An automorphism theorem on certain type B semigroups

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Abstract. Motivated by studying translational hulls in semigroups and rings, and also motivated M. Petrich and J. E. Ault's works in inverse semigroups in terms of translational hulls, we attempt in this article to study the idempotents of the translational hull of a type B semigroup. Our main result is to prove that the idempotent set of the translational hull of an arbitrary type B semigroup is isomorphic to the translational hull of the idempotent set of such a semigroup.

Keywords: translational hulls, type B semigroups, reductive.

1. Introduction

Recall that a mapping λ [resp., ρ] which maps a semigroup S into itself is a left [resp., right] translation of S if $\lambda(ab) = (\lambda a)b$ [resp., $(ab)\rho = a(b\rho)$], for

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all $a, b \in S$; if also $a(\lambda b) = (a\rho)b$ for all $a, b \in S$, then λ and ρ are *linked* and the pair (λ, ρ) is a *bitranslation* of S. As usual, $\Lambda(S)$ and $P(S)$ denote the set of all left and right translations of S, respectively. $\Omega(S)$ denotes the set of all bitranslation of S. If we define a multiplication on $\Omega(S)$ by the rule $(\lambda, \rho)(\lambda', \rho') = (\lambda\lambda', \rho\rho')$ for all $(\lambda, \rho), (\lambda', \rho') \in \Omega(S)$, then it is easy to check that $\Omega(S)$ forms a subsemigroup of $\Lambda(S) \times P(S)$. We call the semigroup $\Omega(S)$ *the translational hull* of S. Further, we denote

$$\widetilde{\Lambda}(S) = \{\lambda \in \Lambda(S) |\ (\lambda, \rho) \in \Omega(S) \quad \text{for some} \quad \rho \in P(S)\},$$

$$\widetilde{P}(S) = \{\rho \in P(S) |\ (\lambda, \rho) \in \Omega(S) \quad \text{for some} \quad \lambda \in \Lambda(S)\} \quad \text{and}$$

$$\Pi(S) = \{(\lambda_a, \rho_a) |\ a \in S,\ \lambda_a x = ax,\ x\rho_a = xa \quad \text{for all} \quad x \in S\}.$$

The translational hull is an important concept in the algebraic theory of semigroups (see, [11-12]). Many papers studying the translational hulls of various classes of semigroups can be found in [1, 4, 5, 7, 10, 14, 15].

Following [2], the relations $\mathcal{L}^*$ and $\mathcal{R}^*$ on a semigroup S are generalizations of Green's relations $\mathcal{L}$ and $\mathcal{R}$ on S, respectively. Two elements a, b of S are $\mathcal{L}^*-$ [resp., $\mathcal{R}^*-$] related on S if and only if they are $\mathcal{L}-$ [resp., $\mathcal{R}-$] related on a semigroup T such that S is a subsemigroup of T. A semigroup S is called *right abundant* [resp., *left abundant*] if each $\mathcal{L}^*$ class [resp., $\mathcal{R}^*$ class] of S contains at least one idempotent. A semigroup S is called *abundant* if it is both left and right abundant. A right [resp., left] abundant semigroup in which the idempotents commute is *right adequate* [resp., *left adequate*]. A semigroup S is called *adequate* if and only if it is both right and left adequate. Usually, we denote by a^+ $[a^*]$ a typical idempotent $\mathcal{R}^*$-related $[\mathcal{L}^*$-related$]$ to a; $E(S)$ denotes the set of idempotents of S. If S is a semigroup and $\mathcal{K}$ is one of the relations $\mathcal{L}^*$ and $\mathcal{R}^*$ on S, then $\mathcal{K}^S$ and $\mathcal{K}^{\Omega(S)}$ denote the $\mathcal{K}$ relation on S and $\Omega(S)$, respectively.

