Blow up of solution for the Kelvin-Voigt type wave equation with Balakrishnan-Taylor damping and acoustic boundary

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Abstract. The purpose of this work is to study the blow up of solutions in finite time, for a nonlinear equation of the kelving voigt type with balakrishnan taylor damping and acoustic boundary in a bounded domain in $\mathbb{R}^n$.

Keywords: Kelvin-Voigt type, energy decay, Balakrishnan-Taylor damping, acoustic boundary, blow up.

1. Introduction

Let Ω be a bounded, connected set in $\mathbb{R}^n (n \geq 1)$ having a smooth boundary $\Gamma = \partial\Omega$ consisting of two parts Γ_0 and Γ_1 such that $\overline{\Gamma_0} \cup \overline{\Gamma_1} = \Gamma$. We consider the following problem:

$$
\begin{aligned}
&\left|u'\right|^m u'' - \left(a^2 + b\int_\Omega |\nabla u|^2\,dx + \sigma\int_\Omega \nabla u.\nabla u' dx\right)\Delta u - 2\lambda\Delta u' \\
&= |u|^{p-2}\,u \ \text{ in } \ \Omega\times\mathbb{R}^+, \\
&u = 0 \ \text{ in } \Gamma_0\times\mathbb{R}^+, \\
&\left(a^2 + b\int_\Omega |\nabla u|^2\,dx + \sigma\int_\Omega \nabla u.\nabla u' dx\right)\frac{\partial u}{\partial v} + 2\lambda\frac{\partial u'}{\partial v} = y' \ \text{ in } \Gamma_1\times\mathbb{R}^+,
\end{aligned}
$$

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$$u' + p(x)y' + q(x)y = 0 \ \text{ in } \Gamma_1 \times \mathbb{R}^+,$$
$$u(0) = v_0, \ u'(0) = u_1 \ \text{ in } \Omega, \tag{1.1}$$
$$y(0) = y_0 \ \text{ in } \Gamma_1.$$

where primes denote the time derivative, Δ the Laplacian in $\mathbb{R}^n$ taken in space variables, v the unit normal of Γ pointing towards exterior of Ω and $\mathbb{R}^+ = (0, \infty)$ The parameters $\lambda > 0$ is a small internal material damping coefficient, $a > 0$, $b > 0$, $\sigma > 0$. are constant real numbers p and q are functions satisfying

$$p(x) > 0, \quad q(x) > 0 \quad fo\ rall\ x \in \Gamma_1. \tag{1.2}$$

Physically, the first integrodifferential equations in (1.1) occurs in the study of vibrations of damped flexible space structures in bounded domain in $\mathbb{R}^n$. The nonlinear term $|u^{'}|^m u^{''}$, where $m > 0$ is expressed materials whose density depends on the velocity $u^{'}$, The term $2\lambda\Delta u^{'}$ is the internal material damping of Kelvin-Voigt type of the structure. The model in hand, with Balakrishnan-Taylor damping ($\sigma > 0$) and $\rho = 1$, was initially proposed by Balakrishnan and Taylor [1] in 1989 and Bass and Zes [2], this problem was studied by Zaraï and Tatar [17,18,19,20] and results concerning global existence, general decay and blow up of solution have been accomplished. The boundary conditions considered here are of mixed Dirichlet and Neumann type and acoustic boundary. The analytical studies in the area of stabilization of distributed parameter system is currently of interest in view of application to vibration control of various structural elements. The phenomenon was first observed by Hunton as reported by Harrison [8]. In the absence of the Kelvin-Voigt damping ($\lambda = 0$) and $m = \sigma = 0$, we have the well known Kirchhoff [11] equation

$$u'' = \left(a^2 + b \int_\Omega |\nabla u|^2 \, dx\right) \Delta u,$$

it was extensively studied by several authors cited in Zaraï and Tatar [20]. Beale and Rosencrans [3] introduced acoustic boundary conditions of the general form

$$\begin{aligned} &\frac{\partial u}{\partial \nu} = y' && on\ \Gamma_1 \times \mathbb{R}^+ \\ &\gamma u' + m(x)y'' + p(x)y' + q(x)y = 0 && on\ \Gamma_1 \times \mathbb{R}^+ \end{aligned}$$

Recently, wave equations with acoustic boundary conditions have been studied by many authors [4,5,9,14,15,16].

