Tripolar fuzzy soft ideals and tripolar fuzzy soft interior ideals over semiring

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Abstract. In this paper, we introduce the notion of tripolar fuzzy soft semiring, tripolar fuzzy soft ideal, tripolar fuzzy soft interior ideal over semiring and study some of their properties and the relations between them.

Keywords: tripolar fuzzy set, soft set, fuzzy soft set, tripolar fuzzy soft ideal, tripolar fuzzy soft interior ideal.

1. Introduction

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was first introduced by Zadeh [21]. Mandal [15] studied fuzzy ideals and fuzzy interior ideals in ordered semirings. Molodtsov [16] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties, only partially resolves the problem is that objects in universal set often does not precisely satisfy the parameters associated to each of the elements in the set. Acar et al. [1] gave the basic concept of soft ring. Aktas and Cagman [3] introduced the concept of fuzzy subgroup which was extended by Aygunoglu

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and Aygun [5]. Majumdar and Samantha [14] extended soft sets to fuzzy soft set. Ghosh et al. [7] initiated the study of fuzzy soft rings and fuzzy soft ideals. Murali Krishna Rao [17] introduced and studied fuzzy soft ideals and fuzzy soft $k-$ideals over a $\Gamma-$semiring. There are many extensions of fuzzy sets, for example, intuitionistic fuzzy sets, interior valued fuzzy sets, vague sets, bipolar fuzzy sets, cubic sets etc. In 2001, Maji et al. [13] combined the concept of fuzzy set theory which was introduced by Zadeh in 1965 and the notion of soft set theory which was introduced by Molodstov [16] in 1999. Soft semirings was introduced by Feng [6] .

In 2000, Lee [10, 11, 12] used the term bipolar valued fuzzy sets and applied it to algebraic structure. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is $[-1, 1]$. In 1994, Zhang [22] initiated the concept bipolar fuzzy sets as a generalization of fuzzy sets. The concepts of bipolar fuzzy soft sets introduced by Akram [2] introduced the notion of bipolar fuzzy soft lie subalgebras and studied some of their properties. Jun et al. [8, 9] introduced the notion of bipolar fuzzy ideals and bipolar fuzzy filters in BCI-algebras.

The ideals of "intuitionistic fuzzy set" was first introduced by Atanassov [4] as a generalization of notion of fuzzy set. Murali Krishna Rao [18] introduced the notion of tripolar fuzzy interior ideal of $\Gamma-$semigroup. Murali Krishna Rao and Venkateswarlu [19] introduced the notion of tripolar fuzzy set to be able to deal with tripoalr information as a generalization of fuzzy set, bipolar fuzzy set and intuitionistic fuzzy set and introduced the notion of tripolar fuzzy ideal and tripolar fuzzy interior ideal of $\Gamma-$semiring. In this paper,we introduce the notion of tripolar fuzzy soft ideals and interior ideals over semiring. We study some of their algebraic properties and the relations between them.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

Definition 2.1. A universal algebra $(M, +, \cdot)$ is called a semiring if and only if $(M, +), (M, \cdot)$ are semigroups which are connected by distributive laws, *i.e.*, $a(b+c) = ab + ac, \quad (a + b)c = ac + bc,$ for all $a, b, c \in M$.

A semiring M is said to be commutative semiring if $xy = yx$, for all $x, y \in M$. A semiring M is said to have zero element if there exists an element $0 \in M$ such that $0+x = x = x+0$ and $0x = x0 = 0,$ for all $x \in M$. An element $1 \in M$ is said to be unity if for each $x \in M$ such that $x1 = 1x = x$. In a semiring M with unity 1, an element $a \in M$ is said to be left invertible (right invertible) if there exists $b \in M$ such that $ba = 1(ab = 1)$. In a semiring M with unity 1, an element $a \in M$ is said to be invertible if there exists $b \in M$ such that $ab = ba = 1$. A semiring M with unity 1 is said to be division semiring if every non zero element of M is invertible. An element $a \in M$ is said to be regular element of M if there exists $x \in M$ such that $a = axa$. If every element of semiring M is a regular then M is said to be regular semiring. An element $a \in M$ is said to be idempotent of M if $a = a^2$. Every element of M is an idempotent of M then M is said to be idempotent semiring M.

Definition 2.2. A non-empty subset A of semiring M is called

(i) a subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$;

(ii) a quasi ideal of M if A is a subsemiring of M and $AM \cap MA \subseteq A$;

(iii) a bi-ideal of M if A is a subsemiring of M and $AMA \subseteq A$;

(iv) an interior ideal of M if A is a subsemiring of M and $MAM \subseteq A$;

(v) a left (right) ideal of M if A is a subsemiring of M and $MA \subseteq A(AM \subseteq A)$;

(vi) an ideal if A is a subsemiring of $M, AM \subseteq A$ and $MA \subseteq A$;

(vii) a $k-$ideal if A is a subsemiring of $M, AM \subseteq A, MA \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$;

(viii) a left (right) bi- quasi ideal of M if A is a subsemiring of M and $MA \cap AMA\ (AM \cap AMA) \subseteq A$;

(ix) a bi-quasi ideal of M if A is a left bi- quasi ideal and a right bi- quasi ideal of M.

