On some weak structures in intuitionistic fuzzy soft topological spaces

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Abstract. Intuitionistic fuzzy set is fruitful and more realistic model of uncertainty than the fuzzy sets. Extending this approach further, intuitionistic fuzzy soft sets were introduced, as a generalization of fuzzy soft sets and standard soft sets. Intuitionistic fuzzy soft sets used to handle the problem of muti-criteria decision making. In this paper, We initiate and establish the topological structures of intuitionistic fuzzy soft semi-open sets and intuitionistic fuzzy soft semi-closed sets. We also define and explore the properties of intuitionistic fuzzy soft semi-interior, intuitionistic fuzzy soft semi-closure and discuss the relationship between them. Moreover, we establish and discuss intuitionistic fuzzy soft semi-open neighborhood systems. We believe that the findings in this paper will enhance the further study proceed towards muti-criteria decision making problems.

Keywords: intuitionistic fuzzy soft sets, intuitionistic fuzzy soft topology, intuitionistic fuzzy soft semi-open(closed), intuitionistic fuzzy soft semi-interior(closure), intuitionistic fuzzy soft semi-open nbd systems.

1. Introduction

Molodtsov [15] introduced soft sets theory as a new mathematical tool to deal with uncertainties, imprecise and unclear defined objects and to overcome incompatibility with the parametrization tools, which were not solved by existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. This theory proved to be useful in different fields not limited to decision making [6][18], data analysis [4][22], forecasting [19] and so on. The topological structures of soft sets are studied and discussed in [7], [9-10]. In [17], Molodtsov et. al mentioned several directions for the applications of soft sets such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement for modelling the problems in engineering, computer science, economics, social sciences and medical sciences. In soft systems a very general frame work has been provided with the involvement of parameters.

Zadeh [21] initiated the concept of fuzzy sets. intuitionistic fuzzy set (IFS) and intuitionistic L-fuzzy sets (ILFS) were initiated and explored by Atanassov [1-3] to generalize the concept of fuzzy set. Maji et al. introduced the concept of intuitionistic fuzzy soft sets[13-15], which is a generalization of fuzzy soft sets[12] and standard soft sets[15]. It is to be noted that the parameters may not always be crisp, rather they may be intuitionistic fuzzy in nature. Different algebraic structures of intuitionistic fuzzy soft sets are studied and explored in [20]. Coker [5] introduced and studied the concept of intuitionistic fuzzy topological spaces. Li et al. [11] initiated intuitionistic fuzzy topological structures of intuitionistic fuzzy soft sets. They explored the notions of intuitionistic fuzzy soft open(closed) sets, intuitionistic fuzzy soft interior(closure) and intuitionistic fuzzy soft base in intuitionistic fuzzy soft topological spaces. Recently, Hussain [8] initiate the concept of intuitionistic fuzzy soft boundary and explored the characterizations and properties of intuitionistic fuzzy soft boundary in general as well as in terms of intuitionistic fuzzy soft interior and intuitionistic fuzzy soft closure.

In this paper, we initiate and establish the topological structures of intuitionistic fuzzy soft semi-open sets and intuitionistic fuzzy soft semi-closed sets. We also define and explore the properties of intuitionistic fuzzy soft semi-interior, intuitionistic fuzzy soft semi-closure and discuss the relationship between them. Moreover, we establish and discuss intuitionistic fuzzy soft semi-open neighborhood systems.

2. Preliminaries

We recall the following:

Definition 2.1 ([21]). A fuzzy set f on X is a mapping $f : X \to I = [0, 1]$. The value of $f(x)$ represents the degree of membership of $x \in X$ in the fuzzy set f, for $x \in X$.

Definition 2.2 ([16]). Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.3 ([12]). Let I^X denotes the set of all fuzzy sets on X and $A \subseteq X$. A pair (f, A) is called a fuzzy soft set over X, where $f : X \to I^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : X \to$ I is a fuzzy set on X .

Definition 2.4 ([2]). An intuitionistic fuzzy set A over the universe X is defined as: $A = \{(x, \mu_A(x), \nu_A(x)); x \in X\}$, where $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ with the property that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, for all $x \in X$. The values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and nonmembership of x to A respectively.

Definition 2.5 ([2]). Let $A = \{(x, \mu_A(x), \nu_A(x)); x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x)); x \in X\}$ are intuitionistic fuzzy set over the universe X. Then

(1) $A^c = \{(x, \nu_A(x), \mu_A(x)); x \in X\}$.

(2) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$.

(3)$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

(4) $A \cap B = \{(x, min\{\mu_A(x), \mu_B(x)\}, max\{\nu_A(x), \nu_B(x)\} : x \in X\}$.

(5) $A \cup B = \{(x, max\{\mu_A(x), \mu_B(x)\}, min\{\nu_A(x), \nu_B(x)\} : x \in X\}$.