The above equation, without the nonlinear source term, has been investigated in Kang [10]. Using the multiplier technique, the author was prove an exponential decay result for the solution.

In this work we prove a finite time blow-up result of solutions, we will see that the direct method introduced and developed by Georgiev and Todorova [6], in 1994 and Salim A. Messaoudi [12,13] is efficient in our case. Combining this method with some necessary modifications due the nature of the problem treated here.

Our paper is organized as follows: in the next section we prepare some material needed in our proofs. Section 3 is devoted to the statement and proof of the finite time blow-up result.

2. Preliminaire

In this section, we present some notations and some material which will be needed in the proof of our results. Let C be the smallest positive constant independent of t (depends only on Ω) satisfying the Poincare inequality

$$\int_{\Omega} u^2 dx \leq C \int_{\Omega} |\nabla u|^2 \, dx. \tag{2.1}$$

And also let $\bar{k}$ be the smallest positive constant independent of t (depends only on Γ_1) satisfying for $u \in H^1(\Omega)$ the embedding inequality

$$\int_{\Gamma_1} u^2 d\Gamma \leq \overline{k} \int_{\Omega} |\nabla u|^2 \, dx. \tag{2.2}$$

Lemma 1. *Suppose u, $v : \mathbb{R}^n \to \mathbb{R}$ are Lebesgue measurable. Then*

$$\|u.v\|_1 < \|u\|_r \|v\|_s \tag{2.3}$$

Lemma 2. *For u, $v > 0$, we have*

$$uv \leq C\left(u^r + v^s\right), \tag{2.4}$$

for all $C > 0$ and $\frac{1}{r} + \frac{1}{s} = 1$.

For the functions p and q, we assume that $p, q \in C(\Gamma_1)$, $p(x) > 0$ and $q(x) > 0$, for all $x \in \Gamma_1$. This assumption implies that there exist positive constants p_i, q_i $(i = 0; 1)$ such that

$$p_0 \leq p(x) \leq p_1, \quad q_0 \leq q(x) \leq q_1, \quad for\ all\ x \in \Gamma_1 \tag{2.5}$$

By using Galerkin's approximation and the methods of Gorain [7] and Park [16], we can obtain existence result for the solution of (1.1) under the conditions on p and q as above.

3. Blowing up property

In this section we consider the blowing up property of the solution to problem (1.1). To this end, we use the method in [6]. We define the energy function of the solution to (1.1) by

$$\begin{aligned} E(t) &= \frac{1}{m+2}\left\|u^{'}\right\|^{m+2} + \frac{1}{2}\left(a^2 + \frac{b}{2}\|\nabla u\|^2\right)\|\nabla u\|^2 \\ &\quad -\frac{1}{p}\|u\|^p + \frac{1}{2}\int_{\Gamma_1} q(x)(y(t))^2 d\Gamma. \end{aligned} \tag{3.1}$$

Lemma 3. *Let the pair of functions (u, y) be the solution of problem (1.1), then $E(t)$ is a nonincreasing function on $[0, t)$ and*

$$\begin{aligned}\frac{dE(t)}{dt} &= -\sigma\left(\frac{1}{2}\frac{d}{dt}\|\nabla u\|^2\right)^2 - 2\lambda\left\|\nabla u'\right\|^2 + \int_{\Gamma_1}\left(u' + q(x)y\right)y'd\Gamma \\ &= -\sigma\left(\frac{1}{2}\frac{d}{dt}\|\nabla u\|^2\right)^2 - 2\lambda\left\|\nabla u'\right\|^2 - \int_{\Gamma_1}p(x)\left(y'\right)^2 d\Gamma \leq 0. \qquad (3.2)\end{aligned}$$

Proof. By multiplying the first equation in (1.1) by u', and integrating over Ω, we get

$$\begin{aligned}\int_\Omega \left|u'\right|^m u'u''dx &= \int_\Omega\left(a^2 + b\int_\Omega|\nabla u|^2dx + \sigma\int_\Omega \nabla u.\nabla u'dx\right)\Delta u u_t dx \\ &\quad +2\lambda\int_\Omega \Delta u' u'dx + \int_\Omega |u|^{p-2}uu'dx\end{aligned}$$

