Hybrid-based mathematical method for enhancing the quantitative research

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Abstract. This study aims to discuss a special structure that can be exploited in the construction of efficient solution. The advantage of their methods usually has been the need to solve larger problems than otherwise would be possible to solve with computer technology. Two methods are discussed in this paper to determine a best method to solve these problems, and to determine which one is has a best result. We used : Network work method and Transportation problem method. Two methods were applied in general system to evaluate the result and compare between them. Also, the researcher discussed an example on a transportation problems. The results of this study indicated which one of the method has a simple steps in the solution.

Keywords: network work method, transportation problem, efficient solution.

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1. Introduction

Operations Research is related with different fields as mathematics, statistics, economics, psychology, engineering. Also operation research used to make a new decision. The commerce field is developing and growing rapidly, and increasing the aware competency between all organizations in the world, and striving to be more successes, survival and achieving to a competitive advantage. The rst of these special method to be analyzed was the transportation problem, which is a particular type of network problem. Also, all organizations need to minimize the cost and time, and increase their margin profit. Recently, the development of method to find the solution of the transportation problem in a simple method has become a major idea in applied mathematical programming For these reasons, this study aims to link between the solution of a transportation problem transshipment problem, and simplex method. For more details about these methods refer to (Mangazarian, 1969), and (Rockefeller, 1970).

2. Literature review

Operations research as a science has been used to help solve decision problems using mathematical and statistical models for a long time, and it has been developed in many scientific fields such as : Mathematics, Engineering and Management. It is one of the areas that has contributed to solving many of mathematical problems and management problems: transportation problem and Network method.

Network method is the most important of the special method in linear programming problems. In this paper, we present a general formulation of network and transportation problems, and we formulate some example to the connect between two methods. So, we used to convert the formulation of these methods to a simplex formulation.

In 1736, Euler discussed a new solution of the problem that such a path does not exist. This solution is considered to be the first theorem of graph theory, specifically of planar graph theory, but the first book of operations researches appeared in 1946 As "Methods of Research Operations" for Morris and Campbell's. Some of the scientists developed a method of problem solving in the Simplex Method and other methods. In (Khachian, 1979) proposed a new method of solving the linear program, but theoretically only. In (Karmarkar,1984) developed a new polynomial-time algorithm for linear programming and introduced an algebraic method with high results but the rest of the simplex is the easiest, and in (C. Simon and L. Blume, 1994) you can study about anything in Mathematics for Economists. Also (Holman and Robert, 1995) introduced a new application about the linear programming and applied a different branch in operation research. There many application are related with these method for example in (K. Fagerholt, 1999), he dicuss an optimal design in a ship routing problem.

The applications continued to be widely period-intensive until (Hillier, 2001), (Hamdy, 2007), and other spresented a new application in operation research. In (Reeb, J. and S. Leavengood, 2002), they studied a special for linear programming problems in a transportation problem. (Wayne Winston, 2004), presented an applications and algorithms in some of operation research models. Also, in (Ahuja, R. K., Magnanti, T. L.Orlin, J. B.,1993), (Baillon, J.B., Cominetti, R., 2008), (Fosgerau, M., Frejinger, E., Karlstrom, A., 2013),(Heuberger, C.,2004), and (Airoldi, E. M., Blocker, A. W., 2013) discussed a different ideas in these topics.

In (Vaidya and Kasturiwale, 2016) discussed a new approach while solving two phase simplex method, and they discussed this subject with respect to a number of iteration. Also, for more studies between Simplex method and other methods see (Ahmed Alsaraireh and et al, 2018), and (Mohammad Almasarweh and et. Al, 2018).

All applications remained separate and all studies discussed the development of simplex method, network and transportation problem, this study discussed the link between all mathematical models in problem solving Through the previous studies were based on separate applications and did not apply some of the methods on the same problem. Therefore, it is necessary to apply mathematical methods in order to compare the results and determine the most appropriate method of the solution. The paper is organized in two sections, section 3 is devoted to general formulate and discuss some steps of methods. In section 4, numerical example for problems are discussed.