Definition 2.6 ([2]). An intuitionistic fuzzy set A over the universe X is said to be intuitionistic fuzzy null set denoted as $\tilde{0}$, and is defined as: $A = \{(x, 0, 1) : x \in X\}$.

Definition 2.7 ([2]). An intuitionistic fuzzy set A over the universe X is said to be intuitionistic fuzzy absolute set denoted as $\tilde{1}$, and is defined as: $A = \{(x, 1, 0) : x \in X\}$.

Definition 2.8 ([14]). Let X be the initial universal set and E be the set of parameters. Let IF^X denotes the set of all intuitionistic fuzzy soft sets on X and $A \subseteq X$. A pair (IF, A) is called a IF soft set over X, where $f : A \to IF^X$ is a function. That is, for each $a \in A$, $f(a) = f_a : A \to IF^X$, is an intuitionistic fuzzy set on X and is defined as: $f(a) = \{(x, \mu_A(x), \nu_A(x)); x \in X\}$.

From now on, for our convenience, we will represent the intuitionistic fuzzy soft set (IF, A) as IF soft set f_A.

Now we give the example of IF soft sets as:

Example 2.9. Let $(IF, A) = f_A$ describe the character of the employees with respect to the given parameters, for finding the best employee of the financial year. Let the set of employees under consideration be $X = \{x_1, x_2, x_3, x_4\}$. Let $E = \{$ regular workload (r), conduct (c), field performances (g), sincerity(s), pleasing personality (p)$\}$ be the set of parameters framed to choose the best employee. Suppose the administrator Mr. X has the parameter set $A = \{r, c, p\} \subseteq E$ to choose the best employee. Then f_A be the an IF soft set over X, defined as follows:

$f(r)(x_1) = (0.8, 0.1), f(r)(x_2) = (0.7, 0.5), f(r)(x_3) = (0.9, 0.1), f(r)(x_4) = (0.7, 0.2)$

$f(c)(x_1) = (0.6, 0.2),\ f(c)(x_2) = (0.7, 0.1),\ f(c)(x_3) = (0.5, 0.3),\ f(c)(x_4) = (0.3, 0.6)$

$f(p)(x_1) = (0.6, 0.2),\ f(p)(x_2) = (0.7, 0.1),\ f(p)(x_3) = (0.5, 0.3),\ f(p)(x_4) =$

(0.3, 0.6)
That is

$f_A = (IF, A)\widetilde{\widetilde{=}}\{F(r) = \{(x_1, 0.8, 0.1), (x_2, 0.7, 0.05), (x_3, 0.9, 0.1), (x_4, 0.7, 0.2)\},$
$F(c) = \{(x_1, 0.6, 0.2), (x_2, 0.7, 0.1), (x_3, 0.5, 0.3), (x_4, 0.3, 0.6)\},$
$F(p) = \{(x_1, 0.6, 0.2), (x_2, 0.7, 0.1), (x_3, 0.5, 0.3), (x_4, 0.3, 0.6)\}\}.$

The tabular representation of IF soft sets f_A is:

	x_1	x_2	x_3	x_4
r	(0.8, 0.1)	(0.7, 0.5)	(0.9, 0.1)	(0.7, 0.2)
c	(0.6, 0.2)	(0.7, 0.1)	(0.5, 0.3)	(0.3, 0.6)
p	(0.6, 0.2)	(0.7, 0.1)	(0.5, 0.3)	(0.3, 0.6)

In short, we will represent f_A as:
$f_A\widetilde{\widetilde{=}}\{\{x_{(0.8,0.1)}, x_{(0.7,0.05)}, x_{(0.9,0.1)}, x_{(0.7,0.2)}\}, \{x_{(0.6,0.2)}, x_{(0.7,0.1)}, x_{(0.5,0.3)}, x_{(0.3,0.6)}\},$
$\{x_{(0.6,0.2)}, x_{(0.7,0.1)}, x_{(0.5,0.3)}, x_{(,0.3,0.6)}\}\}.$

Definition 2.10 ([14]). Two IF soft sets f_A and g_B over a common universe X, we say that f_A is a IF soft subset of g_B if

(1) $A \subseteq B$ and

(2) for all $a \in A$, $f_a \subseteq g_a$; implies f_a is a IF subset of g_a.

We denote it by $f_A\widetilde{\widetilde{\subseteq}}g_B$. f_A is said to be a IF soft super set of g_B, if g_B is a IF soft subset of f_A. We denote it by $f_A\widetilde{\widetilde{\supseteq}}g_B$.

Note that two IF soft sets f_A and g_B over a common universe X are said to be IF soft equal, if f_A is an IF soft subset of g_B and g_B is an If soft subset of f_A.

