Upper and lower nearly (I, J)-continuous multifunctions

E. Rosas[*]
Departamento de Matemáticas
Universidad De Oriente
Cumaná, Venezuela
and
Universidad de la Costa
Departamento de Ciencias Naturales y Exactas
Barranquilla
Colombia
ennisrafael@gmail.com
erosas@cuc.edu.co

C. Carpintero
Departamento de Matemáticas
Universidad de Oriente
Cumaná
Venezuela
and
Corporación Universitaria del Caribe-CECAR
Sincelejo
Colombia
carpintero.carlos@gmail.com

J. Moreno
Corporación Universitaria Remington
Facultad de Ciencias Básicas e Ingeniería
Medellin
Colombia
jbmorenobarrios@gmail.com

Abstract. In this paper the authors introduce and study upper and lower nearly (I, J)-continuous multifunctions. Some characterizations and several properties concerning upper (lower) nearly (I, J)-continuous multifunctions are obtained. The results improves many results in Literature.

Keywords: nearly (I, J)-continuous multifunctions, I-open set, I-closed set, lower nearly (I, J)-continuous multifunctions, upper almost nearly (I, J)-continuous multifunctions.

∗. Corresponding author

1. Introduction

It is well known today, that the notion of multifunction is playing a very important role in general topology, upper and lower continuity have been extensively studied on multifunctions $F : (X, \tau) \to (Y, \sigma)$. Currently using the notion of topological ideal, different types of upper and lower continuity in multifunction $F : (X, \tau, I) \to (Y, \sigma)$ have been studied and characterized [2], [7], [8], [9], [14], [17]. The concept of ideal topological spaces has been introduced and studied by Kuratowski [12] and the local function of a subset A of a topological space (X, τ) was introduced by Vaidyanathaswamy [16] as follows: Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called the local function of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(\tau, I) = \{x \in X \ / \ U \cap A \notin I$ for every $U \in \tau_x\}$, where $\tau_x = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $cl^*(,)$ for a topology $\tau^*(\tau, I)$ called the *-topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\tau, I)$. We will denote $A^*(\tau, I)$ by A^*. In 1990, Jankovic and Hamlett [10], introduced the notion of I-open set in a topological space (X, τ) with an ideal I on X. In 1992, Abd El-Monsef et al. [1] further investigated I-open sets and I-continuous functions. In 2007, Akdag [2], introduce the concept of I-continuous multifunctions in a topological space with and ideal on it. Given a multifunction $F : (X, \tau) \to (Y, \sigma)$, and two ideals I, J on X and Y respectively. Now with the topological spaces (X, τ, I) and (Y, σ, J), consider the multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$. We want to study some type of upper and lower continuity of F. In this paper, we introduce and study a new class of multifunction called a nearly (I, J)-continuous multifunctions in topological spaces. Investigate its relation with another class of continuous multifunctions given in the Literature.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces in which no separation axioms are assumed, unless explicitly stated and if I is an ideal on X, (X, τ, I) mean an ideal topological space. For a subset A of (X, τ), $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. A subset A is said to be regular open [15] (resp. semiopen [11], preopen [13] , semi preopen [4]) if $A = \mathrm{int}(\mathrm{cl}(A))(resp. A \subseteq \mathrm{cl}(\mathrm{int}(A)), A \subseteq \mathrm{int}(\mathrm{cl}(A)), A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))))$. The complement of regular open (resp. semiopen, semi-preopen) set is called regular closed (resp. semiclosed, semi-preclosed) set. A subset S of (X, τ, I) is an I-open [10], if $S \subseteq \mathrm{int}(S^*)$. The complement of an I-open set is called I-closed set. The I-closure and the I-interior, can be defined in the same way as $\mathrm{cl}(A)$ and $\mathrm{int}(A)$, respectively, will be denoted by $I\,\mathrm{cl}(A)$ and $I\mathrm{int}(A)$, respectively. The family of all I-open (resp. I-closed, regular open, regular closed, semiopen, semi closed, preopen, semi-preclosed) subsets of a (X, τ, I), denoted by $IO(X)$ (resp.