A right adequate semigroup S is called *right type B*, if it satisfies the following conditions:

(B1) $(\forall e, f \in E(S^1), a \in S)\ (efa)^* = (ea)^*(fa)^*$, where $S^1 = S$ if S has an identity element 1, otherwise $S^1 = S \cup \{1\}$;

(B2) $(\forall a \in S, e \in E(S))\ e \leq a^* \Longrightarrow (\exists f \in E(S^1))\ \ e = (fa)^*$, where "$\leq$" is a natural partial order on $E(S)$ (i.e., $(\forall x, y \in E(S)) \quad x \leq y \Leftrightarrow x = xy = yx \quad (see, [6])$).

Dually, we can define a *left type B* semigroup. A semigroup which is both right and left type B is called *type B* (see, [3]). Recently, Li and others study some classes of type B semigroups (refer to [7, 8, 9]). In [4], Fountain proved that the translational hull of an adequate semigroup is adequate. For a type B semigroup S, the translational hull of S, $\Omega(S)$, is again a type B semigroup (refer to [7]), and thus the idempotents of $\Omega(S)$ form a semilattice (i.e., the idempotents are commute). How the structure of this semilattice, $E(\Omega(S))$, is influenced by the structure of the semilattice of idempotents of S, $E(S)$, is seen in our main results: $E(\Omega(S)) \cong \Omega(E(S))$. It is well known that an arbitrary inverse semigroup is type B, but the converse is not true. Therefore, our results generalize the corresponding results for inverse semigroups.

2. Preliminaries

We follow the notions in [2, 3, 4, 13]. First, we recall some known results and notations which will be frequently used in this paper.

Lemma 2.1 ([2]). *Let S be a semigroup and $a, b \in S$. Then the following statements are equivalent:*

(1) $a\mathcal{L}^*b$ $[a\mathcal{R}^*b]$;

(2) *for all* $x, y \in S^1$, $ax = ay$ $[xa = ya]$ *if and only if* $bx = by$ $[xb = yb]$.

Corollary 2.2 ([2]). *Let S be a semigroup and $a \in S, e \in E(S)$. Then the following statements are equivalent:*

(1) $a\mathcal{L}^*e$ $[\ a\mathcal{R}^*e]$;

(2) $ae = a$ $[ea = a]$ *and for all* $x, y \in S^1$, $ax = ay$ $[xa = ya]$ *implies* $ex = ey$ $[xe = ye]$.

It is easy to check that $\mathcal{L}^*$ is a right congruence and $\mathcal{R}^*$ is a left congruence. In an arbitrary semigroup, we can show $\mathcal{L} \subseteq \mathcal{L}^*$ and $\mathcal{R} \subseteq \mathcal{R}^*$. If S is a semigroup and a, b, are regular elements of S, then $a\mathcal{L}^*b$ $[a\mathcal{R}^*b]$ if and only if $a\mathcal{L}b$ $[a\mathcal{R}b]$. In particular, for an arbitrary adequate semigroup S, $(ab)^+ = (ab^+)^+$ and $(ab)^* = (a^*b)^*$ for all $a, b \in S$, and S is $\mathcal{L}^*-$unipotent and $\mathcal{R}^*-$ unipotent (i.e., each $\mathcal{L}^*-$ class and each $\mathcal{R}^*-$class of S contains exactly one idempotent).

Let S be an adequate semigroup and $a \in S$, $(\lambda, \rho) \in \Omega(S)$. The authors in [4] define the mappings $\lambda^*, \lambda^+, \rho^*$ and ρ^+ which map S into itself by the rule:

$$\lambda^*a = (\lambda a^+)^*a, \quad \lambda^+a = (a^+\rho)^+a, \quad a\rho^* = a(\lambda a^*)^*, \quad a\rho^+ = a(a^*\rho)^+.$$

It is clear that $(\lambda^*, \rho^*), (\lambda^+, \rho^+) \in \Omega(S)$ and furthermore, those elements are the idempotents of $\Omega(S)$.