Definition 2.11 ([14]). The union of two IF soft sets of f_A and g_B over the common universe X is the IF soft set h_C, where $C = A \cup B$ and for all $c \in C$,

$$h(c) = \begin{cases} f(c), & \text{if } c \in A - B \\ g(c), & \text{if } c \in B - A \\ f(c) \cup g(c), & \text{if } c \in A \cap B \end{cases}$$

We write $f_A\widetilde{\widetilde{\cup}}g_B = h_C$.

Definition 2.12 ([14]). The intersection h_C of two IF soft sets f_A and g_B over a common universe X , denoted $f_A\widetilde{\widetilde{\cap}}g_B$, is defined as $C = A \cap B$, and $h(c) = f(c) \cap g(c)$, for all $c \in C$.

Definition 2.13 ([14]). The relative complement of an IF soft set f_A is an IF soft set f_A^c, which is denoted by $(f_A)^c$ and where $f^c : A \to IF(X)$ is a IF set-valued function that is, for each $x \in A$, $f^c(A)$ is an IF set in X, whose membership function $f_a^c(x) = (f_a(x))^c$, for all $x \in A$. Here f_a^c is the membership function of $f^c(a)$.

Definition 2.14 ([11]). Let τ be the collection of IF soft sets over X, then τ is said to be an IF soft topology on X, if

(1) $\tilde{\tilde{\Phi}}_A$, $\tilde{\tilde{X}}_A$ belong to τ.

(2) If $(f_A)_i \in \tau$, for all $i \in I$, then $\tilde{\tilde{\cup}}_{i\in I}(f_A)_i \in \tau$.

(3) For $f_a, g_b \in \tau$ implies that $f_a \tilde{\tilde{\cap}} g_b \in \tau$.

The triplet (X, τ, A) is called an IF soft topological space over X. Every member of τ is called IF soft open set. An IF soft set is called IF soft closed if and only if its complement is IF soft open.

Example 2.15. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and $\tau = \{\widetilde{\widetilde{\Phi_A}}, \widetilde{\widetilde{X_A}}, f_A, g_A, h_A, k_A\}$, where f_A, g_A, h_A, k_A are IF soft sets over X, defined as follows

$$\begin{aligned}
f(e_1)(x_1) &= (0.2, 0.8), \quad f(e_1)(x_2) = (0.6, 0.3),\\
f(e_2)(x_1) &= (0.2, 0.5), \quad f(e_2)(x_2) = (0.9, 0.1),\\
g(e_1)(x_1) &= (0.1, 0.8), \quad g(e_1)(x_2) = (0.6, 0.1),\\
g(e_2)(x_1) &= (0.2, 0.8), \quad g(e_2)(x_2) = (0.8, 0.1),\\
h(e_1)(x_1) &= (0.2, 0.8), \quad h(e_1)(x_2) = (0.6, 0.1),\\
h(e_2)(x_1) &= (0.2, 0.5), \quad h(e_2)(x_2) = (0.9, 0.1),\\
k(e_1)(x_1) &= (0.1, 0.8), \quad k(e_1)(x_2) = (0.6, 0.3),\\
k(e_2)(x_1) &= (0.2, 0.8), \quad k(e_2)(x_2) = (0.8, 0.1).
\end{aligned}$$

Then

$$\begin{aligned}
\tau =&\{\widetilde{\widetilde{\Phi_A}}, \widetilde{\widetilde{X_A}}, (\{x_{(0.2,0.8)}, x_{(0.6,0.3)}\}, \{x_{(0.2,0.5)}, x_{(0.9,0.1)}\}), (\{x_{(0.1,0.8)}, x_{(0.6,0.1)}\},\\
&\{x_{(0.2,0.8)}, x_{(0.8,0.1)}\}), (\{x_{(0.2,0.8)}, x_{(0.6,0.1)}\}, \{x_{(0.2,0.5)}, x_{(0.9,0.1)}\}), (\{x_{(0.1,0.8)},\\
&x_{(0.6,0.3)}\}, \{x_{(0.2,0.8)}, x_{(0.8,0.1)}\})\}
\end{aligned}$$

is an IF soft topology on X and hence (X, τ, A) is an IF soft topological space over X.

Theorem 2.16 ([11]). *Let τ be the collection of IF soft sets over X. Then*

(1) $\tilde{\tilde{\Phi}}_A$, $\tilde{\tilde{X}}_A$ *are IF soft closed sets over X.*

(2) *The intersection of any number of IF soft closed sets is an IF soft closed set over X.*

(3) *The union of any two IF soft closed sets is an IF soft closed set over X.*

Definition 2.17 ([11]). Let (X, τ, A) be an IF soft topological space over X and f_A be an IF soft set over X. Then IF soft interior of IF soft set f_A over X is denoted by $int(f_A)$ and is defined as the union of all IF soft open sets contained in f_A. Thus $int(f_A)$ is the largest IF soft open set contained in f_A.