A semiring M is a left (right) simple semiring if M has no proper left (right) ideal of M. A semiring M is said to be simple semiring if M has no proper ideals.

Let M be a non-empty set. Then a mapping $f : M \to [0, 1]$ is called a fuzzy subset of M. A fuzzy subset $\mu : M \to\ [0, 1]$ is non-empty if μ is not the constant function. For any two fuzzy subsets λ and μ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$, for all $x \in M$.

Definition 2.3. An intuitionistic fuzzy set f of a non-empty set X is an object having the form $f = (\mu_f, \lambda_f) = \{(x, \mu_f(x), \lambda_f(x)) \mid x \in X\}$, where $\mu_f : X \to [0, 1]; \lambda_f : X \to [0, 1]$ are membership functions, $\mu_f(x)$ is a degree of membership, $\lambda_f(x)$ is a degree of non membership and $0 \leq \mu_f(x) + \lambda_f(x) \leq 1$, for all $x \in X$.

Definition 2.4. A bipolar fuzzy set A of a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \delta_A(x)) \mid x \in X\}$, where $\mu_A : X \to [0, 1]; \delta_A : X \to [-1, 0]$. $\mu_A(x)$ represents degree of satisfaction of an element x to the property corresponding to fuzzy set A and $\delta_A(x)$ represents degree of satisfaction of an element x to the implicit counter property of fuzzy set A.

Definition 2.5. A tripolar fuzzy set A in a universe set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X$ and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1\}$, where $\mu_A : X \to [0, 1]; \lambda_A : X \to [0, 1]; \delta_A : X \to [-1, 0]$ such that $0 \leq \mu_A(x)+\lambda_A(x) \leq$

1. The membership degree $\mu_A(x)$ characterizes the extent that the element x satisfies to the property corresponding to tripolar fuzzy set A, $\lambda_A(x)$ characterizes the extent that the element x satisfies to the not property (irrelevant) corresponding to tripolar fuzzy set A and $\delta_A(x)$ characterizes the extent that the element x satisfies to the implicit counter property of tripolar fuzzy set A. For simplicity $A = (\mu_A, \lambda_A, \delta_A)$ has been used for $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X$ and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1\}$.

Remark 2.6. A tripolar fuzzy set A is a generalization of a bipolar fuzzy set and an intuitionistic fuzzy set. A tripolar fuzzy set $A = \{(x, \mu_A(x), \lambda_A(x), \delta_A(x)) \mid x \in X\}$ represents the taste of food stuffs. Assuming the sweet taste of food stuff as a positive membership value $\mu_A(x)$ i.e. the element x is satisfying the sweet property. Then bitter taste of food stuff as a negative membership value $\delta_A(x)$ i.e. the element x is satisfying the bitter property and the remaining tastes of food stuffs like acidic, chilly etc., as a non memberships value $\lambda_A(x)$. i.e., the element is satisfying irrelevant to the sweet property.

Definition 2.7. A tripolar fuzzy set $A = (\mu_A, \lambda_A, \delta_A)$ of a semiring M is called a tripolar fuzzy subsemiring of M if A satisfies the following conditions.

(i) $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(x)\}$;

(ii) $\lambda_A(x+y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$;

(iii) $\delta_A(x+y) \leq \max\{\delta_A(x), \delta_A(y)\}$;

(iv) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(x)\}$;

(v) $\lambda_A(xy) \leq \max\{\lambda_A(x), \lambda_A(y)\}$;

(vi) $\delta_A(xy) \leq \max\{\delta_A(x), \delta_A(y)\}$, for all $x, y \in M$.

Definition 2.8. A tripolar fuzzy subsemiring $A = (\mu_A, \lambda_A, \delta_A)$ of a semiring M is called a tripolar fuzzy ideal of M if A satisfies the following conditions

(i) $\mu_A(xy) \geq \max\{\mu_A(x), \mu_{A)}(y)\}$;

(ii) $\lambda_A(xy) \leq \min\{\lambda_A(x), \lambda_A(y)\}$;

(iii) $\delta_A(xy) \leq \min\{\delta_A(x), \delta_A(y)\}$, for all $x, y \in M$.

Definition 2.9. A tripolar fuzzy subsemiring $A = (\mu_A, \lambda_A, \delta_A)$ of a semiring M is called a tripolar fuzzy interior ideal of M if A satisfies the following conditions

(i) $\mu_A(xzy) \geq \mu_A(z)$;

(ii) $\lambda_A(xzy) \leq \lambda_A(z)$;

(iii) $\delta_A(xzy) \leq \delta_A(z)$, for all $x, y, z \in M$.

