On normalizers of Sylow subgroups and p-nilpotency of finite groups

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Abstract. In this paper, we prove the p-nilpotency of a finite group under the assumption that some subgroups of Sylow subgroups are partial CAP-or c-supplemented in their normalizers. Our results unify and generalize some earlier results.

Keywords: partial CAP-subgroup, c-supplemented subgroup, maximal subgroup, p-nilpotency, formation.

1. Introduction

In this paper, all groups considered are finite and G stands for a finite group. Let $\pi(G)$ stand for the set of all prime divisors of the order of G. Let $\mathcal{F}$ denote a formation, $\mathcal{U}$ denote the class of supersolvable groups. The other notation and terminology are standard(see [9]).

Let H be a subgroup of G and A/B a G-chief factor. We say, following Gaschütz in [5], that H covers A/B if $HA = HB$; H avoids A/B if $H \cap A = H \cap B$. H is said to be cover-avoiding in G, in brevity, H is a CAP-subgroup of G, if H either covers or avoids any G-chief factor. In [2], H is a partial CAP-subgroup of G if there exists a chief series Γ_A of G such that H either covers or avoids each factor of Γ_A. Partial CAP-subgroups have also been called semi CAP-subgroups in [4]. Many authors presented some conditions for a group to be p-nilpotent or supersolvable under the condition that some subgroups of Sylow subgroups are partial CAP-subgroups, for details, the readers are referred to [7], [11] and [13]. Recall that a subgroup H is said to be c-supplemented (in G) if there exists a subgroup K of G such that $G = HK$ and $H \cap K \leq H_G$. Many authors presented some conditions for a group to be solvable, p-nilpotent

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and supersolvable under the condition that some subgroups of Sylow subgroups are c-supplemented subgroups (see[3], [6] and [14]).

On the other hand, the normalizers of Sylow subgroups of a group play an important role in the structure of groups. A question of particular interest in studying finite groups is the influence of the structure and embedding of the normalizers of the Sylow subgroups on the structure of the group. A nice example is the celebrated Burnside′s Theorem.

Theorem 1.1 (Burnside). *Let P be a Sylow p-subgroup of G. If $N_G(P) = C_G(P)$, then G is p-nilpotent.*

The following extension of Burnside′s Theorem due to Hall is also interesting.

Theorem 1.2 ([8]). *Let P be a Sylow p-subgroup of G. If p'-elements of $N_G(P)$ commute with the elements of P and the nilpotency class of P is less than p, then G is p-nilpotent.*

Wielandt, Ballester-Bolinches and Esteban-Romero proved the following results respectively.

Theorem 1.3 ([15]). *A group G is p-nilpotent if it has a regular Sylow p-subgroup whose G-normalizer is p-nilpotent.*

Theorem 1.4 ([1]). *A group G is p-nilpotent if it has a modular Sylow p-subgroup whose G-normalizer is p-nilpotent.*

The main purpose of the present paper is to analyze the structural consequences of the c-supplementation or the partial cover and avoidance property of the maximal subgroups of the Sylow subgroups in their normalizers.

2. Preliminary results

A subgroup H is said to be π-quasinormal (s-quasinormal or s-permutable) in G if $PH = HP$ for all Sylow subgroups P of G (see [10]).

Lemma 2.1 ([[10] and [12, Lemma 2.2]]). *Suppose that U is a π-quasinormal subgroup of G, $H \leq G$ and $K \trianglelefteq G$. Then*

(a) *If $U \leq H$, then U is π-quasinormal in H;*

(b) *UK is π-quasinormal in G and UK/K is π-quasinormal in G/K;*

(c) *Let $K \leq H$. If H/K is π-quasinormal in G/K, then H is π-quasinormal in G;*

(d) *If P is a π-quasinormal subgroup of G for some prime p, then $N_G(P) \geq O^p(G)$.*

The lemma presented below is crucial in the sequel. The proof is a routine check, and we omit its details.