$IC(X), RO(X), RC(X), SO(X), SC(X), PO(X), SPO(X), SPC(X)$). We set $IO(X,x) = \{A : A \in IO(X)$ and $x \in A\}$. It is well known that in a topological space (X, τ, I), $X^* \subseteq X$ but if the ideal is codense, that is $\tau \cap I = \emptyset$, then $X \subseteq X^*$.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, also we always assume that $F(x) \neq \varnothing$ for all $x \in X$. For a multifunction $F : X \to Y$, the upper and lower inverse of any subset A of Y, denoted by $F^+(A)$ and $F^-(A)$, respectively, that is $F^+(A) = \{x \in X : F(x) \subseteq A\}$ and $F^-(A) = \{x \in X : F(x) \cap A \neq \varnothing\}$. In particular, $F^+(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$.

Definition 2.1 ([2])**.** *A multifunction $F : (X, \tau, I) \to (Y, \sigma)$ is said to be*

1. *upper I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I-open set U containing x such that $U \subseteq F^+(V)$.*

2. *lower I-continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an I-open set U containing x such that $U \subseteq F^-(V)$.*

3. *I-continuous if it is both upper I-continuous and lower I-continuous.*

Definition 2.2 ([6])**.** *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be*

1. *upper semi continuous at a point $x \in X$ if for each open set V of Y with $F(x) \in V$, there exists an open set U containing x such that $F(U) \subseteq V$.*

2. *lower semi continuous at a point $x \in X$ if for each open set V of Y with $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $F(a) \cap V \neq \emptyset$ for all $a \in U$.*

Definition 2.3. *A subset A of a topological space (X, τ) is said to be N-closed [6] if every cover of A by regular open sets of X has a finite subcover.*

Definition 2.4 ([8])**.** *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be:*

1. *upper nearly continuous at a point $x \in X$ if for each open set V containing $F(x)$ and having N-closed complement, there exists an open set U containing x such that $F(U) \subset V$.*

2. *lower nearly continuous at a point $x \in X$ if for each open set V of Y meeting $F(x)$ and having N-closed complement, there exists an open set U of X containing x such that $F(u) \cap V \neq \emptyset$ for each $u \in U$.*

3. *upper (resp. lower) nearly continuous on X if it has this pro-perty at every point of X.*

Definition 2.5 ([9])**.** *A multifunction $F : (X, \tau) \to (Y, \sigma)$ is said to be:*

1. *upper almost nearly continuous at a point* $x \in X$ *if for each open set* V *containing* $F(x)$ *and having* N*-closed complement, there exists an open set* U *containing* x *such that* $F(U) \subset int(\mathrm{cl}(V))$.

2. *lower almost nearly continuous at a point* $x \in X$ *if for each open set* V *of* Y *meeting* $F(x)$ *and having* N*-closed complement, there exists an open set* U *of* X *containing* x *such that* $F(u) \cap int(\mathrm{cl}(V)) \neq \emptyset$ *for each* $u \in U$.

3. *upper (resp. lower) almost nearly continuous on* X *if it has this property at every point of* X.

Definition 2.6 ([5])**.** *A multifunction* $F : (X, \tau, I) \to (Y, \sigma)$ *is said to be:*

1. *upper nearly* I*-continuous at a point* $x \in X$ *if for each open set* V *containing* $F(x)$ *and having* N*-closed complement, there exists an* I*-open set* U *containing* x *such that* $F(U) \subset V$.

2. *lower nearly* I*-continuous at a point* $x \in X$ *if for each open set* V *of* Y *meeting* $F(x)$ *and having* N*-closed complement, there exists an* I*-open set* U *of* X *containing* x *such that* $F(u) \cap V \neq \emptyset$ *for each* $u \in U$.

3. *upper (resp. lower) nearly* I*-continuous on* X *if it has this property at every point of* X.

Definition 2.7 ([7])**.** *A multifunction* $F : (X, \tau, I) \to (Y, \sigma)$ *is said to be:*

1. *upper almost nearly* I*-continuous at a point* $x \in X$ *if for each open set* V *containing* $F(x)$ *and having* N*-closed complement, there exists an* I*-open set* U *containing* x *such that* $F(U) \subset int(\mathrm{cl}(V))$.