3. Problem formulation

A linear programming problem is very important in different fields as Mathematics, Engineering, management, and others. Any linear programming problems consist from an objective function with a single variable or multi-variables,and the constraints with linear qualities or linear inequalities. The computation is a simple of a linear programming problem with respect to a nonlinear programming problem.

Here we will present a general formulation of the transportation problem, formulation of network models to connect the previous models with a simplex model. This formulation is very important for all researcher in an application science, especially in management and business.

Now, Denote the following figure presents a general model a transportation problem. Where $b_1, b_2, ..., b_n \equiv$ Demand quantities, $a_1, a_2, ..., a_m \equiv$ Supply quantities, $c_{ij} \equiv$ unit cost from i to j unit and $a_1, a_2, ..., a_m \equiv$ Number of units transferred.

We must check if the model is balanced or not. If the model is not balanced, add a dummy row or column.

The cost of each cell in a dummy row or column will be zero.

Sources	Destinations				Supply
	1	2	… …..	n	
1	C_{11}	C_{12}		C_{1n}	a_1
2	C_{21}	C_{22}		C_{2n}	a_2
3	C_{31}	C_{32}		C_{3n}	a_3
⋮	⋮	⋮	⋮	⋮	⋮
m	C_{m1}	C_{m2}		C_{mn}	a_m
Demand	b_1	b_2		b_n	**Sum**

Also, the following model presents an equivalent model for transportation problems by simplex model. Also, the following model presents an equivalent model for transportation problems by simplex model.

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \text{ Subject to: } \begin{cases} \sum_{j=1}^{n} x_{ij} \leqslant a_i \\ \sum_{i=1}^{m} x_{ij} \geq b_j \\ x_{ij} \geq 0 \end{cases}$$

If $\sum a_i = \sum b_j$, then model will be

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \text{ Subject to: } \begin{cases} \sum_{j=1}^{n} x_{ij} = a_i \\ \sum_{i=1}^{m} x_{ij} = b_j \\ x_{ij} \geq 0 \end{cases}$$

If $\sum a_i \neq \sum b_j$, then model will be

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \text{ Subject to: } \begin{cases} \sum_{j=1}^{n+1} x_{ij} = a_i \\ \sum_{i=1}^{m} x_{ij} = b_j \\ x_{ij} \geq 0 \end{cases}$$

or

$$\text{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij} \text{ Subject to: } \begin{cases} \sum_{j=1}^{n} x_{ij} = a_i \\ \sum_{i=1}^{m+1} x_{ij} = b_j \\ x_{ij} \geq 0 \end{cases}$$

From previous models, we used all cases to convert from a transportation model to simplex model.

The transport problem studies the method of transport from the source site to the requested site directly, but in some cases, the transport goes through stages. Therefore, we connect this problem the mathematical programming. so, we can present the transport and shipping data in terms of costs, demand quantities, and supply quantities.

The following figure presents the general model of the transshipment model and transportation network in two ways (Direct path and un-direct path), but we will discuss in this paper un-directed path.

Figure (1): Directed Network

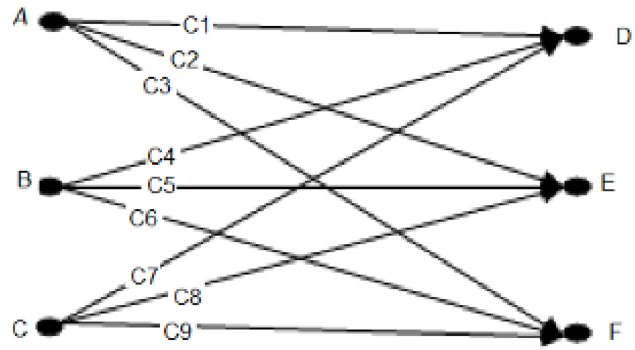
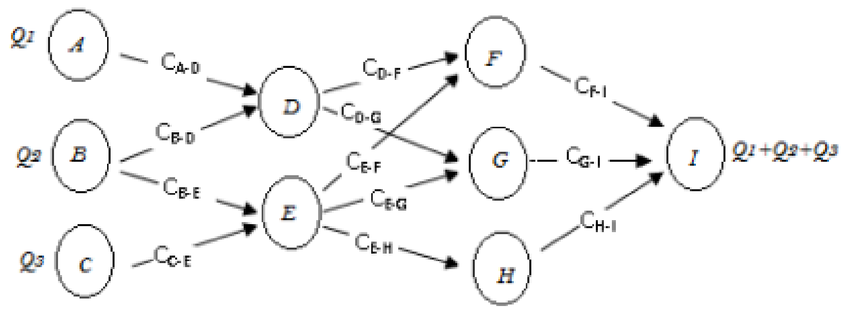