Lemma 2.3 ([4]). *Let S be an abundant semigroup and λ_1, λ_2 [ρ_1, ρ_2] be left [right] translations of S. Then*

(1) $\lambda_1 = \lambda_2 \Longleftrightarrow (\forall e \in E(S)) \quad \lambda_1 e = \lambda_2 e$;

(2) $\rho_1 = \rho_2 \Longleftrightarrow (\forall e \in E(S)) \quad e\rho_1 = e\rho_2$.

Lemma 2.4 ([4]). *Let S be an adequate semigroup. Then the following statements are true:*

(1) $e\rho^* = \lambda^* e = (\lambda e)^* \in E(S)$, *for all* $e \in E(S)$;

(2) $e\rho^+ = \lambda^+ e = (e\rho)^+ \in E(S)$, *for all* $e \in E(S)$;

(3) $(\lambda^*, \rho^*)\mathcal{L}^{*\Omega(S)}(\lambda, \rho)\mathcal{R}^{*\Omega(S)}(\lambda^+, \rho^+)$;

(4) $E(\Omega(S)) = \{ \ (\lambda, \rho) \in \Omega(S) \mid \lambda E(S) \bigcup E(S)\rho \subseteq E(S) \ \}$.

Lemma 2.5 ([7]). *Let S be a type B semigroup such that $(\lambda_1, \rho_1), (\lambda_2, \rho_2) \in \Omega(S)$. Then the following statements hold:*

(1) $(\lambda_1, \rho_1) = (\lambda_2, \rho_2) \Longleftrightarrow \lambda_1 = \lambda_2 \Longleftrightarrow \rho_1 = \rho_2$*;*

(2) $\Omega(S)$ *is a type B semigroup;*

(3) $(\lambda_1, \rho_1)\mathcal{L}^{*\Omega(S)}(\lambda_2, \rho_2) \Longleftrightarrow \lambda_1 e\mathcal{L}^{*S}\lambda_2 e$ *for all* $e \in E(S)$;

(4) $(\lambda_1, \rho_1)\mathcal{R}^{*\Omega(S)}(\lambda_2, \rho_2) \Longleftrightarrow e\rho_1\mathcal{R}^{*S}e\rho_2$ *for all* $e \in E(S)$.

As in [13], a semigroup S is *left reductive [resp., right reductive]* if for any $a, b \in S$, $xa = xb$ [resp., $ax = bx$] for all $x \in S$ implies that $a = b$ and S is *reductive* if it is both left and right reductive.

Lemma 2.6 ([12]). *Let S be a reductive semigroup. Then $S \cong \Pi(S)$ and $\Omega(S) \cong \widetilde{\Lambda}(S) \cong \widetilde{P}(S)$.*

3. Properties

In this section we will consider some properties of the translational hull of a type B semigroup.

Theorem 3.1. *Let S be a type B semigroup. Then*

(1) $S \cong \Pi(S)$ *and* $\Omega(S) \cong \widetilde{\Lambda}(S) \cong \widetilde{P}(S)$;

(2) $\Pi(E(S)) = E(\Pi(S))$.

Proof. (1) By Lemma 2.6, it only remains to show that S is reductive. To see it, let $a, b \in S$ be such that $ax = bx$ for all $x \in S$. Then $(ax)^* = (bx)^*$. Hence, $(a^*x)^* = (b^*x)^*$ since S is type B. Choose an idempotent a^* of S to replace the element x of the above formula, we get $a^* = b^*a^*$. Similarly, $b^* = a^*b^*$. Note that $E(S)$ is a semilattice, we have $a^* = b^*$. Thus, $a = aa^* = ba^* = bb^* = b$. Therefore, S is right reductive. Dually, S is left reductive, as required.