Definition 2.10. Let $(f,A),(g,B)$ be fuzzy soft sets over a semiring M. The intersection of fuzzy soft sets (f,A) and (g,B) is denoted by $(f,A)\cap(g,B)=(h,C)$ where $C=A\cup B$ is defined as

$$h_c=\begin{cases} f_c, & \text{if } c\in A\setminus B;\\ g_c, & \text{if } c\in B\setminus A; \text{ for all } c\in A\cup B.\\ f_c\cap g_c, & \text{if } c\in A\cap B\ ,\end{cases}$$

Definition 2.11. Let $(f,A),(g,B)$ be fuzzy soft sets over a semiring M. The union of fuzzy soft sets (f,A) and (g,B) is denoted by $(f,A)\cup(g,B)=(h,C)$ where $C=A\cup B$ is defined as

$$h_c=\begin{cases} f_c, & \text{if } c\in A\setminus B;\\ g_c, & \text{if } c\in B\setminus A; \text{ for all } c\in A\cup B.\\ f_c\cup g_c, & \text{if } c\in A\cap B,\end{cases}$$

Definition 2.12. Let $(f,A),(g,B)$ be fuzzy soft sets over a semiring M. Then (f,A) and (g,B) is denoted by $(f,A)\wedge(g,B)$ is defined by $(f,A)\wedge(g,B)=(f\cap g,C)=(h,C)$, where $C=A\times B$, $h_c(x)=\min\{f_a(x),g_b(x)\}$, for all $c=(a,b)\in A\times B$ and $x\in M$.

Definition 2.13. Let $(f,A),(g,B)$ be fuzzy soft sets over an ordered semiring M. Then (f,A) or (g,B) is denoted by $(f,A)\vee(g,B)$ is defined by $(f,A)\vee(g,B)=(h,C)$, where $C=A\times B$ and $h_c(x)=\max\{f_a(x),g_b(x)\}$, for all $c=(a,b)\in A\times B, x\in M$.

3. Tripolar fuzzy soft ideals over semiring

In this section, we introduce the notion of tripolar fuzzy sets to be able to deal with tripolar information as a generalization of fuzzy sets, bipolar fuzzy set and intuitionistic fuzzy sets. We introduce the notion of tripolar fuzzy soft ideals and interior ideals over semiring.

Definition 3.1. A tripolar fuzzy soft set (f,A) over semiring M is called a tripolar fuzzy soft semiring over M if $f(a)=\{(\mu_{f(a)}(x),\lambda_{f(a)}(x),\delta_{f(a)}(x))\mid x\in M, a\in A\}$, where $\mu_{f(a)}(x):M\to[0,1]; \lambda_{f(a)}(x):M\to[0,1]; \delta_{f(a)}(x):M\to[-1,0]$ such that $0\le\mu_{f(a)}(x)+\lambda_{f(a)}(x)\le 1$ and for all $x\in M$ satisfying the following conditions

(i) $\mu_{f(a)}(x+y)\ge\min\{\mu_{f(a)}(x),\mu_{f(a)}(x)\}$;

(ii) $\lambda_{f(a)}(x+y)\le\max\{\lambda_{f(a)}(x),\lambda_{f(a)}(y)\}$;

(iii) $\delta_{f(a)}(x+y)\le\max\{\delta_{f(a)}(x),\delta_{f(a)}(y)\}$;

(iv) $\mu_{f(a)}(xy)\ge\min\{\mu_{f(a)}(x),\mu_{f(a)}(x)\}$;

(v) $\lambda_{f(a)}(xy) \leq \max\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}$;

(vi) $\delta_{f(a)}(xy) \leq \max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$, for all $x, y \in M$ and $a \in A$.

Definition 3.2. A tripolar fuzzy soft set (f, A) over a semiring M is called a tripolar fuzzy soft ideal over M if

(i) $\mu_{f(a)}(x + y) \geq \min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}$;

(ii) $\lambda_{f(a)}(x + y) \leq \max\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}$;

(iii) $\delta_{f(a)}(x + y) \leq \max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$;

(iv) $\mu_{f(a)}(xy) \geq \max\{\mu_{f(a)}(x), \mu_{f(a)}(x)\}$;

(v) $\lambda_{f(a)}(xy) \leq \min\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}$;

(vi) $\delta_{f(a)}(xy) \leq \min\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}$, for all $x, y \in M$ and $a \in A$.

Remark 3.3. Every tripolar fuzzy soft ideal (f, A) over a semiring M is a tripolar fuzzy soft semiring (f, A) over M but the converse is not true.

Example 3.4. Let $M = \{x_1, x_2, x_3\}$. Then we define operations with the following tables:

$+$	x_1	x_2	x_3
x_1	x_1	x_2	x_3
x_2	x_2	x_2	x_3
x_3	x_3	x_3	x_2

;

.	x_1	x_2	x_3
x_1	x_1	x_3	x_3
x_2	x_3	x_2	x_3
x_3	x_3	x_3	x_3

Let $E = \{a, b, c\}$ and $B = \{a, b\}$. Then (ϕ, B) is tripolar fuzzy soft set defined as $(\phi, B) = \{\phi(a), \phi(b)\}$, where
$\phi(a) = \{(x_1, 0.2, 0.7, -0.2), (x_2, 0.3, 0.6, -0.3), (x_3, 0.6, 0.3, -0.3)\}$
$\phi(b) = \{(x_1, 0.4, 0.5, -0.3), (x_2, 0.6, 0.3, -0.5), (x_3, 0.5, 0.4, -0.2)\}$.
Then (ϕ, B) is a tripolar fuzzy soft semiring over M but not a tripolar fuzzy soft ideal over M. And (ϕ, B) is tripolar fuzzy soft interior ideal over M.