Lemma 2.2. *Let H be a subgroup of G. Then*

(1) *Let $N \trianglelefteq G$ and $N \leq H$. If H is a partial CAP-or c-supplemented subgroup of G, then H/N is a partial CAP-or c-supplemented subgroup of G/N;*

(2) *Let π be a set of primes, H a π-subgroup and N a normal π'-subgroup of G. If H is a partial CAP-or c-supplemented subgroup of G, then HN/N is a partial CAP-or c-supplemented subgroup of G/N;*

(3) *Let $H \leq K$. If H is a partial CAP-or c-supplemented subgroup of G, then H is a partial CAP-or c-supplemented subgroup of K;*

(4) *Let $N \trianglelefteq G$. If H is a CAP-subgroup of G, then HN/N is a CAP-subgroup of G/N.*

Lemma 2.3 ([18, Theorem 3.1]). *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and G a group with a solvable normal subgroup N such that $G/N \in \mathcal{F}$. Assume that every Sylow subgroup of $F(N)$ is cyclic, then $G \in \mathcal{F}$.*

Lemma 2.4 ([17, Theorem 3.1]). *Let G be a group and p a prime dividing the order of G with $(|G|, p-1) = 1$. If there exists a Sylow p-subgroup P of G such that every maximal subgroup of P is a partial CAP-or c-supplemented subgroup of G, then G is p-nilpotent.*

3. Main results

Theorem 3.1. *Let G be a group and p a prime dividing the order of G with $(|G|, p-1) = 1$. If there exists a Sylow p-subgroup P of G such that every maximal subgroup of P is partial CAP-or c-supplemented subgroup of $N_G(P)$ and P' is π-quasinormal in G, then G is p-nilpotent.*

Proof. Assume that the result is false and let G be a counterexample of minimal order.

(1) Let L be a normal subgroup of G contained in P. Then G/L satisfies the hypothesis.

It is clear that $(|G/L|, p-1) = 1$. For any maximal subgroup P_1/L of P/L, $p = |P/L : P_1/L| = |P : P_1|$, so P_1 is a maximal subgroup of P. By the hypothesis, P_1 is a partial CAP-or c-supplemented subgroup of $N_G(P)$ and P' is π-quasinormal in G. By Lemma 2.2 (1), P_1/L is a partial CAP-or c-supplemented subgroup of $N_G(P)/L = N_{G/L}(P/L)$, and $(P/L)' = P'L/L$ is π-quasinormal in G/L by Lemma 2.1 (b), thus we have (1).

(2) $1 \neq P' \leq O_p(G)$ and G is solvable.

For any $Q \in Syl_q(N_G(P))$, where $q \neq p$. It is easy to see that all maximal subgroups of P are partial CAP-or c-supplemented subgroups of PQ by Lemma

2.2 (3). Thus PQ satisfies the hypothesis of Lemma 2.4, so PQ is p-nilpotent, hence $Q \leq C_G(P)$. Assume that P is abelian, then $N_G(P) = C_G(P)$, hence G is p-nilpotent by Burnside's Theorem, a contradiction. So $P' \neq 1$. By the hypothesis, P' is π-quasinormal in G, thus $P' \lhd \lhd G$, hence $O_p(G) \neq 1$. By (1), we have $G/O_p(G)$ is p-nilpotent. Since $(|G|, p-1) = 1$, we conclude that $G/O_p(G)$ is solvable, thus G is solvable.

(3) End of the proof.

Let $\{G_r \mid r \in \pi(G)\}$ be a Sylow system of G and $H = G_pG_r$ for any $r \in \pi(G)$ with $r \neq p$. By Lemma 2.1(a) and Lemma 2.2 (3), the hypothesis is still true for H. If $|\pi(G)| > 2$, then $G_r \trianglelefteq H$, which implies that G_p normalizes G_r for any $r \in \pi(G)$, hence G is p-nilpotent, a contradiction. Thus we may assume that $|G| = p^a q^b$.