(2) Straightforward. □

Example 3.1 ([7]). Let $\mathbb{N}$ be the set of all non-negative integers and put $S = \{(m, n) \in \mathbb{N} \times \mathbb{N} \mid m \geq n \ \}$. Define a multiplication "$\bullet$" on S by

$$(m, n) \bullet (p, q) = (m - n + t, q - p + t),$$

where $t = \max\{n, p\}$. Then, it is easy to check that $(S, \bullet)$ is a semigroup and $E(S) = \{(m, m) \in \mathbb{N} \times \mathbb{N}\}$. As in [7], Li and Wang have proved that S is a type B semigroup such that $(n, n)\mathcal{L}^*(m, n)\ \mathcal{R}^*(m, m)$ for all $(m, n) \in S$. Hence, $(m, n)^* = (n, n)$ and $(m, n)^+ = (m, m)$.

Next, we show that S is reductive. Let $(m, n), (h, k) \in S$ be such that $(m, n) \bullet (p, q) = (h, k) \bullet (p, q)$ for all $(p, q) \in S$. Then

$$(m - n + t,\ q - p + t) = (h - k + s,\ q - p + s), \tag{1.1}$$

where $t = \max\{n, p\}, s = \max\{k, p\}$. Hence, $s = t$ and $m - n = h - k$. Choose the idempotent $(0, 0)$ of S to replace the element (p, q) of the above formula (1.1), we have $n = t = s = k$, and thus $m = h$. Therefore, $(m, n) = (h, k)$, this gives that S is right reductive. Dually, S is left reductive.

Theorem 3.2. *Let S be a type B semigroup. Then the following statements are true:*

(1) *if λ is a mapping of S into itself, then $\lambda \in \Lambda(S)$ if and only if $(\lambda e)f = \lambda f$ for all $e, f \in E(S)$ with $f \leq e$, and $\lambda x = (\lambda x^+)x$ for all $x \in S$;*

(2) *if ρ is a mapping of S into itself, then $\rho \in P(S)$ if and only if $f(e\rho) = f\rho$ for all $e, f \in E(S)$ with $f \leq e$, and $x\rho = x(x^*\rho)$ for all $x \in S$;*

(3) *for $\lambda \in \Lambda(S), \rho \in P(S)$, $(\lambda, \rho) \in \Omega(S)$ if and only if $e(\lambda f) = (e\rho)f$ for all $e, f \in E(S)$.*

Proof. (1) Let $\lambda x = (\lambda x^+)x$ for all $x \in S$ and $(\lambda e)f = \lambda f$ for all $e, f \in E(S)$ with $f \leq e$. Then, for all $x, y \in S$, we have

$$\begin{aligned} \lambda(xy) &= (\lambda(xy)^+)xy && (\text{ by hypothesis }) \\ &= (\lambda x^+)(xy)^+xy && (\text{ from } (xy)^+ \leq x^+ \text{and hypothesis }) \\ &= (\lambda x^+)xy && \\ &= (\lambda x)y && (\text{ by hypothesis }) \end{aligned}$$

This gives $\lambda \in \Lambda(S)$. The converse is clear.

(2) It is the dual of (1).

(3) Let $\lambda \in \Lambda(S), \rho \in P(S)$ be such that $e(\lambda f) = (e\rho)f$ for all $e, f \in E(S)$. Then, for all $x, y \in S$, we have

$$\begin{aligned} x(\lambda y) &= xx^*(\lambda y^+)y && (\text{ by (1) }) \\ &= x(x^*\rho)y^+y && (\text{ by hypothesis }) \\ &= (x\rho)y && (\text{ by (2) }) \end{aligned}$$

Therefore, $(\lambda, \rho) \in \Omega(S)$. The converse is clear. □

4. Main results

In this section, we shall give characterizations of the semilattice of idempotents of $\Omega(S)$ for an arbitrary type B semigroup S. Furthermore, we shall give the proof of our main result: $E(\Omega(S)) \cong \Omega(E(S))$.