Therefore, using integration by parts, the conditions in the borders and some manipulations, we get

$$\begin{aligned}&\frac{d}{dt}\left[\frac{1}{m+2}\left\|u'\right\|_{m+2}^{m+2} + \frac{a^2}{2}\frac{d}{dt}\|\nabla u\|^2 + \frac{b}{4}\|\nabla u\|^4 - \frac{1}{p}\|u\|_p^p + \frac{1}{2}\int_{\Gamma_1}q(x)y^2d\Gamma\right] \\ &= -\frac{\sigma}{4}\left(\frac{d}{dt}\|\nabla u\|^2\right)^2 - 2\lambda\left\|\nabla u'\right\|^2 - \int_{\Gamma_1}p(x)\left(y'\right)^2d\Gamma.\end{aligned}$$

□

Theorem 1. *Suppose that $m > 1$, $p > 3\beta(m+1)$, assume that*

$$\begin{aligned}E_0 &= \frac{1}{m+2}\|u_1\|_{m+2}^{m+2} + \frac{1}{2}\left(a^2 + \frac{b}{2}\|\nabla u_0\|^2\right)\|\nabla u_0\|^2 \\ &\quad -\frac{1}{p}\|u_0\|^p + \frac{1}{2}\int_{\Gamma_1}q(x)(y(0))^2d\Gamma < 0. \qquad (3.3)\end{aligned}$$

Then the solution of problem (1.1) blow up in finite time

$$T^* \leq \frac{2(1-\alpha)}{(2\alpha-1)\gamma[L(0)]^{\frac{2\alpha-1}{2(1-\alpha)}}},$$

where

$$\beta = \max\left(\frac{1}{2} + \frac{\overline{k}q_1}{(p_0 a)^2}, 1\right).$$

Lemma 4. *There exists a positive constant $C > 1$ depending on Ω only such that*

$$\|u\|_p^s \leq C\left(\|\nabla u\|^2 + \|u\|_p^p\right), \qquad (3.4)$$

for any $2 \leq s \leq p$ and $u \in H^1(\Omega)$.

Proof. If $\|u\|_p \leq 1$ then $\|u\|_p^s \leq \|u\|_p^2 \leq C \|\nabla u\|_2^2$ by Soboleve embeding theorem.

If $\|u\|_p > 1$ then $\|u\|_p^s \leq \|u\|_p^p$. Therefore (3.4) follows. □

We set

$$H(t) = -E(t).$$

That's mean

$$H'(t) = \frac{\sigma}{4}\left(\frac{d}{dt}\|\nabla u\|^2\right)^2 + 2\lambda\left\|\nabla u'\right\|^2 + \int_{\Gamma_1} p(x)\left(y'\right)^2 d\Gamma \geq 0. \tag{3.5}$$

Consequently we have

$$0 < H(0) \leq H(t) \leq \frac{1}{p}\|u\|_p^p, \tag{3.6}$$

As a result of (3.1) and (3.4), we have

Corollary 1. *Let the assumptions of the lemma 4 hold. Then we have the following*

$$\begin{aligned} \|u\|_p^s \leq\; & C\left(-\frac{1}{m+2}\left\|u'\right\|_{m+2}^{m+2} - \|\nabla u\|^4\right. \\ & \left. - \int_{\Gamma_1} p(x)\left(y'\right)^2 d\Gamma - H(t) + \|u\|_p^p\right). \end{aligned} \tag{3.7}$$

for all $t \in (0,T)$, *for any* $u(.,t) \in H^1(\Omega)$ *and* $2 \leq s \leq p$.

By virtue of (3.1) and (3.3). We then define

$$L(t) = H^{2(1-\alpha)}(t) + \varepsilon\int_\Omega \left|u'\right|^m u'u dx. \tag{3.8}$$

for ε small positive to be chosen later and

$$\frac{1}{2} < \alpha < \frac{m(p-1)+3p-2}{2p(m+2)}. \tag{3.9}$$

By taking a derivative of (3.8) we get

$$\begin{aligned} L'(t) = {} & 2(1-\alpha)H^{(1-2\alpha)}(t)H'(t) + \varepsilon\int_\Omega \left|u'\right|^{m+2} dx \\ & + \varepsilon(m+1)\int_\Omega \left|u'\right|^m u'' u dx. \end{aligned} \tag{3.10}$$

