\mathcal{V}$. Then B^C is nano closed in $\mathcal{V}$. By (ii) $f^{-1}(B^C)$ is nano open in $\mathcal{U}$. Hence $f^{-1}(B)$ is nano closed in $\mathcal{U}$. Hence f is nano contra continuous.

$(ii) \Rightarrow (iii)$. Let B be a nano closed set such that $f(x) \in B$. By (ii) $x \in f^{-1}(B)$ which is nano open. Let A $= f^{-1}(B)$. Then $x \in A$ and $f(A) \subseteq B$.

$(iii) \Rightarrow (ii)$. Let B be any nano closed set in $\mathcal{V}$ and $x \in f^{-1}(B)$. Then $f(x) \in B$ and there exists a nano open set $\mathrm{U}_x \in \mathcal{NO}(X, x)$ such that $f(\mathrm{U}_x) \subseteq B$. Therefore $f^{-1}(B) = \cup\{\mathrm{U}_x.x \in f^{-1}(B)\} \in \mathcal{N}O(X)$.

$(iii) \Rightarrow (iv)$. Let A be any subset of $\mathcal{U}$. If $y \notin kerf(A)$, then by Theorem 3.3 there exists $B \in \mathcal{N} \subseteq (\mathcal{V}, f(x))$ such that $f(A) \cap (B) = \emptyset$. Thus we have $A \cap f^{-1}(B)=\emptyset$ and since $f^{-1}(B)$ is nano open we have $\mathcal{N}cl(A) \cap f^{-1}(B) = \emptyset$. Therefore, we obtain $f(\mathcal{N}(cl(A)) \cap B = \emptyset$ and hence $y \notin f(\mathcal{N}cl(A))$. This implies that $f[\mathcal{N}(cl(A)] \subseteq \mathcal{N}Ker[f(A)]$.

$(iv) \Rightarrow (v)$. Let B be any subset of $\mathcal{V}$. By(iv) and Theorem 3.3 we have $f[\mathcal{N}cl(f^{-1}(B))] \subseteq kerf[f^{-1}(B)] \subseteq kerB$. Thus $\mathcal{N}cl[f^{-1}(B)] \subseteq f^{-1}[\mathcal{N}Ker(B)]$.

$(v) \Rightarrow (i)$. Let B be any nano open set of $\mathcal{V}$. Then by, Theorem 3.3 we have $\mathcal{N}cl[f^{-1}(B)] \subseteq f^{-1}[\mathcal{N}Ker(B)]$ and $\mathcal{N}cl[f^{-1}(B)] = f^{-1}(B)$. This proves that $f^{-1}(B)$ is nano closed in $\mathcal{U}$. □

Remark 5.5. The concept of nano continuity and nano contra continuity are independent of each other as shown in the following example.

Example 5.6. Let $\mathcal{U} = \{a, b, c, d, e\}$ with $\mathcal{U}/R = \{\{a, c\}, \{b\}, \{d\}, \{e\}\}$ and let $X = \{a, d, e\} \subseteq \mathcal{U}$. Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{d, e\}, \{a, c, d, e\}, \{a, c\}\}$. The nano closed sets are $\mathcal{U}, \emptyset, \{a, b, c\}, \{b\}, \{a, c\}$. Let $\mathcal{V} = \{x, y, z, w\}$ with $\mathcal{V}/R' = \{\{x\}, \{y, z\}, \{w\}\}$. Let $Y = \{x, z\} \subseteq \mathcal{V}$.

Then $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{x\}, \{x, y, z\}, \{y, z\}\}$. Define $f : \mathcal{U} \to \mathcal{V}$ as $f(a) = x, f(b) = w, f(c) = x, f(d) = z, f(e) = y.$ Then $f^{-1}(\{x\}) = \{a, c\}$, $f^{-1}(\{x, y, z\}) = \{a, c, d, e\}$, $f^{-1}(\{y, z\}) = \{d, e\}$. Hence f is a nano continuous function, but not nano contra continuous function. Because, $f^{-1}(\{x\}) = \{a, c\}$ is not nano closed in $\mathcal{U}$,where $\{x\}$ is nano open in $\mathcal{U}$. Therefore f is nano continuous but not nano contra continuous.