Let L be a minimal normal subgroup of G. Since P' is π-qusinormal in G, by Lemma 2.1(b), we have $P'L$ is π-qusinormal in G and $P'L/L$ is π-qusinormal in G/L. If L is a q-group, then we consider the quotient group G/L. Evidently, $PL/L \in Syl_p(G/L)$. For any maximal subgroup T/L of PL/L, we have $p = |(PL/L) : (T/L)|$, and $T = PL \cap T = (P \cap T)L$. Let $P_1 = P \cap T$. Then $P_1 \cap L = P \cap T \cap L = P \cap L$, so

$$p = |PL : T| = |PL : (P \cap T)L| = |P : P \cap T| = |P : P_1|.$$

Thus P_1 is a maximal subgroup of P. By the hypothesis, P_1 is a partial CAP-or c-supplemented subgroup of $N_G(P)$. If P_1 is a partial CAP-subgroup of $N_G(P)$, by Lemma 2.2, it is easy to check that P_1L/L is also a partial CAP-subgroup of $N_{G/L}(PL/L) = N_G(P)L/L$. If P_1 is a c-supplemented subgroup of $N_G(P)$, then there exists a subgroup K of $N_G(P)$ such that $N_G(P) = P_1K$ and $P_1 \cap K \leq (P_1)_{N_G(P)}$. Obviously, we have $N_G(P)L/L = (P_1L/L)(KL/L)$. Since $(|P_1|, |L|) = 1$, we get

$$|P_1 \cap KL| = \frac{|P_1| \cdot |KL|_p}{|P_1KL|_p} = \frac{|P_1| \cdot |K|_p}{|N_G(P)L|_p} = \frac{|P_1| \cdot |K|_p}{|N_G(P)|_p} = |P_1 \cap K|.$$

This implies that $P_1 \cap KL = P_1 \cap K$, thus

$$\begin{aligned}(P_1L/L) \cap (KL/L) &= (P_1L \cap KL)/L = (P_1 \cap KL)L/L \\ &= (P_1 \cap K)L/L \leq (P_1)_{N_G(P)}L/L \leq (P_1L/L)_{N_{G/L}(PL/L)}.\end{aligned}$$

So P_1L/L is a partial CAP-or c-supplemented subgroup of $N_{G/L}(PL/L)$. By the minimality of G, G/L is p-nilpotent and so is G, a contradiction. Hence L is a p-group and $L \leq P$. By (1), G/L is p-nilpotent. Similarly, if N is another minimal normal subgroup of G, then $N \leq P$ and so G/N is also p-nilpotent. Now it follows that $G \cong G/N \cap L$ is p-nilpotent, a contradiction. Thus L must be the unique minimal normal subgroup of G. Since the class of p-nilpotent groups is a saturated formation, $L \nleq \Phi(G)$, hence $\Phi(G) = 1$. By [16, Thorem 5.3], we get $O_p(G) = F(G) = L$.

Since $P^{'}$ is π-quasinormal in G, we have $N_G(P^{'}) \geq O^p(G)$ by Lemma 2.1 (d), and P normalizes $P^{'}$, we get $P^{'} \trianglelefteq G$, so $P^{'} = O_p(G) = L$ by the unique minimal normality of L. By (1), we have $G/O_p(G)$ is p-nilpotent, hence $O_p(G)Q \trianglelefteq G$, where $Q \in Syl_q(G)$. Since $O_p(G)Q \cap P = O_p(G) = P^{'} \leq \Phi(P)$, $O_p(G)Q$ is p-nilpotent by J. Tate Theorem([9, Theorem 4.4.7]). Thus $Q \trianglelefteq O_p(G)Q \trianglelefteq G$, which implies that $Q \trianglelefteq G$, so G is p-nilpotent, a contradiction. This final contradiction completes our proof. □

Theorem 3.2. *Let G be a group and p a prime dividing the order of G with $(|G|, p-1) = 1$. Suppose that H is a normal subgroup of G such that G/H is p-nilpotent. If there exists a Sylow p-subgroup P of H such that every maximal subgroup of P is a partial CAP-or c-supplemented subgroup of $N_G(P)$ and $P^{'}$ is π-quasinormal in G, then G is p-nilpotent.*

Proof. Assume that the result is false. Let (G, H) be a counterexample with $|G| + |H|$ minimal.