Definition 3.5. A tripolar fuzzy soft (f, A) over semiring M is called a tripolar fuzzy soft interior ideal of M if

(i) $\mu_{f(a)}(xzy) \geq \mu_{f(a)}(z)$;

(ii) $\lambda_{f(a)}(xzy) \leq \lambda_{f(a)}(z)$;

(iii) $\delta_{f(a)}(xzy) \leq \delta_{f(a)}(z)$, for all $x, y, z \in M$ and $a \in A$.

Theorem 3.6. *Every tripolar fuzzy soft ideal over a semiring M is a tripolar fuzzy soft interior ideal over a semiring M.*

Proof. Let (f, A) be tripolar fuzzy soft ideal over a semiring M. Then $f(a) = \{\mu(a), \lambda(a), f(a)\}$ is a tripolar fuzzy ideal of $M, a \in A$. Then

(i) $\mu_{f(a)}(xzy) \geq \mu_{f(a)}(xz) \geq \mu_{f(a)}(z)$;

(ii) $\lambda_{f(a)}(xzy) \leq \lambda_{f(a)}(xz) \leq \lambda_{f(a)}(z)$;

(iii) $\delta_{f(a)}(xzy) \leq \delta_{f(a)}(xz) \leq \delta_{f(a)}(z)$, for all $x, y, z \in M$ and $a \in A$.

Hence (f, A) is a tripolar fuzzy soft interior ideal over M. □

Theorem 3.7. *Every tripolar fuzzy soft interior ideal over a regular semiring M is a tripolar fuzzy soft ideal over M.*

Proof. Let (f, A) be tripolar fuzzy soft interior ideal over a regular semiring M. Then $f(a) = \{\mu_{f(a)}, \lambda_{f(a)}, \delta_{f(a)}\}$ is a tripolar fuzzy ideal of $M, a \in A$.

Suppose $x, y \in M$. Then $xy \in M$. Then there exists $z \in M$ such that $xy = xyzxy$

$$\mu_{f(a)}(xy) = \mu_{f(a)}(xyzxy) = \mu_{f(a)}\big(xy(zxy)\big) \geq \mu_{f(a)}(y),$$
$$\mu_{f(a)}(xy) = \mu_{f(a)}\big((xyz)xy\big) \geq \mu_{f(a)}(x).$$

Hence $\mu_{f(a)}$ is a fuzzy ideal of M.

$$\lambda_{f(a)}(xy) = \lambda_{f(a)}(xyzxy) \leq \lambda_{f(a)}(y),$$
$$\lambda_{f(a)}(xy) = \lambda_{f(a)}\big((xyz)xy\big) \leq \lambda_{f(a)}(x).$$

Hence $\lambda_{f(a)}$ is a fuzzy ideal of M.

$$\delta_{f(a)}(xy) = \delta_{f(a)}(xyzxy) \leq \delta_{f(a)}(y),$$
$$\delta_{f(a)}(xy) \leq \delta_{f(a)}(x).$$

Hence $\delta_{f(a)}$ is a fuzzy ideal of M.

Therefore $f(a)$ is a tripolar fuzzy ideal of the semiring M.

Thus (f, A) is a tripolar fuzzy soft ideal over semiring M. □

Theorem 3.8. *If (f, A) and (g, B) are two tripolar fuzzy soft ideals over a semiring M then $(f, A) \wedge (g, B)$ is a tripolar fuzzy soft ideal over a semiring M.*

Proof. Suppose (f, A) and (g, B) are two tripolar fuzzy soft ideals over a semiring M. Then by Definition 2.12, $(f, A) \wedge (g, B) = (f \cap g, C)$, where $C = A \times B$ and $(f \wedge g)(a, b) = f(a) \cap g(b)$, for all $(a, b) \in C$. Then

$$\begin{aligned}\mu_{f(a)\cap g(b)}(x+y) &= \min\{\mu_{f(a)}(x+y), \mu_{g(b)}(x+y)\}\\ &\geq \min\big\{\min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}, \min\{\mu_{g(b)}(x), \mu_{g(b)}(y)\}\big\}\\ &= \min\big\{\min\{\mu_{f(a)}(x), \mu_{g(b)}(x)\}, \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\}\big\}\\ &= \min\{\mu_{f(a)\cap g(b)}(x), \mu_{f(a)\cap g(b)}(y)\},\end{aligned}$$