Example 5.7. Let $\mathcal{U} = \{a, b, c, d, e\}$ with $\mathcal{U}/R = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$ and let $X = \{a, b\} \subseteq \mathcal{U}$. Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$. The nano closed sets are $\mathcal{U}, \emptyset, \{c, d, e\}$. Let $\mathcal{V} = \{x, y, z, u, v\}$ with $\mathcal{V}/R' = \{\{u, v\}, \{x, z\}, \{y\}\}$. Let $Y = \{x, y, z\} \subseteq \mathcal{V}$. Then $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{x, y, z\}\}$. Define f: $\mathcal{U} \to \mathcal{V}$ as $f(a) = u, f(b) = v, f(c) = x, f(d) = y, f(e) = z$. Thus $f^{-1}(\{x, y, z\}) = \{c, d, e\}$ is a nano closed in $\mathcal{U}$ and not a nano open set in $\mathcal{U}$. Hence f is nano contra continuous function, but not nano continuous function.

Theorem 5.8. *Every nano strongly continuous function is nano contra continuous.*

Proof. Let B be a subset of $\mathcal{V}$. Since f is nano strongly continuous, $f^{-1}(B)$ is a nano clopen in $\mathcal{U}$. That is $f^{-1}(B)$ is both nano closed and also nano open in $\mathcal{U}$. Since B is any subset of $\mathcal{V}$ and $f^{-1}(B)$ is nano closed in $\mathcal{U}$. Then f is nano contra continuous. □

Remark 5.9. The converse of Theorem 5.8 need not be true which can be shown by the following example.

Example 5.10. Consider Example 5.7, the function f: $\mathcal{U} \to \mathcal{V}$ is nano contra continuous. Consider $\{y, z\}$ to be any subset of $\mathcal{U}$. Then $f^{-1}(\{y, z\}) = \{d, e\}$ where it is not nano clopen in $\mathcal{U}$. Therefore f is nano contra continuous but not strongly continuous.

Theorem 5.11. *Every nano perfectly continuous function is nano contra continuous function.*

Proof. Let B be a nano open subset of $\mathcal{V}$. Since f is nano perfectly continuous, $f^{-1}(B)$ is a nano clopen in $\mathcal{U}$. That is $f^{-1}(B)$ is nano closed in $\mathcal{U}$. Since B is a nano open subset of $\mathcal{V}$ and $f^{-1}(B)$ is nano closed in $\mathcal{U}$. Then f is nano contra continuous. □

Remark 5.12. The converse of Theorem 5.11 need not be true as shown in the following example.

Example 5.13. Let $\mathcal{U} = \{a, b, c, d\}$ with $\mathcal{U}/R = \{\{a\}, \{b, c\}, \{d\}\}$ and let $X = \{a, d\} \subseteq \mathcal{U}$. Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{a, d\}\}$. Then nano closed sets are $\mathcal{U}, \emptyset, \{b, c\}$. Let $\mathcal{V} = \{x, y, z, w\}$ with $\mathcal{V}/R' = \{\{x, z\}, \{y, w\}\}$. Let $Y = \{x, z\} \subseteq \mathcal{V}$. Then $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{x, z\}\}$. Define $f \colon \mathcal{U} \to \mathcal{V}$ as $f(a) = y, f(b) = x, f(c) = z, f(d) = w$ then $f^{-1}(\{x, z\}) = \{b, c\}$ which is a nano closed set and not a nano open set in $\mathcal{U}$. Hence f is nano contra continuous and not nano perfectly continuous.

Theorem 5.14. *For a function $f \colon \mathcal{U} \to \mathcal{V}$ the following conditions are equivalent*

- (i) *f is nano perfectly continuous.*
- (ii) *f is nano continuous and nano contra continuous.*
- (iii) *f is nano α-continuous and nano contra continuous.*
- (iv) *f is nano pre-continuous and nano contra continuous.*

Proof. $(i) \Rightarrow (ii)$. Let B be a nano open set in $\mathcal{V}$. Since f is nano perfectly continuous. $f^{-1}(B)$ is a nano clopen set in $\mathcal{U}$. Hence $f^{-1}(B)$ is nano open and nano closed in $\mathcal{U}$. Therefore f is both nano continuous and nano contra continuous.