Figure (2): Undirected Network



In this paper, we will discuss undirected path and will try to connect it with a transportation table and simplex table. The first step, we will present the previous network by a linear-programming formulation through the simplex table. From the table we can write an equivalent a transportation problems to

Nodes	x_{AD}	x_{BD}	x_{BE}	x_{CE}	x_{DF}	x_{DG}	x_{EF}	x_{EG}	x_{EH}	x_{FI}	x_{GI}	x_{HI}	*RHS*
A	1												Q_1
B	1		1										Q_2
C				1									Q_3
D	−1	−1			1	1							0
E		−1		−1			1	1	1				0
F					−1		−1			1			0
G						−1		−1			1		0
H									−1			1	0
I										−1	−1	−1	$-\sum_{i=1}^{3} Q_i$
Obj. fun.	c_{AD}	c_{BD}	c_{BE}	c_{CE}	c_{DF}	c_{DG}	c_{EF}	c_{EG}	c_{EH}	c_{FI}	c_{GI}	c_{HI}	

the first model by the following equations. At each node we will depend on the following formula:

$$Flow\ out\ of\ a\ node - Flow\ into\ of\ a\ node = 0$$

The following equation presents the initial points A,B,C:

$$x_{AD} = Q_1, \tag{1}$$

$$x_{BD} + x_{BE} = Q_2, \tag{2}$$

$$x_{CE} = Q_3, \tag{3}$$

But the following equation presents the other nodes D,E,..,H:

$$x_{AD} + x_{BD} = x_{DF} + x_{DG}, \tag{4}$$

$$\vdots$$

$$x_{EH} = x_{HI}. \tag{5}$$

The equation of the last node I:

$$x_{FI} + x_{GI} + x_{HI} = Q_1 + Q_2 + Q_3. \tag{6}$$

Entering a dummy variable with a nonnegative variable and suppose a positive quantity(w) for the internal nodes to make all the equations are balanced, as:

$$x_{AD} + x_{BD} = x_{DF} + x_{DG} = W, \tag{7}$$

$$\vdots$$

$$x_{EH} = x_{HI} = W. \tag{8}$$

Now, we will display the equations in a transportation table to find an optimal solution of the system, and we notice that the table represents the general model to display any network in a transportation table. From the previous table, there

	D	E	F	G	H	I	**Supply**
A	C_{AD}	k	k	k	k	k	Q_1
B	C_{BD}	C_{BE}	k	k	k	k	Q_2
C	k	C_{CE}	k	k	k	k	Q_3
D	0	k	C_{DF}	C_{DG}	k	k	W
E	K	0	C_{EF}	C_{EG}	C_{EH}	k	W
F	k	k	0	k	k	C_{FI}	W
G	k	k	k	0	k	C_{GI}	W
H	k	k	k	k	0	C_{HI}	W
Demand	W	W	W	W	W	$\sum_1^3 Q_i$	$\sum_1^3 Q_i + 5W$

exist some variable such that,

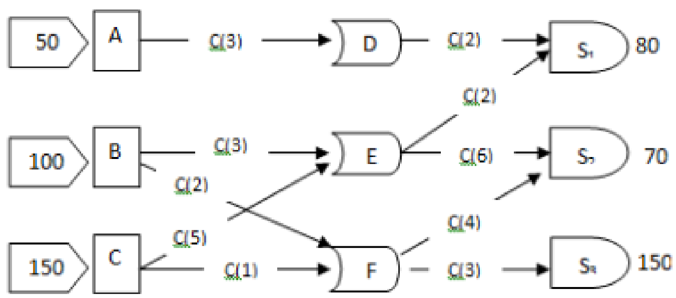
W ≡ The large positive number to make a problem is balanced,

K ≡ The large cost to fill all missing cost.