$$\begin{aligned}
\lambda_{f(a)\cap g(b)}(x+y) &= \min\{\lambda_{f(a)}(x+y), \lambda_{g(b)}(x+y)\} \\
&\geq \min\big\{\max\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}, \max\{\lambda_{g(b)}(x), \lambda_{g(b)}(y)\}\big\} \\
&= \max\big\{\min\{\lambda_{f(a)}(x), \lambda_{g(b)}(x)\}, \min\{\lambda_{f(a)}(y), \lambda_{g(b)}(y)\}\big\} \\
&= \max\{\lambda_{f(a)\cap g(b)}(x), \lambda_{f(a)\cap g(b)}(y)\}. \\
\delta_{f(a)\cap g(b)}(x+y) &= \min\{\delta_{f(a)}(x+y), \delta_{g(b)}(x+y)\} \\
&\geq \min\big\{\max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}, \max\{\delta_{g(b)}(x), \delta_{g(b)}(y)\}\big\} \\
&= \max\big\{\min\{\delta_{f(a)}(x), \delta_{g(b)}(x)\}, \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\}\big\} \\
&= \max\{\delta_{f(a)\cap g(b)}(x), \delta_{f(a)\cap g(b)}(y)\}, \\
\mu_{f(a)\cap g(b)}(xy) &= \min\{\mu_{f(a)}(xy), \mu_{g(b)}(xy)\} \\
&\geq \min\big\{\min\{\mu_{f(a)}(x), \mu_{f(a)}(y)\}, \min\{\mu_{g(b)}(x), \mu_{g(b)}(y)\}\big\} \\
&= \min\big\{\min\{\mu_{f(a)}(x), \mu_{g(b)}(x)\}, \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\}\big\} \\
&= \min\{\mu_{f(a)\cap g(b)}(x), \mu_{f(a)\cap g(b)}(y)\}, \\
\lambda_{f(a)\cap g(b)}(xy) &= \min\{\lambda_{f(a)}(xy), \lambda_{g(b)}(xy)\} \\
&\leq \min\big\{\max\{\lambda_{f(a)}(x), \lambda_{f(a)}(y)\}, \max\{\lambda_{g(b)}(x), \lambda_{g(b)}(y)\}\big\} \\
&= \max\big\{\min\{\lambda_{f(a)}(x), \lambda_{g(b)}(x)\}, \min\{\lambda_{f(a)}(y), \lambda_{g(b)}(y)\}\big\} \\
&= \max\{\lambda_{f(a)\cap g(b)}(x), \lambda_{f(a)\cap g(b)}(y)\}, \\
\delta_{f(a)\cap g(b)}(xy) &= \min\{\delta_{f(a)}(xy), \delta_{g(b)}(xy)\} \\
&\leq \min\big\{\max\{\delta_{f(a)}(x), \delta_{f(a)}(y)\}, \max\{\delta_{g(b)}(x), \delta_{g(b)}(y)\}\big\} \\
&= \max\big\{\min\{\delta_{f(a)}(x), \delta_{g(b)}(x)\}, \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\}\big\} \\
&= \max\{\delta_{f(a)\cap g(b)}(x), \delta_{f(a)\cap g(b)}(y)\}.
\end{aligned}$$

Hence $(f, A) \wedge (g, B)$ is a tripolar fuzzy soft ideal over a semiring M. □

Theorem 3.9. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semiring M then $(f, A) \wedge (g, B)$ is a tripolar fuzzy interior ideal over semiring M.*

Proof. Suppose (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semiring M. Obviously $(f, A) \wedge (g, B)$ is a soft tripolar fuzzy subsemiring of M.

By Definition 2.12, $(f, A) \wedge (g, B) = (f \cap g, C)$, where $C = A \times B$.

Suppose $(a, b) \in C, x, y \in M$. Then

$$\begin{aligned}
\mu_{f\wedge g(a,b)}(xyz) &= \mu_{f(a)\cap g(b)}(xyz) \geq \min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\} = \mu_{f(a)\cap g(b)}(y) \\
&= \mu_{f\wedge g(a,b)}(y), \\
\lambda_{f\wedge g(a,b)}(xyz) &= \lambda_{f(a)\cap g(b)}(xyz) = \min\{\lambda_{f(a)}(xyz), \lambda_{g(b)}(xyz)\} \\
&\leq \min\{\lambda_{f(a)}(y), \lambda_{g(b)}(y)\} = \lambda_{f(a)\cap g(b)}(y) = \lambda_{f\wedge g(a,b)}(y),
\end{aligned}$$

$$\begin{aligned}\delta_{f\wedge g(a,b)}(xyz) &= \delta_{f(a)\cap g(b)}(xyz) = \min\{\delta_{f(a)}(xyz), \delta_{g(b)}(xyz)\}\\ &\leq \min\{\delta_{f(a)}(y), \delta_{g(b)}(y)\} = \delta_{f(a)\cap g(b)}(y) = \delta_{f\wedge g(a,b)}(y).\end{aligned}$$

Hence $(f, A) \wedge (g, B)$ is a soft tripolar fuzzy interior ideal of the semiring M. □

The proofs of following theorems are similar to Theorem 3.9.