Definition 2.18 ([11])**.** Let (X, τ, A) be an IF soft topological space over X and f_A be an IF soft set over X. Then the IF soft closure of f_A, denoted by $cl(f_A)$ is the intersection of all IF soft closed super sets of f_A. Clearly $cl(f_A)$ is the smallest IF soft closed set over X which contains f_A.

3. Properties of intuitionistic fuzzy (IF) soft semi-open(closed) sets

We define:

Definition 3.1. Let (X, τ, A) be an IF soft topological space over X, where X is a nonempty set and τ is a family of IF soft sets. An IF soft set f_A in an IF topological space (X, τ) is called IF soft semi-open, if there exists an IF soft open set g_A such that $g_A \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} cl(g_A)$. The class of all IF soft semi-open sets in X is denoted by $\widetilde{\widetilde{SSO}}(X)$.

Definition 3.2. An IF soft set f_A in an IF soft topological space (X, τ, A) is IF soft semi-closed if and only if its complement $(f_A)^c$ is IF soft semi-open. The class of an IF soft semi-closed sets is denoted by $\widetilde{\widetilde{SSC}}(X)$.

Remark 3.3. From the definition, it is obvious that an IF soft open set is IF soft semi-open in an IF soft topological space (X, τ, A). The following example shows that the converse is not true in general.

Example 3.4. Let us consider an IF soft topological space (X, τ, A) of Example 2.15. The IF soft closed sets are

$$\begin{aligned}&\widetilde{\widetilde{\Phi}}_A \tilde{\tilde{=}} (\{x_{(0,1)}, x_{(0,1)}\}, \{x_{(0,1)}, x_{(0,1)}\}), \widetilde{\widetilde{X}}_A \tilde{\tilde{=}} (\{x_{(1,0)}, x_{(1,0)}\}, \{x_{(1,0)}, x_{(1,0)}\}),\\ &\quad (\{x_{(0.8,0.2)}, x_{(0.3,0.6)}\}, \{x_{(0.5,0.2)}, x_{(0.1,0.9)}\}), (\{x_{(0.8,0.1)}, x_{(0.1,0.6)}\},\\ &\quad \{x_{(0.8,0.2)}, x_{(0.1,0.8)}\}), (\{x_{(0.8,0.2)}, x_{(0.1,0.6)}\}, \{x_{(0.5,0.2)}, x_{(0.1,0.9)}\}),\\ &\quad (\{x_{(0.8,0.1)}, x_{(0.3,0.6)}\}, \{x_{(0.8,0.2)}, x_{(0.1,0.8)}\}).\end{aligned}$$

Now we consider an IF soft set l_A over X defined by
$l(e_1)(x_1) = (0.2, 0.7)$, $l(e_1)(x_2) = (0.6, 0.2)$,
$l(e_2)(x_1) = (0.2, 0.4)$, $l(e_2)(x_2) = (0.9, 0.1)$,

That is, $l_A \tilde{\tilde{=}} (\{x_{(0.2,0.7)}, x_{(0.6,0.2)}\}, \{x_{(0.2,0.4)}, x_{(0.9,0.1)}\})$. Then there exists an IF soft open set $f_A \tilde{\tilde{=}} (\{x_{(0.2,0.8)}, x_{(0.6,0.3)}\}, \{x_{(0.2,0.5)}, x_{(0.9,0.1)}\})$ such that $f_A \tilde{\subseteq} l_A \tilde{\subseteq} cl(f_A) \tilde{\tilde{=}} (\{x_{(1,0)}, x_{(1,0)}\}, \{x_{(1,0)}, x_{(1,0)}\}) \tilde{\tilde{=}} \widetilde{\widetilde{X}}_A$.

Hence l_A is an IF soft semi-open set, but l_A is not an IF soft open set.

Proposition 3.5. *Let f_A be an IF soft set in an IF soft topological space (X, τ, A). Then f_A is IF soft semi-open if and only if $f_A \tilde{\tilde{\subseteq}} cl(int(f_A))$.*

Proof. $(\Rightarrow)$ Suppose that f_A is IF soft semi-open, then the there exists an IF soft open set g_A such that $g_A \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} cl(g_A)$. Now $g_A \tilde{\tilde{\subseteq}} int(f_A)$ implies that $cl(g_A) \tilde{\tilde{\subseteq}} cl(int(f_A))$. Therefore $f_A \tilde{\tilde{\subseteq}} cl(g_A) \tilde{\tilde{\subseteq}} cl(int(f_A))$.

$(\Leftarrow)$ Suppose that $f_A \tilde{\tilde{\subseteq}} cl(int(f_A))$. Take $g_A \tilde{\tilde{=}} int(f_A)$, we have

$$g_A \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} cl(g_A).$$

This completes the proof.