By using the first equation in (1.1), we have

$$\begin{aligned}
&\varepsilon(m+1)\int_{\Omega}\left|u'\right|^{m}u''udx\\
&=-\varepsilon(m+1)a^{2}\left\|\nabla u\right\|^{2}-\varepsilon(m+1)\frac{\sigma}{2}\left(\frac{d}{dt}\left\|\nabla u\right\|^{2}\right)\left\|\nabla u\right\|^{2}\\
&-\varepsilon(m+1)b\left\|\nabla u\right\|^{4}-\varepsilon(m+1)-2\lambda\varepsilon(m+1)\int_{\Omega}\nabla u\nabla u'dx\\
&+\varepsilon(m+1)\int_{\Omega}|u|^{p}dx+\varepsilon(m+1)\int_{\Gamma_1}uy'dx. \qquad (3.11)
\end{aligned}$$

By using the fourth equation in (1.1) we have

$$\varepsilon(m+1)\int_{\Gamma_1}uy'd\Gamma=-\varepsilon(m+1)\int_{\Gamma_1}\frac{1}{p(x)}u'ud\Gamma-\varepsilon(m+1)\int_{\Gamma_1}\frac{q(x)}{p(x)}yud\Gamma. \qquad (3.12)$$

By replacing (3.11) and (3.12) in (3.10) we get

$$\begin{aligned}
L'(t) &= 2(1-\alpha)H^{(1-2\alpha)}(t)H'(t)+\varepsilon\left\|u'\right\|_{m+2}^{m+2}+\varepsilon(m+1)\left\|u\right\|_{p}^{p}\\
&-\varepsilon(m+1)a^{2}\left\|\nabla u\right\|^{2}-\varepsilon(m+1)\frac{\sigma}{2}\left(\frac{d}{dt}\left\|\nabla u\right\|^{2}\right)\left\|\nabla u\right\|^{2}\\
&+\varepsilon(m+1)\int_{\Omega}|u|^{p}dx-2\lambda\varepsilon(m+1)\int_{\Omega}\nabla u\nabla u'dx\\
&-\varepsilon(m+1)b\left\|\nabla u\right\|^{4}-\varepsilon(m+1)\int_{\Gamma_1}\frac{1}{p(x)}u'ud\Gamma\\
&-\varepsilon(m+1)\int_{\Gamma_1}\frac{q(x)}{p(x)}yud\Gamma. \qquad (3.13)
\end{aligned}$$

from (3.1) we have

$$\begin{aligned}
2\varepsilon\left\|u\right\|_{p}^{p} &= 2\varepsilon pH(t)+\frac{2\varepsilon p}{m+2}\left\|u'\right\|_{m+2}^{m+2}+\varepsilon pa^{2}\left\|\nabla u\right\|^{2}\\
&+\varepsilon\frac{pb}{2}\left\|\nabla u\right\|^{4}+\varepsilon p\int_{\Gamma_1}q(x)y^{2}d\Gamma,
\end{aligned}$$

so, we can write

$$\begin{aligned}
\varepsilon(m+1)\left\|u\right\|_{p}^{p} &= \varepsilon(m-1)\left\|u\right\|_{p}^{p}+2\varepsilon pH(t)+\frac{2\varepsilon p}{m+2}\left\|u'\right\|_{m+2}^{m+2}+\varepsilon pa^{2}\left\|\nabla u\right\|^{2}\\
&+\varepsilon\frac{pb}{2}\left\|\nabla u\right\|^{4}+\varepsilon p\int_{\Gamma_1}q(x)y^{2}d\Gamma. \qquad (3.14)
\end{aligned}$$

By replacing (3.14) in (3.13) we get

$$\begin{aligned} L^{'}(t) &= 2(1-\alpha)H^{(1-2\alpha)}(t)H^{'}(t)+\varepsilon\left(1+\frac{2p}{m+2}\right)\left\|u^{'}\right\|_{m+2}^{m+2} \\ &+\varepsilon a^{2}\left[p-(m+1)\right]\|\nabla u\|^{2}+\varepsilon b\left[\frac{p}{2}-(m+1)\right]\|\nabla u\|^{4} \\ &+2\varepsilon pH(t)+\varepsilon\left(m-1\right)\|u\|_{p}^{p}-\varepsilon(m+1)\frac{\sigma}{2}\left(\frac{d}{dt}\|\nabla u\|^{2}\right)\|\nabla u\|^{2} \\ &-2\lambda\varepsilon(m+1)\int_{\Omega}\nabla u\nabla u^{'}dx-\varepsilon(m+1)\int_{\Gamma_1}\frac{q(x)}{p(x)}yud\Gamma \\ &-\varepsilon(m+1)\int_{\Gamma_1}\frac{1}{p(x)}u^{'}ud\Gamma+\varepsilon p\int_{\Gamma_1}q(x)y^{2}dx. \end{aligned} \tag{3.15}$$