Theorem 3.10. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semiring M then $(f, A) \cup (g, B)$ is a tripolar fuzzy interior ideal of a semiring M.*

Theorem 3.11. *If (f, A) and (g, B) are two tripolar fuzzy soft interior ideals over a semiring M then $(f, A) \cap (g, B)$ is a tripolar fuzzy interior ideal of a semiring M.*

Theorem 3.12. *If (f, A) and (g, B) are two tripolar fuzzy soft ideals over semiring M then $(f, A) \cup (g, B)$ is a tripolar fuzzy soft ideal over M.*

Proof. Suppose (f, A) and (g, B) are two tripolar fuzzy soft ideals over semiring M. Then by Definition 2.11, we have $(f, A) \cup (g, B) = (h, C)$, where $C = A \cup B$, and

$$h(c) = f \cup g(c) = \begin{cases} f(c), & \text{if } c \in A \setminus B;\\ g(c), & \text{if } c \in B \setminus A;\\ f(c), \cup g(c) & \text{if } c \in A \cap B, \quad \text{for all} \quad c \in A \cup B.\end{cases}$$

Case (i). If $c \in A \setminus B$ then $f \cup g(c) = f(c)$. Thus we have

$$\begin{aligned}\mu_{f\cup g(c)}(x+y) &= \mu_{f(c)}(x+y) \geq \min\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}\\ &= \min\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\},\\ \lambda_{f\cup g(c)}(x+y) &= \lambda_{f(c)}(x+y) \leq \max\{\lambda_{f(c)}(x), \lambda_{f(c)}(y)\}\\ &= \max\{\lambda_{f\cup g(c)}(x), \lambda_{f\cup g(c)}(y)\},\\ \delta_{f\cup g(c)}(x+y) &= \delta_{f(c)}(x+y) \leq \max\{\delta_{f(c)}(x), \delta_{f(c)}(y)\}\\ &= \max\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\},\\ \mu_{f\cup g(c)}(xy) &= \mu_{f(c)}(xy) \geq \max\{\mu_{f(c)}(xy), \mu_{f(c)}(xy)\}\\ &= \max\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\},\\ \lambda_{f\cup g(c)}(xy) &= \lambda_{f(c)}(xy) \leq \min\{\lambda_{f(c)}(x), \lambda_{f(c)}(y)\}\\ &= \min\{\lambda_{f\cup g(c)}(x), \lambda_{f\cup g(c)}(y)\},\\ \delta_{f\cup g(c)}(xy) &= \delta_{f(c)}(xy) \leq \min\{\delta_{f(c)}(x), \delta_{f(c)}(y)\}\\ &= \min\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.\end{aligned}$$

Case (ii). If $c \in B \setminus A$ then $f \cup g(c) = g(c)$.

Since $g(c)$ is a tripolar fuzzy ideal of semiring $M, f \cup g(c)$ is a tripolar fuzzy ideal of semiring M.

Case (iii). If $c \in A \cap B$ then $f \cup g(c) = f(c) \cup g(c)$.

$$\begin{aligned}
\mu_{f\cup g(c)}(x+y) &= \max\{\mu_{f(c)}(x+y), \mu_{g(c)}(x+y)\} \\
&\geq \max\{\min\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}, \min\{\mu_{g(c)}(x), \mu_{g(c)}(y)\}\} \\
&= \min\{\max\{\mu_{f(c)}(x), \mu_{g(c)}(x)\}, \max\{\mu_{f(c)}(y), \mu_{g(c)}(y)\}\} \\
&= \min\{\mu_{f(c)\cup g(c)}(x), \mu_{f(c)\cup g(c)}(y)\} \\
&= \min\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}
\end{aligned}$$

Similarly we can prove

$$\begin{aligned}
\lambda_{f\cup g(c)}(x+y) &\leq \max\{\lambda_{f\cup g(c)}(x), \lambda_{f\cup g(c)}(y)\} \\
\delta_{f\cup g(c)}(x+y) &\leq \max\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}. \\
\mu_{f\cup g(c)}(xy) &= \mu_{f(c)\cup g(c)}(xy) \\
&= \max\{\mu_{f(c)}(xy), \mu_{g(c)}(xy)\} \\
&\geq \max\{\max\{\mu_{f(c)}(x), \mu_{f(c)}(y)\}, \max\{\mu_{g(c)}(x), \mu_{g(c)}(y)\}\} \\
&= \max\{\max\{\mu_{f(c)}(x), \mu_{g(c)}(x)\}, \max\{\mu_{f(c)}(y), \mu_{g(c)}(y)\}\} \\
&= \max\{\mu_{f(c)\cup g(c)}(x), \mu_{f(c)\cup g(c)}(y)\} \\
&= \max\{\mu_{f\cup g(c)}(x), \mu_{f\cup g(c)}(y)\}
\end{aligned}$$

Similarly we can prove

$$\begin{aligned}
\lambda_{f\cup g(c)}(xy) &\leq \min\{\lambda_{f\cup g(c)}(x), \lambda_{f\cup g(c)}(y)\} \\
\delta_{f\cup g(c)}(xy) &\leq \min\{\delta_{f\cup g(c)}(x), \delta_{f\cup g(c)}(y)\}.
\end{aligned}$$

Therefore $f \cup g(c)$ is a tripolar fuzzy ideal of M.