By Lemma 2.1 (a) and Lemma 2.2 (3), it is easy to see that every maximal subgroup of P is a partial CAP-or c-supplemented subgroup of $N_H(P)$ and $P^{'}$ is π-quasinormal in H, then H is p-nilpotent by Theorem 3.1. Let M be a normal p-complement of H. Then $M \trianglelefteq G$. Assume that $M \neq 1$. We consider the quotient group G/M. Similar to the proof of (3) in Theorem 3.1, it is easy to see that the hypothesis is still true for $(G/M, H/M)$, hence G/M is p-nilpotent and so is G, a contradiction. Thus we conclude that $M = 1$. Now $H = P$ is a p-subgroup. Let T/P be a normal p-complement of G/P. It is clear that every maximal subgroup of P is a partial CAP-or c-supplemented subgroup of $N_T(P)$ and $P^{'}$ is π-quasinormal in T, then T is p-nilpotent by Theorem 3.1, so $T_{p^{'}} \trianglelefteq T \trianglelefteq G$ and $T_{p^{'}}$ is also a Hall $p^{'}$-subgroup of G, thus $T_{p^{'}} \trianglelefteq G$, hence G is p-nilpotent, a contradiction. This final contradiction completes our proof. □

Corollary 3.1. *Let G be a group and p a prime dividing the order of G with $(|G|, p-1) = 1$. Suppose that H is a normal subgroup of G such that G/H is p-nilpotent. If there exists a Sylow p-subgroup P of H such that every maximal subgroup of P is a partial CAP-subgroup of $N_G(P)$ and $P^{'}$ is π-quasinormal in G, then G is p-nilpotent.*

Corollary 3.2. *Let G be a group and p a prime dividing the order of G with $(|G|, p-1) = 1$. Suppose that H is a normal subgroup of G such that G/H is p-nilpotent. If there exists a Sylow p-subgroup P of H such that every maximal subgroup of P is a c-supplemented subgroup of $N_G(P)$ and $P^{'}$ is π-quasinormal in G, then G is p-nilpotent.*

Theorem 3.3. *Let G be a group. For any prime factor p of $|G|$, there exists a Sylow p-subgroup P of G such that every maximal subgroup of P is a partial CAP-or c-supplemented subgroup of $N_G(P)$ and $P^{'}$ is π-quasinormal in G, then G is supersolvable.*

Proof. Assume that the result is false and let G be a counterexample of minimal order.