2. *lower almost nearly* I*-continuous at a point* $x \in X$ *if for each open set* V *of* Y *meeting* $F(x)$ *and having* N*-closed complement, there exists an* I*-open set* U *of* X *containing* x *such that* $F(u) \cap int(\mathrm{cl}(V)) \neq \emptyset$ *for each* $u \in U$.

3. *upper (resp. lower) almost nearly* I*-continuous on* X *if it has this property at every point of* X.

3. Upper and lower nearly (I, J)-continuous multifunctions

Definition 3.1. *A multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$ *is said to be:*

1. *upper nearly* (I, J)*-continuous at a point* $x \in X$ *if for each* J*-open set* V *containing* $F(x)$ *and having* N*-closed complement, there exists an* I*-open set* U *containing* x *such that* $F(U) \subset V$.

2. *lower nearly* (I, J)*-continuous at a point* $x \in X$ *if for each* J*-open set* V *of* Y *meeting* $F(x)$ *and having* N*-closed complement, there exists an* I*-open set* U *of* X *containing* x *such that* $F(u) \cap V \neq \emptyset$ *for each* $u \in U$.

3. *upper (resp. lower) nearly (I,J)-continuous on X if it has this property at every point of X.*

Example 3.2. Let $X = Y = \{a,b,c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$, $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $f : (X,\tau,I) \to (Y,\sigma,J)$ as follows: $f(a) = \{a\}$, $f(b) = \{c\}$ and $f(c) = \{b\}$. It is easy to see that:

The set of all I-open is $\{\emptyset, X, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$.

The set of all J-open is $\{\emptyset, \{a,b\}, \{a,c\}\}$.

In consequence, f is upper nearly (I,J)-continuous on X.

Example 3.3. Let $X = Y = \{a,b,c\}$ with two topologies $\tau = \{\emptyset, X, \{b,c\}\} = \sigma$ and two ideals $I = \{\emptyset, \{b\}\} = J$. Define a multifunction $f : (X,\tau,I) \to (Y,\sigma,J)$ as follows: $f(a) = \{a\}$, $f(b) = \{c\}$ and $f(c) = \{b\}$. It is easy to see that:

The set of all I-open is $\{\emptyset, X\{c\}, \{a,c\}, \{b,c\}\}$.

In consequence, f is not upper nearly (I,J)-continuous.

Recall that if (X,τ,I) is an ideal topological space and I is the empty ideal, then for each $A \subseteq X$, $A^* = cl(A)$, that is to said, every I-open set is a pre-open set, in consequence, if $f : (X,\tau,I) \to (Y,\sigma,\{\emptyset\})$ is upper nearly $(I,\{\emptyset\})$-continuous, then f is upper nearly I-continuous.

Example 3.4. $f : (X,\tau,I) \to (Y,\sigma)$ upper nearly I-continuous but $f : (X,\tau,I) \to (Y,\sigma,\{\emptyset\})$ is not upper nearly $(I,\{\emptyset\})$-continuous.

Now consider (X,τ,I) and (Y,σ,J) two ideals topological spaces, If $J \neq \{\emptyset\}$, then the concepts of upper nearly (I,J)-continuous and upper nearly I-continuous are independent, as we can see in the following examples.

Example 3.5. In the Example 3.2, the multifunction f is upper nearly (I,J)-continuous on X but is not upper nearly I-continuous on X.

Example 3.6. In the Example 3.3, the multifunction f is upper nearly I-continuous on X but is not upper nearly (I,J)-continuous on X.

Example 3.7. Let $\mathbb{R}$ be the set of real numbers with the discrete topology τ_d and $I = \{\emptyset\}$=J. Consider the multifunction $F : (\mathbb{R},\tau_d,I) \to (\mathbb{R},\tau_d,J)$ defined as follows: $F(x) = \{x\}$ for all $x \in \mathbb{R}$. It is easy to see that: F is upper (resp. lower) nearly (I,J)-continuous on X.