Hence $(f, A) \cup (g, B)$ is a tripolar fuzzy soft ideal over M. □

Corollary 3.13. *If (f, A) and (g, B) are two tripolar fuzzy soft ideals over semiring M then $(f, A) \cap (g, B)$ is a tripolar fuzzy soft ideal over M.*

Definition 3.14. Let (f, A) and (g, B) be two tripolar fuzzy soft sets over a semiring M. The the product (f, A) and (g, B) is defined as $(f, A) \circ (g, B) = (f \circ g, C)$, where $C = A \cup B$. Then by Definition 2.13, we have $(f, A) \cup (g, B) = (h, C)$, where $C = A \cup B$; and

$$\mu_{(f\cup g)(c)}(x) = \begin{cases} \mu_{f(c)}(x), & \text{if } c \in A \setminus B; \\ \mu_{g(c)}(x), & \text{if } c \in B \setminus A; \\ \sup_{x=yz} \big\{ \min\{\mu_{f(c)}(y), \mu_{g(c)}(z)\} \big\}, & \text{if } c \in A \cap B. \end{cases}$$

$$\lambda_{(f\cup g)(c)}(x) = \begin{cases} \lambda_{f(c)}(x), & \text{if } c \in A \setminus B; \\ \lambda_{g(c)}(x), & \text{if } c \in B \setminus A; \\ \inf_{x=yz} \big\{ \max\{\lambda_{f(c)}(y), \lambda_{g(c)}(z)\} \big\}, & \text{if } c \in A \cap B. \end{cases}$$

$$\delta_{(f\cup g)(c)}(x) = \begin{cases} \delta_{f(c)}(x), & \text{if } c \in A \setminus B; \\ \delta_{g(c)}(x), & \text{if } c \in B \setminus A; \\ \inf_{x=yz}\big\{\max\{\delta_{f(c)}(y), \delta_{g(c)}(z)\}\big\}, & \text{if } c \in A \cap B. \end{cases}$$

Theorem 3.15. *If (f, A) and (g, B) are tripolar fuzzy soft interior ideals over semiring M then $(f, A)\circ(g, B)$ is a tripolar fuzzy soft interior ideal over semiring M.*

Proof. Obviously $(f, A)\circ(g, B)$ is a tripolar fuzzy soft subsemiring over M. Let $x, y, z \in M$. By Definition 3.14, $(f, A) \circ (g, B) = (f \circ g, C)$, where $C = A \cup B$ and $c \in C, x \in M$.

Case (i). If $c \in A \setminus B$ then

$$\mu_{f\circ g(c)} = \mu_{f(c)};\ \lambda_{f\circ g(c)} = \lambda_{f(c)};\ \delta_{f\circ g(c)} = \delta_{f(c)}.$$

Since (f, A) is a tripolar fuzzy soft interior ideal over M, $f \circ g(c)$ is a tripolar fuzzy soft interior ideal of M.

Case (ii). If $c \in B \setminus A$ then

$$\mu_{f\circ g(c)} = \mu_{g(c)}; \lambda_{f\circ g(c)} = \lambda_{g(c)}; \delta_{f\circ g(c)} = \delta_{g(c)}.$$

Since (g, B) is a tripolar fuzzy soft interior ideal over M, $f \circ g(c)$ is a tripolar fuzzy soft interior ideal of M.

Case (iii). If $c \in A \cap B$ then

$$\begin{aligned} \mu_{f\circ g(c)}(x) &= \sup_{x=ab}\big\{\min\{\mu_{f(a)}(x), \mu_{g(b)}(x)\}\big\}. \\ \mu_{f\circ g(c)}(xyz) &= \sup_{x=ab}\big\{\min\{\mu_{f(a)}(xyz), \mu_{g(b)}(xyz)\}\big\} \\ &\geq \sup_{x=ab}\big\{\min\{\mu_{f(a)}(y), \mu_{g(b)}(y)\}\big\} = \mu_{f\circ g(c)}(y). \\ \lambda_{f\circ g(c)}(xyz) &= \inf_{x=ab}\big\{\max\{\lambda_{f(a)}(xyz), \lambda_{g(b)}(xyz)\}\big\} \\ &\leq \sup_{x=ab}\big\{\max\{\lambda_{f(a)}(y), \lambda_{g(b)}(y)\}\big\} = \lambda_{f\circ g(c)}(y). \\ \delta_{f\circ g(c)}(xyz) &= \inf_{x=ab}\big\{\max\{\delta_{f(a)}(xyz), \delta_{g(b)}(xyz)\big\} \\ &\leq \inf_{x=ab}\big\{\max\{\delta_{f(a)}(y), \delta_{g(b)}(y)\}\big\} = \delta_{f\circ g(c)}(y). \end{aligned}$$

Hence $f \circ g(c)$ is a tripolar fuzzy interior ideal of M.