Theorem 3.6. *Let (X, τ, A) be an IF soft topological space. Then an arbitrary union of IF soft semi-open sets is an IF soft semi-open set.*

Proof. Let $\{g_{A_\alpha} : \alpha \in I\}$ be a collection of IF soft semi-open sets and $h_A \tilde{\tilde{=}} \tilde{\tilde{\cup}}_{\alpha \in I} (g_A)_\alpha$. Since each g_{A_α} is IF soft semi-open, then there exist an IF soft open set f_{A_α} such that $f_{A_\alpha} \tilde{\tilde{\subseteq}} g_{A_\alpha} \tilde{\subseteq} cl(f_{A_\alpha})$ and so $\tilde{\tilde{\cup}}_{\alpha \in I} f_{A_\alpha} \tilde{\tilde{\subseteq}} \tilde{\tilde{\cup}}_{\alpha \in I} g_{A_\alpha}$ $\tilde{\tilde{\subseteq}} \tilde{\tilde{\cup}}_{\alpha \in I} cl(f_{A_\alpha}) \tilde{\tilde{\subseteq}} cl(\tilde{\tilde{\cup}}_{\alpha \in I} f_{A_\alpha})$. Let $f_A \tilde{\tilde{=}} \tilde{\tilde{\cup}}_{\alpha \in I} f_{A_\alpha}$. Then f_A is IF soft open and $f_A \tilde{\tilde{\subseteq}} \tilde{\tilde{\cup}}_{\alpha \in I} g_{A_\alpha} \tilde{\subseteq} cl(f_A)$. Therefore, $\tilde{\tilde{\cup}}_{\alpha \in I} g_{A_\alpha}$ is an IF soft semi-open set. Hence the proof.

Proposition 3.7. *Let f_A be an IF soft semi-open set and h_A be an IF soft set in an IF soft topological space (X, τ, A). Suppose $f_A \tilde{\tilde{\subseteq}} h_A \tilde{\tilde{\subseteq}} cl(f_A)$. Then h_A is an IF soft semi-open set in X.*

Proof. f_A be an IF soft semi-open set implies that there exists an IF soft open set g_A such that $g_A \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} cl(g_A)$. Now $g_A \tilde{\tilde{\subseteq}} h_A$ and $cl(f_A) \tilde{\tilde{\subseteq}} cl(g_A)$ implies that $h_A \tilde{\tilde{\subseteq}} cl(g_A)$. Therefore $g_A \tilde{\tilde{\subseteq}} h_A \tilde{\tilde{\subseteq}} cl(g_A)$. Hence h_A is an IF soft semi-open set in X. This completes the proof.

Proposition 3.8. *Let f_A be an IF soft set in an IF soft topological space (X, τ, A). Then f_A is IF soft semi-closed if and only if there exists an IF soft closed set h_A such that $int(h_A) \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} h_A$.*

Proof. This follows directly from the definition of an IF soft semi-closed set.

The following proposition is obvious.

Proposition 3.9. *Every IF soft closed set is IF soft semi-closed in an IF soft topological space (X, τ, A).*

The following example shows that the converse of the above proposition is not true in general.

Example 3.10. Consider an IF soft topological space (X, τ, A) over X of Example 3.4. Take an IF soft set $m_A \tilde{\tilde{=}} (l_A)^c$ over X as:

$m_A \tilde{\tilde{=}} (\{x_{(0.7,0.2)}, x_{(0.2,0.6)}\}, \{x_{(0.4,0.2)}, x_{(0.1,0.9)}\})$.

We observe that m_A is an IF soft semi-closed, but m_A is not an IF soft closed sets.

Next we give the characterization of an IF soft semi-closed set.

Theorem 3.11. *Let f_A be an IF soft set in an IF soft topological space (X, τ, A). Then f_A is IF soft semi-closed if and only if $int(cl(f_A)) \tilde{\tilde{\subseteq}} f_A$.*

Proof. $(\Rightarrow)$ Suppose that f_A is IF soft semi-closed, then by Proposition 3.8, there exists an IF soft closed set h_A such that $int(h_A) \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} h_A$. This follows that $cl(f_A) \tilde{\tilde{\subseteq}} cl(h_A) \tilde{\tilde{=}} h_A$. Thus $int(cl(f_A)) \tilde{\tilde{\subseteq}} int(h_A)$.

Therefore, $int(cl(f_A)) \tilde{\tilde{\subseteq}} int(h_A) \tilde{\tilde{\subseteq}} f_A$.

$(\Leftarrow)$ Suppose that f_A be an IF soft set in (X, τ, A) such that $int(cl(f_A)) \tilde{\tilde{\subseteq}} f_A$. Take $cl(f_A) \tilde{\tilde{=}} h_A$. Then $int(h_A) \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} h_A$. This implies that f_A is an IF soft semi-closed set. Hence the proof.