Remark 3.8. It is easy to see that if $F : (X,\tau,I) \to (Y,\sigma,J)$ is a multifunction and $JO(Y) \subset \sigma$. If F is upper (lower) nearly I-continuous, then F is upper (lower) nearly (I,J)-continuous. Even more, if $F : (X,\tau,I) \to (Y,\sigma,J)$ is a multifunction and $JO(Y) \subset \sigma$, we can find upper (resp. lower) nearly (I,J)-continuous on X that are not upper (lower) nearly I-continuous.

The following theorem characterize the upper nearly (I,J) continuous multifunctions.

Theorem 3.9. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is upper nearly (I, J)-continuous.*
2. *$F^+(V)$ is I-open for each J-open set V of Y having N-closed complement.*
3. *$F^-(K)$ is I-closed for every N-closed and J-closed subset K of Y.*
4. *$I\,\mathrm{cl}(F^-(B)) \subset F^-(J\,\mathrm{cl}(B))$ for every subset B of Y having N-closed J-closure.*
5. *$F^+(Jint(B)) \subset Iint(F^+(B))$ for every subset B of Y such that $Y \backslash Jint(B)$ is N-closed.*

Proof. (1)⇒(2): Let $x \in F^+(V)$ and V be any J- open set of Y having N-closed complement. From (1), there exists an I-open set U_x containing x such that $U_x \subset F^+(V)$. It follows that $F^+(V) = \bigcup_{x \in F^+(V)} U_x$. Since any union of I-open sets is I-open, $F^+(V)$ is I-open in (X, τ).

(2)⇒(3): Let K be any N-closed and J- closed set of Y. Then by (2), $F^+(Y \backslash K) = X \backslash F^-(K)$ is an I-open set. Then it is obtained that $F^-(K)$ is an I-closed set.

(3)⇒(4): Let B be any subset of Y having N-closed J- closure. By (3), we have $F^-(B) \subset F^-(J\,\mathrm{cl}(B)) = I\,\mathrm{cl}(F^-(J\,\mathrm{cl}(B)))$. Hence $I\,\mathrm{cl}(F^-(B)) \subset I\,\mathrm{cl}(F^-(J\,\mathrm{cl}(B))) = F^-(J\,\mathrm{cl}(B))$.

(4)⇒(5): Let B be a subset of Y such that $Y \backslash J\mathrm{int}(B)$ is N-closed.

Then by (4), we have $X \backslash I\mathrm{int}(F^+(B)) = I\,\mathrm{cl}(X \backslash F^+(B)) = I\,\mathrm{cl}(F^-(Y \backslash B)) \subset F^-(J\,\mathrm{cl}(Y \backslash B)) = F^-(Y \backslash J\mathrm{int}(B)) = X \backslash F^+(J\mathrm{int}(B))$. Therefore, we obtain $F^+(J\mathrm{int}(B)) \subset I\mathrm{int}(F^+(B))$.

(5)⇒(1): Let $x \in X$ and V be any J-open set of Y containing $F(x)$ and having N-closed complement. Then by (5), $x \in F^+(V) = F^+(J\mathrm{int}(V)) \subset I\mathrm{int}(F^+(V))$. In consequence, there exists an I-open set U containing x such that $U \subset F^+(V)$; hence $F(U) \subset V$. This shows that F is upper nearly I-continuous. □

Theorem 3.10. *For a multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$, the following statements are equivalent:*

1. *F is lower nearly (I, J)-continuous.*
2. *$F^-(V)$ is I-open for each J-open set V of Y having N-closed complement.*
3. *$F^+(K)$ is I-closed for every N-closed and J-closed set K of Y.*
4. *$I\,\mathrm{cl}(F^+(B)) \subset F^+(Jcl(B))$ for every subset B of Y having N-closed closure.*

5. $F^-(Jint(B)) \subset Iint(F^-(B))$ *for every subset* B *of* Y *such that* $Y\setminus Jint(B)$ *is* N*-closed.*