Theorem 4.1. *Let S be a type B semigroup. Then*

$$E(\Omega(S)) \cong \{\ \lambda \in \Lambda(S) \mid \lambda E(S) \subseteq E(S)\ \}.$$

Proof. By Lemma 2.5(2), $E(\Omega(S))$ is a semilattice. Consider the mapping

$$\psi : E(\Omega(S)) \longrightarrow \{\ \lambda \in \Lambda(S) \mid \lambda E(S) \subseteq E(S)\ \},\ (\lambda, \rho) \longmapsto \lambda.$$

By Lemma 2.4(4), it is easy to see that ψ is well-defined. Clearly ψ is a homomorphism.

Let $\lambda \in \Lambda(S)$ with $\lambda E(S) \subseteq E(S)$. Then define a mapping ρ on S by

$$x\rho = x\lambda(x^*) \quad \text{for all} \quad x \in S.$$

Hence, for any $e \in E(S)$, $e\rho = e(\lambda e) = (\lambda e)e = \lambda e$ since $\lambda e \in E(S)$ and $E(S)$ is a semilattice. That is, $e\rho \in E(S)$ for all $e \in E(S)$. Note that $E(S)$ is a semilattice. For all $e, f \in E(S)$, we have

$$(fe)\rho = \lambda(fe) = \lambda(ef) = (\lambda e)f = f(\lambda e) = f(e\rho).$$

In particular, $f\rho = f(e\rho)$, for all $e, f \in E(S)$ with $f \leq e$; also, for $x \in S$, $x(x^*\rho) = x(\lambda x^*) = x\rho$. By Theorem 3.2(2), $\rho \in P(S)$. Hence, for all $e, f \in E(S)$,

$$e(\lambda f) = e(f\rho) = (ef)\rho = (fe)\rho = f(e\rho) = (e\rho)f,$$

and so $(\lambda, \rho) \in \Omega(S)$ from Theorem 3.2(3). This together with the fact $\lambda e = e\rho \in E(S)$ for all $e \in E(S)$ and with Lemma 2.4(4), yields that $(\lambda, \rho) \in E(\Omega(S))$. Therefore, ψ is surjective.

On the other hand, let $\psi[(\lambda_1, \rho_1)] = \psi[(\lambda_2, \rho_2)]$ for some $(\lambda_1, \rho_1), (\lambda_2, \rho_2) \in E(\Omega(S))$, that is, $\lambda_1 = \lambda_2$. By Lemma 2.5(1), $(\lambda_1, \rho_1) = (\lambda_2, \rho_2)$. This gives that ψ is injective. Therefore, $E(\Omega(S)) \cong \{\ \lambda \in \Lambda(S) \mid \lambda E(S) \subseteq E(S)\ \}$. □

Proposition 4.2. *Let S be a type B semigroup and $\lambda \in \Lambda(S), \rho \in P(S)$. Then $(\lambda, \rho) \in E(\Omega(S))$ if and only if $\lambda^2 = \lambda, \rho^2 = \rho$ and $\lambda e = e\rho$ for all $e \in E(S)$.*

Proof. By Lemma 2.5(2), $\Omega(S)$ is a type B semigroup and $\Omega(S)$ is $\mathcal{L}^*-$unipotent. Let $(\lambda, \rho) \in E(\Omega(S))$. Then $\lambda^2 = \lambda, \rho^2 = \rho$. By Lemma 2.4(3), $(\lambda^*, \rho^*)\mathcal{L}^*(\lambda, \rho)$. Hence, $(\lambda, \rho) = (\lambda^*, \rho^*)$ since $\Omega(S)$ is $\mathcal{L}^*-$unipotent. Thus, by Lemma 2.4(1), $\lambda e = \lambda^* e = e\rho^* = e\rho$, as required.