Theorem 3.12. *Let $\{f_{A_\alpha} : \alpha \in I\}$ be a collection of an IF soft semi-closed sets in an IF soft topological space (X, τ, A). Then the intersection $\tilde{\tilde{\cap}}_{\alpha \in I}(f_{A_\alpha})$ is an IF soft semi-closed set in (X, τ, A).*

Proof. Since each $\alpha \in I$, f_{A_α} is an IF soft semi-closed set, then by Proposition 3.8, there exists an IF soft closed set h_{A_α} such that $int(h_{A_\alpha}) \tilde{\tilde{\subseteq}} (f_{A_\alpha}) \tilde{\tilde{\subseteq}} (h_{A_\alpha})$. This follows that $\tilde{\tilde{\cap}}_{\alpha \in I}(int(h_{A_\alpha})) \tilde{\tilde{\subseteq}} \tilde{\tilde{\cap}}_{\alpha \in I}(int(h_{A_\alpha})) \tilde{\tilde{\subseteq}} \tilde{\tilde{\cap}}_{\alpha \in I}(f_{A_\alpha}) \tilde{\subseteq} \tilde{\tilde{\cap}}_{\alpha \in I}(h_{A_\alpha})$. Take $\tilde{\tilde{\cap}}_{\alpha \in I}(h_{A_\alpha}) \tilde{\tilde{=}} (h_A)$. Then by Theorem 2.16, h_A is an IF soft closed and hence $\tilde{\tilde{\cap}}_{\alpha \in I}(f_{A_\alpha})$ is an IF soft semi-closed. This completes the proof.

Theorem 3.13. *Let f_A be an IF soft semi-closed set and g_A be an IF soft set in an IF soft topological space (X, τ, A). If $int(f_A) \tilde{\tilde{\subseteq}} g_A \tilde{\tilde{\subseteq}} f_A$, then g_A is an IF soft semi-closed set.*

Proof. Since f_A is an IF soft semi-closed, then by Proposition 3.8, there exists an IF soft closed set h_A such that $int(h_A) \tilde{\tilde{\subseteq}} f_A \tilde{\tilde{\subseteq}} h_A$. Then $g_A \tilde{\tilde{\subseteq}} h_A$. Also $int(int(h_A)) \tilde{\tilde{=}} int(h_A) \tilde{\tilde{\subseteq}} int(f_A)$. This implies that $int(h_A) \tilde{\tilde{\subseteq}} g_A$.

Therefore, $int(h_A) \tilde{\tilde{\subseteq}} g_A \tilde{\tilde{\subseteq}} h_A$. Hence g_A is an IF soft semi-closed set. Hence the proof.

Definition 3.14. Let f_A be an IF soft sets in an IF soft topological spaces (X, τ, A). Then

(1) the IF soft semi-interior of f_A is denoted by $\widetilde{\widetilde{sint}}(f_A)$ and is defined as the union of all IF soft semi-open subsets of f_A.

Clearly, $\widetilde{\widetilde{sint}}(f_A)$ is the largest IF soft semi-open set over X contained in f_A.

(2) the IF soft semi-closure of f_A is denoted by $\widetilde{\widetilde{scl}}(f_A)$ and is defined as the intersection of all IF soft semi-closed supersets of f_A.

Note that $\widetilde{\widetilde{scl}}(f_A)$ is the smallest IF soft semi-closed set over X which contains f_A.

Example 3.15. Let us consider an IF soft topological space (X, τ, A) over X and an IF soft closed sets of Example 3.10. It is clear that for any IF soft closed set k_A, $\widetilde{\widetilde{scl}}(k_A)\tilde{\tilde{=}}k_A$.

Example 3.16. Let us consider an IF soft topological space (X, τ, A) over X and an IF soft open sets of Example 3.4. It is clear that for any IF sot open set f_A, $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{=}}(f_A)$.

From Remark 3.3, Proposition 3.9 and Definition 3.14, we have:

Remark 3.17. Clearly, if f_A be an IF soft set in an IF soft topological space (X, τ, A), then $int(f_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\subseteq}}(f_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{scl}}(f_A)\tilde{\tilde{\subseteq}}cl(f_A)$.

Theorem 3.18. *Let f_A and g_A are two IF soft sets in an IF soft topological space (X, τ, A) over X. Then*

(1) $\widetilde{\widetilde{sint}}(\widetilde{\widetilde{\Phi_A}})\tilde{\tilde{=}}\widetilde{\widetilde{scl}}(\widetilde{\widetilde{\Phi_A}})\tilde{\tilde{=}}\widetilde{\widetilde{\Phi_A}}$ *and* $\widetilde{\widetilde{sint}}(\widetilde{\widetilde{X_A}})\tilde{\tilde{=}}\widetilde{\widetilde{scl}}(\widetilde{\widetilde{X_A}})\tilde{\tilde{=}}\widetilde{\widetilde{X_A}}$.