Proof. The proof is similar to that of Theorem 3.9. □

Corollary 3.11. *A multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$ *is upper nearly* (I, J)*-continuous (resp. lower nearly* (I, J)*-continuous) if* $F^-(K)$ *is* I*-closed (resp.* $F^+(K)$ *is* I*-closed) for every* N*-closed set* K *of* Y*.*

Proof. Let G be any J-open set of Y having N-closed complement. Then $Y\setminus G$ is N-closed. By the hypothesis, $X\setminus F^+(G) = F^-(Y\setminus G) = I\mathrm{int}(F^-(Y\setminus G)) = I\,\mathrm{cl}(X\setminus F^+(G)) = X\setminus I\mathrm{int}(F^+(G))$ and hence, $F^+(G) = I\mathrm{int}(F^+(G))$. It follows from Theorem 3.9, that F is upper nearly (I, J)-continuous. The proof of lower nearly (I, J)-continuous is entirely similar. □

Definition 3.12. *A multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$ *is said to be:*

1. *upper* (I, J)*-continuous at a point* $x \in X$ *if for each* J*-open set* V *containing* $F(x)$*, there exists an* I*-open set* U *containing* x *such that* $F(U) \subset V$*.*

2. *lower* (I, J)*-continuous at a point* $x \in X$ *if for each* J*-open set* V *of* Y *meeting* $F(x)$*, there exists an* I*-open set* U *of* X *containing* x *such that* $F(u) \cap V \neq \emptyset$ *for each* $u \in U$*.*

3. *upper (resp. lower)* (I, J)*-continuous on* X *if it has this property at every point of* X*.*

Example 3.13. The Multifunction defined in Example 3.7 is upper nearly (I, J)-continuous on X but is not upper (I, J)-continuous on X.

Remark 3.14. Every upper (resp. lower) (I, J)-continuous multifunction on X is upper (resp. lower) nearly (I, J)-continuous multifunction on X, but the converse is not necessarily true, as we can see in the following example.

Example 3.15. The Multifunction defined in Example 3.2 is upper nearly (I, J)-continuous on X but is not upper (I, J)-continuous.

Theorem 3.16. *For a multifunction* $F : (X, \tau, I) \to (Y, \sigma, J)$*, the following statements are equivalent:*

1. F *is upper* (I, J)*-continuous.*

2. $F^+(V)$ *is* I*-open for each* J*-open set* V *of* Y*.*

3. $F^-(K)$ *is* I*-closed for every* J*-closed subset* K *of* Y*.*

4. $I\,\mathrm{cl}(F^-(B)) \subset F^-(J\,\mathrm{cl}(B))$ *for every subset* B *of* Y*.*

5. *For each point* $x \in X$ *and each* J*-open set* V *containing* $F(x)$*,* $F^+(V)$ *is an* I*-open containing* x*.*

6. *For each point $x \in X$ and each J-open set containing $F(x)$, there exist an I-open set U containing x such that $F(U) \subseteq V$.*

Proof. The proof is similar to that of Theorem 3.9. □

Theorem 3.17. *Let $F : (X, \tau, I) \to (Y, \sigma, J)$ and $F : (Y, \sigma, J) \to (Z, \beta, K)$ be multifunctions. If F is upper nearly (I, J)-continuous (upper (I, J)-continuous) and G upper (I, J)-continuous (upper nearly (I, J)-continuous), then $F \circ G : (X, \tau, I) \to (Z, \beta, K)$ is upper nearly (I, J)-continuous.*

Definition 3.18. *An ideal topological space (X, τ, I) is said to be I-compact [3] if every cover of X by I-open sets have a finite subcover.*

Definition 3.19. *A multifunction $F : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:*

1. *upper (I, J)-irresolute at a point $x \in X$ if for each I-open set U containing x, there exists an I-open set V containing $F(x)$ such that $F(U) \subset V$.*
2. *lower (I, J)-irresolute at a point $x \in X$ if for each j-open set V of Y meeting $F(x)$, there exists an I-open set U containing x such that $U \subseteq F^{-}(V)$.*
3. *upper (resp. lower) (I, J)-irresolute on X if it has this property at every point of X.*