On the other hand by using (2.1) and (2.2), we find

$$\begin{gathered} -\varepsilon(m+1)\int_{\Gamma_1}\tfrac{1}{p(x)}u^{'}ud\Gamma\geq \\ -\varepsilon(m+1)\tfrac{\varepsilon_1\overline{k}}{2p_0}\left\|\nabla u^{'}\right\|^{2}-\varepsilon(m+1)\tfrac{\overline{k}}{2\varepsilon_1p_0}\|\nabla u\|^{2}, \end{gathered} \tag{3.16}$$

$$\begin{gathered} -\varepsilon(m+1)\int_{\Gamma_1}\tfrac{q(x)}{p(x)}uyd\Gamma\geq \\ -\varepsilon(m+1)\tfrac{q_1\varepsilon_2}{2p_0}\overline{k}\|\nabla u\|^{2}-\varepsilon(m+1)\tfrac{1}{2p_0\varepsilon_2}\int_{\Gamma_1}q(x)y^{2}dx, \end{gathered} \tag{3.17}$$

$$\begin{gathered} -\varepsilon\left(m+1\right)\tfrac{\sigma}{2}\left(\tfrac{d}{dt}\|\nabla u\|^{2}\right)\|\nabla u\|^{2}\geq \\ -\varepsilon\left(m+1\right)\tfrac{\varepsilon_3}{4}\sigma\|\nabla u\|^{4}-\varepsilon\left(m+1\right)\tfrac{\sigma}{4\varepsilon_3}\left(\tfrac{d}{dt}\|\nabla u\|^{2}\right)^{2}, \end{gathered} \tag{3.18}$$

and

$$\begin{gathered} -2\varepsilon\left(m+1\right)\lambda\int_{\Omega}\nabla u\nabla u^{'}dx\geq \\ -\varepsilon\left(m+1\right)\lambda\varepsilon_4\|\nabla u\|^{2}-\varepsilon\left(m+1\right)\lambda\tfrac{1}{\varepsilon_4}\left\|\nabla u^{'}\right\|^{2}, \end{gathered} \tag{3.19}$$

by putting the last inequalities in (3.15), we get

$$\begin{aligned} L^{'}(t) &\geq \left[2\left(1-\alpha\right)H^{1-2\alpha}(0)-\varepsilon\left(m+1\right)\frac{1}{\varepsilon_3}\right]\left(\frac{d}{dt}\|\nabla u\|^{2}\right)^{2}+\varepsilon\|u\|_{p}^{p} \\ &+\varepsilon\left\|u^{'}\right\|_{m+2}^{m+2}+\varepsilon pH(t)+(m+1)\,b\left[\frac{p}{2\left(m+1\right)}-\left(1+\frac{\varepsilon_3}{4b}\sigma\right)\right]\|\nabla u\|^{4} \\ &+\lambda\left[2\left(1-\alpha\right)H^{1-2\alpha}(0)-\varepsilon\left(m+1\right)\left(\frac{\varepsilon_1\overline{k}}{2p_0}+\lambda\frac{1}{\varepsilon_4}\right)\right]\left\|\nabla u^{'}\right\|^{2} \\ &+\varepsilon\left(m+1\right)\left[a^{2}\left(\frac{p}{(m+1)}-1\right)-\frac{\overline{k}}{2p_0}\left(\frac{1}{\varepsilon_1}+q_1\varepsilon_2+\frac{2p_0}{\overline{k}}\lambda\varepsilon_4\right)\right]\|\nabla u\|^{2} \\ &+\varepsilon\left[p-(m+1)\frac{1}{2p_0\varepsilon_2}\right]\int_{\Gamma_1}q(x)y^{2}d\Gamma. \end{aligned} \tag{3.20}$$