(2) *f_A is an IF soft semi-open (respt. IF soft semi-closed) set if and only if* $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{=}}f_A$ *(respt.* $\widetilde{\widetilde{scl}}(f_A)\tilde{\tilde{=}}f_A$*).*

(3) $\widetilde{\widetilde{sint}}(\widetilde{\widetilde{sint}}(f_A))\tilde{\tilde{=}}f_A$.

(4) $f_A\tilde{\tilde{\subseteq}}(g_A)$ *implies* $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(g_A)$ *and* $\widetilde{\widetilde{scl}}(f_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{scl}}(g_A)$.

(5) $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{sint}}(g_A)\tilde{\tilde{=}}\widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cap}}g_A)$ *and* $\widetilde{\widetilde{scl}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{scl}}(g_A)\tilde{\tilde{\supseteq}}\widetilde{\widetilde{scl}}(f_A\tilde{\tilde{\cap}}g_A)$.

(6) $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cup}}\widetilde{\widetilde{sint}}(g_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cup}}g_A)$ *and* $\widetilde{\widetilde{scl}}(f_A)\tilde{\tilde{\cup}}\widetilde{\widetilde{scl}}(g_A)\tilde{\tilde{=}}\widetilde{\widetilde{scl}}(f_A\tilde{\tilde{\cup}}g_A)$.

Proof.(1)–(4) follow directly from the definitions of IF soft semi-interior and If soft semi-closure and are therefore omitted.

(5) (i) Using (4), we have $(f_A\tilde{\tilde{\cap}}g_A)\tilde{\subseteq}f_A$, $(f_A\tilde{\tilde{\cap}}g_A)\tilde{\tilde{\subseteq}}g_A$ implies

$$\widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cap}}g_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(f_A), \widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cap}}g_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(g_A),$$

so that $\widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cap}}g_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{sint}}(g_A)$. Also, since $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\subseteq}}f_A$ and $\widetilde{\widetilde{sint}}(g_A)\tilde{\tilde{\subseteq}}g_A$ implies $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{sint}}(g_A)\tilde{\tilde{\subseteq}}(f_A\tilde{\tilde{\cap}}g_A)$.

Thus $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{sint}}(g_A)$ is a IF soft semi-open subset of $(f_A\tilde{\tilde{\cap}}g_A)$. Hence $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{sint}}(g_A)\tilde{\tilde{\subseteq}}\widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cap}}g_A)$. Thus $\widetilde{\widetilde{sint}}(f_A)\tilde{\tilde{\cap}}\widetilde{\widetilde{sint}}(g_A) = \widetilde{\widetilde{sint}}(f_A\tilde{\tilde{\cap}}g_A)$.

(ii) Using (4), we have $(f_A \tilde{\tilde{\cap}} g_A) \tilde{\tilde{\subseteq}} f_A$, $(f_A \tilde{\tilde{\cap}} g_A) \tilde{\tilde{\subseteq}} g_A$ implies $\widetilde{\widetilde{scl}}(f_A \tilde{\tilde{\cap}} g_A) \tilde{\tilde{\subseteq}} \widetilde{\widetilde{scl}}(f_A)$, $\widetilde{\widetilde{scl}}(f_A \tilde{\tilde{\cap}} g_A) \tilde{\tilde{\subseteq}} \widetilde{\widetilde{scl}}(g_A)$, so that $\widetilde{\widetilde{scl}}(f_A \tilde{\tilde{\cap}} g_A) \tilde{\tilde{\subseteq}} \widetilde{\widetilde{scl}}(f_A) \tilde{\tilde{\cap}} \widetilde{\widetilde{scl}}(g_A)$.

(6) The proof is similar to (5) by using the property that $f_A \tilde{\subseteq} (f_A \tilde{\tilde{\cup}} g_A)$, $g_A \tilde{\subseteq} (f_A \tilde{\tilde{\cup}} g_A)$. Hence the proof.

Theorem 3.19. *Let f_A be an IF soft set in an IF soft topological space (X, τ, A) over X. Then*

(1) $(\widetilde{\widetilde{sint}}(f_A))^c \tilde{\tilde{=}} \widetilde{\widetilde{scl}}((f_A)^c)$.

(2) $(\widetilde{\widetilde{scl}}(f_A))^c \tilde{\tilde{=}} \widetilde{\widetilde{sint}}((f_A)^c)$.

(3) $\widetilde{\widetilde{sint}}(int(f_A)) \tilde{\tilde{=}} int(\widetilde{\widetilde{sint}}(f_A)) \tilde{\tilde{=}} int(f_A)$.

(4) $\widetilde{\widetilde{scl}}(cl(f_A)) \tilde{\tilde{=}} cl((\widetilde{\widetilde{scl}}(f_A))) \tilde{\tilde{=}} cl(f_A)$.