Therefore $(f, A) \circ (g, B)$ is a tripolar fuzzy interior ideal over M. □

Theorem 3.16. *Let E be a parameter set and $\sum_E(M)$ be the set of all tripolar fuzzy soft interior ideals over semiring M. Then $(\sum_E(M), \cup, \cap)$ forms complete distributive lattices along with the relation $\subseteq$.*

Proof. Suppose (f, A) and (g, B) be soft interior ideals over M such that $A \subseteq E, B \subseteq E$.

By Theorems 3.10 and 3.11, $(f, A) \cap (g, B)$ and $(f, A) \cup (g, B)$ are tripolar fuzzy soft interior ideals over M.

Obviously $(f, A) \cap (g, B)$ is lub of $\{(f, A), (g, B)\}$ and $(f, A) \cup (g, B)$ is glb of $\{(f, A), (g, B)\}$.

Every sub collection of $\sum_E(M)$ has lub and glb.

Hence $\sum_E(M)$ is a complete lattice.

We can prove $(f, A) \cap \big((g, B) \cup (h, C)\big) = \big((f, A) \cap (g, B)\big) \cup \big((f, A) \cap (h, C)\big)$.

Therefore $(\sum_E(M), \cup, \cap)$ forms a complete distributive lattice. □

4. Conclusion

In this paper,we introduced the notion of tripolar fuzzy soft subsemiring, tripolar fuzzy soft ideal, tripolar fuzzy soft interior ideals over semiring and studied some of their algebraic properties and relations between them.

References

[1] U. Acar, F. Koyuncu and B. Tanay, *Soft sets and soft rings*, Comput. Math. Appli., 59 (2010), 3458-3463.

[2] M. Akram, *Bipolar fuzzy soft lie algebras*, Quasigroups and related Systems, 21 (2013), 1-10

[3] H. Aktas and N. Cagman, *Soft sets and soft groups*, Information Science, 177 (2007), 2726-2735.

[4] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy sets and systems, 20 (1986), 87-96.

[5] A. Aygunoglu and H. Aygun, *Introduction to fuzzy soft group*, Comput. Math. Appli., 58 (2009), 1279-1286.

[6] F. Feng, Y. B. Jun and X. Zhao, *Soft semirings*, Comput. Math. Appli., 56 (2008), 2621-2628.

[7] J. Ghosh, B. Dinda and T. K. Samanta, *Fuzzy soft rings and fuzzy soft ideals,* Int. J. P. App. Sci. Tech., 2 (2011), 66-74.

[8] Y. B. Jun, H.S. Kim and K. J. Lee, *Bipolar fuzzy translation in BCK/BCI algebra*, J. of the Chungcheong Math. Soc., 22 (2009), 399-408.

[9] K. H. Kim and Y. B. Jun, *Intuitionistic fuzzy interior ideals of semigroups*, Int. J. of Math. and Math. Sci., 27 (2001), 261-267.

[10] K. J. Lee, *Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI algebras,* Bull. Malays. Math. Soc., 32 (2009), 361-373.

[11] K. M. Lee, *Bipolar valued fuzzy sets and their applications*, Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, 2000, 307-312.

[12] K. M. Lee, *Comparison of interval valued fuzzy sets, intuitionistic fuzzy sets and bipolar valued fuzzy sets,* J. Fuzzy Logic Intelligent Systems, 14 (2004), 125-129.

[13] P. K. Maji, R. Biswas and A. R. Roy, *Fuzzy soft sets*, The J. of Fuzzy Math., 9 (2001), 589-602.

[14] P. Majumdar and S. Samanta, *Generalized fuzzy soft set*, Comput. Math. Appl., 58 (2010), 1279-1286.

[15] D. Mandal, *Fuzzy ideals and fuzzy interior ideals in ordered semirings,* Fuzzy Inf. Engg., 6 (2014), 101-114

[16] D. A. Molodtsov, *Soft set theory-first results*, Computers and Math. with Appli., 37 (1999), 19-31.

[17] M. Murali Krishna Rao, *Fuzzy soft ideal, Fuzzy soft bi-ideal, Fuzzy soft quasi-ideal and fuzzy soft interior ideal over ordered* $\Gamma-$*Semiring,* Asia Pac. J. Math., 5 (2018), 60-84.

[18] M. Murali Krishna Rao, *Tripolar fuzzy interior ideal of* Γ*-semigroup*, Anl. of Fuzzy Math. and Info., 15 (2018), 199206.

[19] M. Murali Krishna Rao and B. Venkateswarlu, *Tripolar fuzzy interior ideal of* Γ*-semiring*, Asia Pac. J. Math., 5 (2018), 192-207.

[20] A. Rosenfeld, *Fuzzy groups,* J. Math. Anal. Appl., 35 (1971), 512-517.

[21] L. A. Zadeh, *Fuzzy sets*, Information and control, 8 (1965), 338-353.

[22] W. R. Zhang, *Bipolar fuzzy sets*, Proc. of FUZZ-IEEE, 1998, 835-840.

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