$(ii) \Rightarrow (iii)$. Given that f is nano continuous and nano contra continuous. Since, every nano continuous map is nano α-continuous, f is nano α-continuous.

$(iii) \Rightarrow (iv)$. Given that f is nano α-continuous and nano contra continuous. Since every nano α-continuous map is nano pre-continuous, f is nano pre-continuous.

$(iv) \Rightarrow (i)$. Given that f is nano pre-continuous and nano contra continuous. To prove that f is nano perfectly continuous. Let B be a nano open set in $\mathcal{V}$. by(iv) $f^{-1}(B)$ is both nano pre-open and nano closed in $\mathcal{V}$. Hence $f^{-1}(B) \subseteq \mathcal{N}Int\mathcal{N}cl[f^{-1}(B)] \subseteq \mathcal{N}Int[f^{-1}(B)]$. Since $f^{-1}(B)$ is nano closed in $\mathcal{V}$. But $\mathcal{N}Int[f^{-1}(B)] \subseteq f^{-1}(B)$. Therefore $\mathcal{N}Int[f^{-1}(B)] = f^{-1}(B)$ and hence $f^{-1}(B)$ is nano open in $\mathcal{V}$. So, $f^{-1}(B)$ is nano open and nano closed in $\mathcal{V}$. Thus f is nano perfectly continuous. □

Theorem 5.15. *Every nano strongly continuous function is both nano continuous and nano contra continuous.*

Proof. Let B be an arbitrary set in $\mathcal{V}$. Since f is nano strongly continuous. $f^{-1}(B)$ is nano clopen in $\mathcal{U}$. That is $f^{-1}(B)$ is both nano open as well as nano closed in $\mathcal{U}$. Since it holds for every subset of $\mathcal{V}$, it is also true for all the nano open sets in $\mathcal{V}$. Therefore clearly f is nano continuous and nano contra continuous. □

Remark 5.16. From the above discussion we have the following table which gives the relationship between different types of nano continuous functions. The symbol "1" in a cell means that a function corresponding row implies a function

Functions	A	B	C	D
A	1	0	0	0
B	1	1	0	1
C	1	1	1	1
D	0	0	0	1

(A) Nano continuous function (B) Nano perfectly continuous function (C) Nano strongly continuous function (D) Nano contra continuous function

on the corresponding column. The symbol "0" means that a function on the corresponding row does not imply a function on the corresponding column.

Remark 5.17. The composition of two nano contra continuous function need not be nano contra continuous as the following example shows.

Example 5.18. Let $\mathcal{U} = \{a, b, c, d, e\}$ with $\mathcal{U}/R' = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$, $\mathcal{U}/R" = \{\{a, b\}, \{c, e\}, \{d\}\}, \mathcal{U}/R''' = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}\}$. Let $X = \{d, e\} \subseteq \mathcal{U}$, $Y = \{c, d, e\} \subseteq \mathcal{U}$, $Z = \{a, b\} \subseteq \mathcal{U}$. Then $\tau_{R'}(X) = \{\mathcal{U}, \emptyset, \{d, e\}\}$, $\tau_{R"}(X) = \{\mathcal{U}, \emptyset, \{c, d, e\}\}$, $\tau_{R'''}(X) = \{\mathcal{U}, \emptyset, \{a, b\}\}$. Then $f : (\mathcal{U}, \tau_{R'}(X)) \to (\mathcal{U}, \tau_{R''}(Y))$ defined by $f(a) = c, f(b) = d, f(c) = e, f(d) = a, f(e) = b$. $g : (\mathcal{U}, \tau_{R''}(Y)) \to (\mathcal{U}, \tau_{R'''}(Z))$ be defined as an identity function. Here f and g are nano contra continuous functions. But $(gof)^{-1}(\{a, b\}) = f^{-1}(\{g^{-1}(\{a, b\})\}) = f^{-1}(\{a, b\}) = \{d, e\}$, which is not nano closed in $[\mathcal{U}, \tau_{R'}(X)]$. Hence gof is not nano contra continuous.