By Theorem 3.1, we know that G is a Sylow tower group of supersolvable type, so G is solvable. Let L be a minimal normal subgroup of G. Then L is an elementary r-subgroup, where $r \in \pi(G)$. By Lemma 2.2 (1) and (2), we have G/L satisfies the hypothesis, thus G/L is supersolvable by the minimal choice of G. Since the class of supersolvable subgroups is a saturated formation, we may assume that L is the unique minimal normal subgroup of G and $L \nleq \Phi(G)$. Hence there exists a maximal subgroup M of G such that $G = LM$ and $L \cap M = 1$. Let $q = max\pi(G)$ and $Q \in Syl_q(G)$. Then $Q \trianglelefteq G$, thus $L \leq Q$ by the unique minimal normality of L. Since $Q = O_q(G) \leq F(G) \leq C_G(L)$, L and M normalize $Q \cap M$, thus $Q \cap M \lhd G$. So $Q \cap M = 1$ or $L \leq Q \cap M$. If the later happens, then $L \leq M$, that is, $G = LM = M$, a contradiction. So $Q \cap M = 1$, and $L \cap M = 1$. This implies that $|Q| = |G : M| = |L|$, hence $L = Q$ and $N_G(Q) = G$. Let Q_1 be a maximal subgroup of Q. By the hypothesis, Q_1 is a partial CAP-or c-supplemented subgroup of $N_G(Q)$. Assume that Q_1 is a partial CAP-subgroup of $N_G(Q)$. By the unique minimal normality of L, $L/1$ is a chief factor of every chief series. Since $L = Q$, clearly, Q_1 can not covers $Q/1$, so Q_1 avoids $Q/1$, that is, $Q_1 \cap Q = 1$, thus $Q_1 = 1$, hence $|L| = |Q| = q$. By Lemma 2.3, we have G is supersolvable, a contradiction. So Q_1 is c-supplemented in $N_G(Q)$. By $N_G(Q) = G$, there exists a subgroup K of G such that $G = Q_1K$ and $Q_1 \cap K \leq (Q_1)_G$. By the unique minimal normality of L, we have $(Q_1)_G = 1$, so $Q_1 \cap K = 1$. On the other hand, since $Q \cap K \trianglelefteq \langle Q, K\rangle = G$, we have $Q \cap K = L = Q$ by the unique minimal normality of L, so $Q \leq K$. Thus $K = G$. So $Q_1 = Q_1 \cap K \leq (Q_1)_G = 1$, thus $|L| = |Q| = q$. By Lemma 2.3, we have G is supersolvable, a contradiction. This final contradiction completes our proof. □

Theorem 3.4. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and G a group with a normal subgroup H such that $G/H \in \mathcal{F}$. For any prime factor p of $|H|$, there exists a Sylow p-subgroup P of H such that every maximal subgroup of P is a CAP-or c-supplemented subgroup of $N_G(P)$ and P' is π-quasinormal in G, then $G \in \mathcal{F}$.*

Proof. Assume that the result is false. Let (G, H) be a counterexample with $|G| + |H|$ minimal.

Since a CAP-subgroup is also a partial CAP-subgroup, H satisfies the hypothesis of Theorem 3.3, hence H is supersolvable. Let $p = max\pi(H)$ and $P \in Syl_p(H)$. Then $P \trianglelefteq G$. We consider the quotient group G/P, then $G/H \cong (G/P)/(H/P) \in \mathcal{F}$. By Lemma 2.1 ($a$) and Lemma 2.2 (4), we have $(G/P, H/P)$ satisfies the hypothesis, thus $G/P \in \mathcal{F}$. Hence we may assume that $H = P$.

Let N be a minimal normal subgroup of G contained in P. By Lemma 2.1 (a) and Lemma 2.2 (4), the hypothesis is still true for $(G/N, P/N)$, so $G/N \in \mathcal{F}$. Since $\mathcal{F}$ is a saturated formation, $N \nleq \Phi(G)$. So there exists a maximal subgroup M of G such that $G = NM$ and $N \cap M = 1$. On the other hand, we can conclude that $\Phi(P) = 1$. Otherwise, we have $G/\Phi(P) \in \mathcal{F}$, then $G/\Phi(G) \cong (G/\Phi(P))/(\Phi(G)/\Phi(P)) \in \mathcal{F}$, so $G \in \mathcal{F}$, a contradiction. Therefore, P is an elementary abelian subgroup.