Conversely, suppose that $\lambda \in \Lambda(S), \rho \in P(S)$ with $\lambda^2 = \lambda, \rho^2 = \rho$, and $\lambda e = e\rho$ for all $e \in E(S)$. Then, for all $e, f \in E(S)$, we have

$$(\lambda e)^2 = (\lambda e)(\lambda e) = (\lambda e)(e\rho) = \lambda(e(e\rho)) = \lambda(e\rho) = \lambda(\lambda e) = \lambda^2 e = \lambda e.$$

Hence, $\lambda e = e\rho \in E(S)$, and so $(e\rho)f = f(e\rho) = (fe)\rho = (ef)\rho = e(f\rho) = e(\lambda f)$ since $E(S)$ is a semilattice. By Theorem 3.2(3), $(\lambda, \rho) \in \Omega(S)$. This together with the facts $\lambda^2 = \lambda, \rho^2 = \rho$, yields that $(\lambda, \rho) \in E(\Omega(S))$. □

Theorem 4.3. *Let S be a type B semigroup. Then $E(\Omega(S)) \cong \Omega(E(S))$.*

Proof. By Lemma 2.4(4), it is easy to see that $\Omega((E(S))$ is indeed a semilattice. Consider a mapping θ as follows:

$$\theta : E(\Omega(S)) \longrightarrow \Omega(E(S)), \qquad (\lambda, \rho)\theta = (\lambda|_{E(S)}, \rho|_{E(S)}),$$

where $\lambda|_{E(S)}$ [resp., $\rho|_{E(S)}$] is the restriction of λ [resp., ρ] to the set $E(S)$. By Lemma 2.4(4), $\lambda|_{E(S)} \in \Lambda(E(S)), \rho|_{E(S)} \in P(E(S))$ and $\lambda|_{E(S)}$ is linked to $\rho|_{E(S)}$. Hence θ maps into $\Omega(E(S))$.

Next, we prove that θ is injective. Let $(\lambda|_{E(S)}, \rho|_{E(S)}) = (\lambda'|_{E(S)}, \rho'|_{E(S)})$. Then, by Lemma 2.3, $(\lambda, \rho) = (\lambda', \rho')$, this gives that θ is injective.

To see that θ is surjective, let $(\lambda^\ddagger, \rho^\ddagger) \in \Omega(E(S))$. Define λ and ρ on S as follows:

$$\lambda x = (\lambda^\ddagger x^+)x, \qquad x\rho = x(x^*\rho^\ddagger).$$

Then

$$\lambda x = (\lambda^\ddagger x^+)x = (\lambda^\ddagger x^+)x^+x = (\lambda x^+)x,$$

and

$$(\lambda e)f = ((\lambda^\ddagger e)e)f = \lambda^\ddagger(ef) = \lambda^\ddagger f = (\lambda^\ddagger f)f = \lambda f$$

for all $e, f \in E(S)$ with $f \leq e$. By Theorem 3.2(1), $\lambda \in \Lambda(S)$. Dually, $\rho \in P(S)$. Furthermore,

$$e(\lambda f) = e(\lambda^\ddagger f)f = e(\lambda^\ddagger f) = (e\rho^\ddagger)f = e(e\rho^\ddagger)f = (e\rho)f,$$

for all $e, f \in E(S)$. By Theorem 3.2(3), $(\lambda, \rho) \in \Omega(S)$. Note that $\lambda|_{E(S)} = \lambda^\ddagger$ and $\rho|_{E(S)} = \rho^\ddagger$. We have

$$\lambda^2 e = \lambda(\lambda e) = \lambda(\lambda^\ddagger e) = \lambda^\ddagger(\lambda^\ddagger e) = {\lambda^\ddagger}^2 e = \lambda^\ddagger e = \lambda e,$$

and similarly $e\rho^2 = e\rho$ and so $\lambda^2 = \lambda, \rho^2 = \rho$. Hence,

$$\lambda e = (\lambda e)e = e(\lambda e) = (e\rho)e = e(e\rho) = e\rho.$$

By Proposition 4.2, $(\lambda, \rho) \in E(\Omega(S))$. Thus, θ is surjective. That θ is a homomorphism is clear. Therefore, $E(\Omega(S)) \cong \Omega(E(S))$. □

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