Theorem 3.20. *Let $F : (X, \tau, I) \to (Y, \sigma, J)$ be a surjective (I, J)-irresolute multifunction such that $F(x)$ is J-compact for each $x \in X$. If (X, τ, I) is I-compact, then (Y, σ, J) is J-compact.*

Proof. Let $\{V_i : i \in \triangle\}$ be a J-open cover of Y. For each $x \in X$, there exists a finite subset $\triangle(x)$ of $\triangle$ such that $F(x) \subseteq \bigcup\{V_i : i \in \triangle(x)\}$. Consider $V(x) = \bigcup\{V_i : i \in \triangle(x)\}$. Then $F(x) \subseteq V(x) \in JO(Y)$. using the fact that F is (I, J)-irresolute, then there exist an $U(x) \in IO(X)$ such that $F(U(x)) \subset V(x)$. Now using the that F is surjective, then the collection $\{U(x) : x \in X\}$ is an I-open cover of X. In consequence, there exists a finite number of points of X, say, $x_1, x_2, ...x_n$ such that $X = \bigcup_{i=1}^{n}\{U(x_i)\}$. It follows that $F(X) = F(\bigcup_{i=1}^{n}\{U(x_i)\} \subseteq \bigcup_{i=1}^{n}\{F(U(x_i))\}) \subseteq \bigcup_{i=1}^{n}\{V(x_i)\} \subseteq \bigcup_{i=1}^{n} \bigcup_{i \in \triangle(x_i)} U(x_i)$. It follows that Y is J-compact. □

Definition 3.21. *A multifunction $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be:*

1. *upper almost nearly (I, J)-continuous at a point $x \in X$ if for each J-open set V containing $F(x)$ and having N-closed complement, there exists an I-open set U containing x such that $F(U) \subset int(J\,\mathrm{cl}(V)))$.*
2. *lower almost nearly (I, J)-continuous at a point $x \in X$ if for each J-open set V of Y meeting $F(x)$ and having N-closed complement, there exists an I-open set U of X containing x such that $F(u) \cap int(J\,\mathrm{cl}(V)) \neq \emptyset$ for each $u \in U$.*

3. *upper (resp. lower) almost nearly (I, J)-continuous on X if it has this property at every point of X.*

Example 3.22. Let $X = \mathbb{R}$ the set of real numbers with the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$, $Y = \mathbb{R}$ with the topology $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $I = \{\emptyset\}$=J. Define $F : (X, \tau, I) \to (Y, \sigma, J)$ as follows: $F(x) = \mathbb{Q}$ if $x \in \mathbb{Q}$ and $F(x) = \mathbb{R} \setminus \mathbb{Q}$ if $x \in \mathbb{R} \setminus \mathbb{Q}$. It is easy to see that F is upper (resp. lower) almost nearly (I, J)-continuous on X.

It is clear that every upper (resp. lower) (I, J)-continuous multifunction is upper (resp. lower) nearly (I, J)-continuous multifunction and every upper (resp. lower) nearly (I, J)-continuous multifunction is upper (resp. lower) almost nearly (I, J)-continuous multifunction but the converse in both cases is not true in general as shown in the following examples.

Example 3.23. Let $\mathbb{R}$ with the finite complement topology τ_c and with the discrete topology, take $I = \{\emptyset\} = J$. Consider the multifunction $F : (\mathbb{R}, \tau_c, I) \to (\mathbb{R}, \tau_d, J)$ defined as follows: $F(x) = \{x\}$ for all $x \in \mathbb{R}$. It is easy to see that: F is upper (resp. lower) nearly (I, J)-continuous on X but is not upper (resp. lower) (I, J)-continuous on X.

Example 3.24. The multifunction F defined in Example 3.22 is upper (resp. lower) nearly almost (I, J)-continuous on X but is not upper (resp. lower) nearly (I, J)-continuous on X.

At this point, there are a question. Given a multifunction $F : (X, \tau, I) \to (Y, \sigma)$. It is possible to write a characterization for upper (resp. lower) nearly almost (I, J)-continuous on X.

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