To find the solution to this problem, we can write this problem in a simplex model, or we can solve it directly by transportation methods.

4. Numerical example

The following table presents the distribution plan from three companies A(50 units), B(100 units), and C(150 units) to four customers S1(80 units), S2(70 units), and S3(150 units). The distribution through four intermediary companies. Find the optimal plan for distribution from companies to customers such that we get the lowest transfer cost. Now, we will display the previous problem



in a simplex model

$$x_{AD} = 50, \tag{9}$$
$$x_{BE} + x_{BF} = 100, \tag{10}$$
$$x_{CE} + x_{CF} = 150 \tag{11}$$

and,

$$x_{DS_1} = x_{AD}, \tag{12}$$
$$x_{ES_1} + x_{ES_1} = x_{BE} + x_{CE}, \tag{13}$$
$$x_{FS_2} + x_{FS_3} = x_{BF} + x_{CF}, \tag{14}$$
$$x_{DS_1} + x_{ES_1} = 80, \tag{15}$$
$$x_{ES_2} + x_{FS_2} = 80, \tag{16}$$
$$x_{FS_3} = 150. \tag{17}$$

	x_{AD}	x_{BE}	x_{BF}	x_{CE}	x_{CF}	x_{DS_1}	x_{ES_1}	x_{ES_2}	x_{FS_2}	x_{FS_3}	R.H.S
A	1										50
B		1	1								100
C				1	1						150
D	-1					1					0
E		-1		-1			1	1			0
F			-1		-1				1	1	0
S_1						-1	-1				-80
S_2								-1	-1		-70
S_3										-1	-150
Obj. Fun.	3	3	2	5	1	2	2	6	4	3	

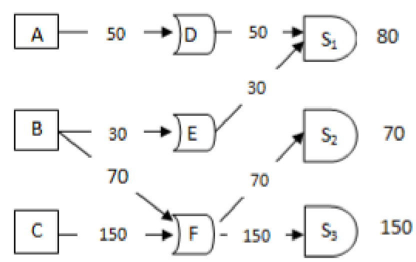
The following table represents the previous problem in a transportation table such that, k means missing link. We have a different methods to solve this

	D	E	F	S_1	S_2	S_3	Supply
A	3	k	k	k	k	k	50
B	k	3	2	k	k	k	100
C	k	5	1	k	k	k	150
D	0	k	k	2	k	k	300
E	K	0	k	2	6	k	300
F	k	k	0	k	4	3	300
Demand	300	300	300	80	70	150	

problem and to compare it with a solution of network.So, we used a maximum flow to solve this problem.

	From	To	Shipment
1	A	D	50
2	B	E	30
3	B	F	70
4	C	F	150
5	D	D	250
6	D	S1	50
7	E	E	270
8	E	S1	30
9	F	F	80
10	F	S2	70
11	F	S3	150

Total Obj. Fun. = 1420



5. Conclusion

In this study we discussed the link between network method, simplex method and transportation problem. Also, we presented the main idea to write any transportation problem in a network and simplex model. The general of formulation for three models is related to gather in a simple procedure. Network method and simplex method are suitable methods to solve any transportation problem. Also, we explained the main steps and the important conditions in simplex model and network model. Simplex model is important to write a transportation problem and network in a simplex model. Also we discussed an example on a linear programming problem. This study attempts to investigate the application of more than one mathematical model in problem solving and work on discussion. From the result of this paper we notice that the three methods are related to gather in a strong procedure, and every one can used these steps to compare it with another methods.

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