Theorem 5.19. *Let f: $\mathcal{U} \to \mathcal{V}$ and g: $\mathcal{V} \to \mathcal{W}$ be the functions then gof is nano contra continuous if g is nano continuous and f is nano contra continuous.*

Proof. Let g be a nano continuous and f a nano contra continuous function. Suppose B is a nano open set in $\mathcal{W}$. Since g is nano continuous $g^{-1}(B)$ is nano open in $\mathcal{V}$. Since f is nano contra continuous, $f^{-1}(g^{-1}((B)))$ is nano closed in $\mathcal{U}$. That is $(gof)^{-1}(B)$ is nano closed in $\mathcal{U}$. Hence gof is nano contra continuous. □

6. Nano Bi-contra continuity

In this section, we define nano bi-contra continuous functions and derive some results involving its characterizations.

Definition 6.1. Let f: $\mathcal{U} \to \mathcal{V}$ be a surjective map. Then f is called a nano bi-contra continuous map if f is nano contra continuous and $f^{-1}(B)$ is nano open in $\mathcal{U}$ implies B is nano closed in $\mathcal{V}$.

Example 6.2. Let $\mathcal{U} = \{a, b, c, d, e\}$ with $\mathcal{U}/R = \{\{a, b\}, \{c, d\}, \{e\}\}$ and let $X = \{c, d\} \subseteq \mathcal{U}$. Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{c, d\}\}$. The nano closed sets are $\mathcal{U}, \emptyset, \{a, b, e\}$. Let $\mathcal{V} = \{x, y, z, u, v\}$ with $\mathcal{V}/R' = \{\{x, y\}, \{z, u, v\}\}$. Let $Y =$

$\{z, u, v\} \subseteq \mathcal{V}$. Then $\tau_{R'}(Y) = \{\mathcal{V}, \emptyset, \{z, u, v\}\}$. Define $f\colon \mathcal{U} \to \mathcal{V}$ by $f(a) = u, f(b) = v, f(c) = x, f(d) = y, f(e) = z$. Now the inverse image of nano open set in $\mathcal{V}$ is $f^{-1}(\{z, u, v\}) = \{a, b, e\}$ which is a nano closed set in $\mathcal{U}$. Hence f is nano contra continuous. That is $f^{-1}(\{x, y\}) = \{c, d\}$ which is a nano open set in $\mathcal{U}$. Hence f is nano bi-contra continuous.

Theorem 6.3. *Let $f\colon \mathcal{U} \to \mathcal{V}$ be a nano bi-contra continuous and an onto mapping. If A is a subset of $\mathcal{U}$ which is both nano closed and nano open, then the restriction $f_A : \{A, \tau_R A(X)\} \to \{\mathcal{V}, \tau_{R'}(Y)\}$ is a nano bi-contra continuous map.*

Proof. Given that f is nano bi-contra continuous. Let B be a nano open set in $\mathcal{V}$, then $f^{-1}(B)$ is nano closed in $\mathcal{U}$. Because f is nano contra continuous. Since A is nano closed and nano open, $f^{-1}(B) \cap A$ is nano closed in A. Also $f_A^{-1}(B) = f^{-1}(B) \cap A$ is nano closed in A. Hence $f_A^{-1}(B)$ be nano open in A and $f_A^{-1}(B) = f^{-1}(B) \cap A$. Since A is nano clopen, $f^{-1}(B)$ is nano open in $\mathcal{U}$ and since f is nano bi-contra continuous, B is nano closed in $\mathcal{V}$. □

Theorem 6.4. *Let $f\colon \mathcal{U} \to \mathcal{V}$ be nano open and nano bi-contra continuous, and $g\colon \mathcal{V} \to \mathcal{W}$ is nano continuous and nano bi-contra continuous, then the composition $gof\colon \mathcal{U} \to \mathcal{V}$ is nano bi-contra continuous map.*