We choose $L(0) = H^{2(1-\alpha)}(0) + \varepsilon \int_\Omega u_0 u_1 > 0$, and

$$\begin{aligned}
a_1 &= 2(1-\alpha) H^{1-2\alpha}(0) - \varepsilon (m+1) \frac{1}{\varepsilon_3}, \\
a_2 &= 2(1-\alpha) H^{1-2\alpha}(0) - \varepsilon (m+1) \left(\frac{\varepsilon_1 \overline{k}}{2p_0} + \lambda \frac{1}{\varepsilon_4} \right), \\
a_3 &= a^2 \left(\frac{p}{(m+1)} - 1 \right) - \frac{\overline{k}}{2p_0} \left(\frac{1}{\varepsilon_1} + q_1 \varepsilon_2 + \frac{2p_0}{\overline{k}} \lambda \varepsilon_4 \right), \\
a_4 &= \frac{p}{2(m+1)} - \left(1 + \frac{\varepsilon_3}{4b} \sigma \right), \\
a_5 &= p - (m+1) \frac{1}{2p_0 \varepsilon_2},
\end{aligned}$$

We choose also $\varepsilon_1 = \frac{2\overline{k}}{a^2 p_0}, \varepsilon_2 = \frac{2}{p_0}, \varepsilon_3 = \frac{2b}{\sigma}, \varepsilon_4 = \frac{a^2}{4\lambda}$, and

$$\varepsilon < \frac{2(1-\alpha) H^{1-2\alpha}(0)}{(m+1)} \min \left(\frac{2b}{\sigma}, \frac{(p_0 a)^2}{\overline{k}^2 + (2\lambda p_0 a)^2} \right),$$

so a_i $(i = 1,2,3,4,5)$ are positive.

Finely, we get

$$\begin{aligned}
L'(t) \geq\ & C \left(\left(\frac{d}{dt} \|\nabla u\|^2 \right)^2 + \|\nabla u\|^4 + \left\| \nabla u' \right\|^2 \right. \\
& \left. + \|\nabla u\|^2 + \left\| u' \right\|_{m+2}^{m+2} + H(t) + \|u\|_p^p \right). \qquad (3.21)
\end{aligned}$$

This implies that

$$L(t) \geq L(0) > 0, \quad \text{for all } t \geq 0. \qquad (3.22)$$

Now, we estimate $|\int_\Omega |u'|^m u_t u dx|^{\frac{1}{2(1-\alpha)}}$, by using the Cauchy-Schwarz's inequality, Holder inequality and Young's inequality, we get

$$\left| \int_\Omega \left| u' \right|^m u_t u dx \right|^{\frac{1}{2(1-\alpha)}} \leq C_2 \left[\left\| u' \right\|_{m+2}^{m+2} + \|u\|_p^{\left(\frac{m+2}{[m(1-2\alpha)-(4\alpha-3)]} \right)} \right]. \qquad (3.23)$$

For $m > 1$ we have $2 \leq s = \frac{m+2}{[m(1-2\alpha)-(4\alpha-3)]} \leq p$, so by using of Corollary 1, we have

$$\left| \int_\Omega \left| u' \right|^{m+1} u dx \right|^{\frac{1}{2(1-\alpha)}} \leq C_2 \left[\left\| u' \right\|_{m+2}^{m+2} + \|u\|_p^p \right]. \qquad (3.24)$$

Therefore, we conclude

$$\begin{aligned} L^{\frac{1}{2(1-\alpha)}}(t) &= \left(H^{2(1-\alpha)}(t) + \varepsilon \int_\Omega u u_t dx\right)^{\frac{1}{2(1-\alpha)}} \\ &\leq \delta \left(H(t) + \left\|u'\right\|_{m+2}^{m+2} + \|u\|_p^p\right), \quad \text{for all } t \geq 0. \end{aligned} \tag{3.25}$$

By combining (3.21) and (3.25) we arrive for some positive cnstant $\gamma > C > \delta$

$$L'(t) \geq \gamma L^{\frac{1}{2(1-\alpha)}}(t), \quad \text{for all } t \geq 0. \tag{3.26}$$

A simple integration of (3.26) over $(0, t)$ then yields

$$L^{\frac{2\alpha-1}{2(1-\alpha)}}(t) \geq \frac{1}{-\frac{2\alpha-1}{2(1-\alpha)}\gamma t + L^{-\frac{2\alpha-1}{2(1-\alpha)}}(0)}. \tag{3.27}$$

Shows that $L(t)$ blows up in a finite time. The proof of Theorem 1 is now completed.

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