By $N \leq P$, we have $G = NM = PM$ and $P \cap M \lhd G$. If $P \cap M \neq 1$, then $N \leq P \cap M$, $N \leq M$, we get $G = NM = M$, a contradiction. Thus $P \cap M = 1$, so $P = N$ is a minimal normal subgroup of G. Let P_1 be a maximal subgroup of P. By the hypothesis, P_1 is a CAP-or c-supplemented subgroup of $N_G(P)$. Assume that P_1 is c-supplemented in $N_G(P)$. By $G = N_G(P)$, there exists a subgroup K of G such that $G = P_1K$ and $P_1 \cap K \leq (P_1)_G$. By the minimal normality of N and $N = P$, we have $(P_1)_G = 1$, $P_1 \cap K = 1$. On the other hand, we have $P = P \cap P_1K = P_1(P \cap K)$. Since $P \cap K$ is normal in K, P is abelian and $G = PK$, we have $P \cap K$ is a normal subgroup of G. The minimality of N implies that $P \cap K = P$, thus $K = G$. So $P_1 = 1$. This shows that P is a cyclic subgroup of order p, thus $G \in \mathcal{F}$ by Lemma 2.3, a contradiction. Hence P_1 is a CAP-subgroup of $N_G(P)$, then P_1 either covers or avoids the chief factor $N/1 = P/1$ of G. Clearly, P_1 can not cover $P/1$, so P_1 avoids $P/1$, that is, $P_1 \cap P = 1$, thus $P_1 = 1$, hence $|P| = p$. By Lemma 2.3, we have G is supersolvable, a contradiction. This final contradiction completes our proof. □

Recall that a group G was called an A-group if all of its Sylow subgroups are abelian. Let G be an A-group. Then for any $P \in Syl_p(G)$, we have $P^{'} = 1$, of course, it is π-quasinormal in G, so we have the following corollaries.

Corollary 3.3 ([13, Theorem 4.3])**.** *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and G a group with a normal A-subgroup H such that $G/H \in \mathcal{F}$. If every maximal subgroup of each Sylow subgroup P of H is cover-avoiding in $N_G(P)$, then $G \in \mathcal{F}$.*

Corollary 3.4. *Let $\mathcal{F}$ be a saturated formation containing $\mathcal{U}$ and G a group with a normal A-subgroup H such that $G/H \in \mathcal{F}$. If every maximal subgroup of each Sylow subgroup P of H is a c-supplemented subgroup of $N_G(P)$, then $G \in \mathcal{F}$.*

4. Remarks

Remark 4.1. The following example illustrates that the hypothesis in Theorem 3.1 that "$P^{'}$ is π-quasinormal in G" can not be removed.

Example Let $G = PSL_2(q)$, where $q \equiv \pm 1 \pmod 8$. Let P be a Sylow 2-subgroup of G. By [9, II, Theorem 8.27], we have the Sylow 2-subgroup of $PSL_2(q)$ is selfnormalizing in $PSL_2(q)$. Evidently, every maximal subgroup of P is normal in $N_G(P) = P$, so every maximal subgroup of P is a partial CAP-or c-supplemented subgroup of $N_G(P)$. However, G is not 2-nilpotent.

Remark 4.2. Even if G is a solvable group and p is an odd prime, the hypothesis in Theorem 3.1 that "P' is π-quasinormal in G" could not be omitted, either.

Example Let $H = Z_3 \times Z_3 \times Z_3$ be an elementary abelian 3-group. It is clear that $Aut(H)$ has a subgroup $Z_{13} \rtimes Z_3$. Now suppose that

$$G = (Z_3 \times Z_3 \times Z_3) \rtimes (Z_{13} \rtimes Z_3).$$

Let P_3 be a Sylow 3-subgroup of G. It is clear that $N_G(P_3) = P_3$, so every maximal subgroup of P_3 is a partial CAP-or c-supplemented subgroup of $N_G(P_3) = P_3$. However, G is not 3-nilpotent.

Acknowledgements

The authors want to express their gratitude to the referees for their helpful suggestions and comments. This work was supported by the National Natural Science Foundation of China (Grant N. 11601225, 11871360), the China Postdoctoral Science Foundation (N. 2015M582492) and the China Scholarship Council.

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Accepted: 12.01.2018