Proof. Let B be nano open in $\mathcal{W}$. Since g is nano continuous $g^{-1}(B)$ is open in $\mathcal{V}$. Since f is nano bi-contra continuous $f^{-1}[g^{-1}(B)]$ is nano closed in $\mathcal{U}$. $(gof)^{-1}(B) = f^{-1}[g^{-1}(B)]$, so $(gof)^{-1}(B)$ is nano closed in $\mathcal{U}$. $f^{-1}[g^{-1}(B)]$ is nano open in $\mathcal{U}$, since f is nano open, $f(f^{-1}(g)^{-1}(B)) = g^{-1}(B)$ is nano open in $\mathcal{V}$. Again g is nano contra continuous. So B is nano closed in $\mathcal{W}$. Hence $(gof)^{-1}(B)$ is nano bi-contra continuous. □

7. Application

Nano contra continuous and nano bi-contra continuous function can be used to define contra and two way contra effects in bio-technology. To illustrate this we have shown a real life example.

Consider the anti-viruses as a treatment for the disease causing viruses. Let $\mathcal{V} = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ be the universe of viruses of three diseases namely Flu-virus, Polio-virus and Hepatitis-virus. In the sequel, $v_1 - A/H5N1Flu$, $v_2 - A/HN1 - Flu$, $v_3 - Polio$, $v_4 - Hepatitis - C$, $v_5 - Hepatitis - B$, $v_6 - Hepatitis - D$ and let $\mathcal{A} = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ be the universe of anti-viruses for three diseases namely Flu-virus, Polio-virus and Hepatitis-virus are a_1 as Arbidol, a_2 as Amantadine, a_3 as Sabin, a_4 as Interferon, a_5 as Rebetol, a_6 as Alpha-Interferon. We know that anti-virus depends on the disease causing virus. Let $\mathcal{U}/R = \{\{v_1, v_2, v_3\}, \{v_4, v_5, v_6\}\}$ and $X = \{v_1, v_2, v_3\} \subseteq \mathcal{V}$. Then $\tau_R(X) = \{\mathcal{U}, \emptyset, \{v_1, v_2, v_3\}\}$. The nano closed sets are $\mathcal{V}, \emptyset, \{v_4, v_5, v_6\}$. Let $\mathcal{A}/R' = \{\{a_1, a_2\}, \{a_3\}, \{a_4, a_5, a_6\}\}$ instead and $Y = \{a_4, a_5, a_6\} \subseteq \mathcal{A}$. Then $\tau_{R'}(Y) = \{\mathcal{A}, \emptyset, \{a_4, a_5, a_6\}\}$. Define $f\colon \mathcal{V} \to \mathcal{A}$ by $f(v_1) = a_1, f(v_2) = a_2$,

$f(v_3) = a_3, f(v_4) = a_4, f(v_5) = a_5, f(v_6) = a_6$. Then $f^{-1}(\mathcal{A}) = \mathcal{V}$, $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(\{a_4, a_5, a_6\}) = \{v_4, v_5, v_6\}$. That is, the inverse image of every nano open set in $\mathcal{A}$ is nano closed in $\mathcal{V}$. Therefore f is nano contra continuous. Also, $f^{-1}(\{a_1, a_2, a_3\}) = \{v_1, v_2, v_3\}$ which is a nano open set in $\mathcal{V}$. That is, the inverse image of every nano closed set in $\mathcal{A}$ is nano open in $\mathcal{V}$. Therefore, the anti-viruses as a function of treatment for disease causing viruses is both nano contra continuous and nano bi-contra continuous.

Conclusion

These types of mappings (nano contra continuous and nano bi-contra continuous mappings) will be of much use in biotechnology, where they need contra or two way contra effects. This mapping is obtained by fixing a contra mapping between a set of viruses to a set of anti-viruses and another contra mapping between the negative viruses to the positive viruses of the anti-virus set.

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