Proof. (1) $\widetilde{\widetilde{sint}}(f_A) \tilde{\subseteq} f_A$ implies that $(f_A)^c \tilde{\subseteq} (\widetilde{\widetilde{sint}}(f_A))^c$.

Now by Theorem 3.18(2) and since $(\widetilde{\widetilde{sint}}(f_A))^c$ is a IF soft semi-closed set, we have $\widetilde{\widetilde{scl}}((f_A)^c) \tilde{\subseteq} \widetilde{\widetilde{scl}}((\widetilde{\widetilde{sint}}(f_A))^c) \tilde{\tilde{=}} (\widetilde{\widetilde{sint}}(f_A))^c$. For the reverse inclusion, $(f_A)^c \tilde{\subseteq} \widetilde{\widetilde{scl}}((f_A)^c)$ implies that $(\widetilde{\widetilde{scl}}((f_A)^c))^c \tilde{\subseteq} ((f_A)^c)^c \tilde{\tilde{=}} f_A$. $\widetilde{\widetilde{scl}}((f_A)^c)$ being IF soft semi-closed implies that $(\widetilde{\widetilde{scl}}((f_A)^c))^c$ is IF soft semi-open. Thus $(\widetilde{\widetilde{scl}}((f_A)^c))^c \tilde{\subseteq} \widetilde{\widetilde{sint}}(f_A)$ and hence $(\widetilde{\widetilde{sint}}(f_A))^c \tilde{\subseteq} ((\widetilde{\widetilde{scl}}((f_A)^c))^c)^c \tilde{\tilde{=}} \widetilde{\widetilde{scl}}((f_A)^c)$.

(2) The proof is same as (1).

(3) By Remark 3.3, $int(f_A)$ is an IF soft open implies that it is IF soft semi-open. Therefore, by Theorem 3.18(2), $\widetilde{\widetilde{sint}}(int(f_A)) \tilde{\tilde{=}} int(f_A)$. Now by Remark 3.17, $int(f_A) \tilde{\subseteq} \widetilde{\widetilde{sint}}(f_A) \tilde{\tilde{=}} f_A$. This implies that $\widetilde{\widetilde{sint}}(int(f_A)) \tilde{\tilde{=}} int(f_A)$.

(4) By Proposition 3.9, $cl(f_A)$ being an IF soft closed implies that it is IF soft semi-closed. Therefore, by Theorem 3.18(2), $\widetilde{\widetilde{scl}}(cl(f_A)) \tilde{\tilde{=}} cl(f_A)$. Now by Remark 3.17, $f_A \tilde{\subseteq} \widetilde{\widetilde{scl}}(f_A) \tilde{\tilde{\subseteq}} cl(f_A)$. Hence $\widetilde{\widetilde{scl}}(f_A) \tilde{\tilde{\subseteq}} cl(\widetilde{\widetilde{scl}}(f_A)) \tilde{\tilde{\subseteq}} \widetilde{\widetilde{scl}}(f_A)$. This implies that $cl((\widetilde{\widetilde{scl}}(f_A))) \tilde{\tilde{=}} cl(f_A)$. This completes the proof.

4. Intuitionistic fuzzy (IF) soft semi-neighborhood systems

Definition 4.1. An IF soft set f_A is called an IF soft point over X, denoted by e_f, if for the element $e \in A$, $f(e) \neq (\Phi, \Phi)$ and $f(e') = (\Phi, \Phi)$, for all $e' \in A - \{e\}$.

Definition 4.2. An IF soft point e_f is said to be in an IF soft set g_A, denoted by $e_f \tilde{\tilde{\in}} g_A$, if for the element $e \in A$, $f(e) \subseteq g(e)$.

Example 4.3. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and let us consider an IF soft point as: $e_{1f} \tilde{\tilde{=}} (e_1, \{x_{(0.1,0.8)}, x_{(0.6,0.3)}\})$ and an IF soft set g_A defined by:

$g(e_1)(x_1) = (0.1, 0.7)$, $g(e_1)(x_2) = (0.6, 0.1)$,
$g(e_2)(x_1) = (0.1, 0.8)$, $g(e_2)(x_2) = (0.9, 0.1)$.

Then $g_A \tilde{\tilde{=}} (\{x_{(0.1,0.7)}, x_{(0.6,0.1)}\}, \{x_{(0.1,0.8)}, x_{(0.9,0.1)}\})$. It is clear that $e_{1f} \tilde{\tilde{\in}} g_A$, since $f(e_1) \subseteq g(e_1)$. Obviously, $f(e_2) = \Phi = \{x_{(0,1)}, x_{(0,1)}\} \subseteq g(e_2)$.

Proposition 4.4. *Let e_F be an IF soft point over X and g_A be an IF soft set over X. If $e_f \tilde{\tilde{\in}} g_A$, then $e_f \tilde{\tilde{\notin}} (g_A)^c$.*

Proof. If $e_f \tilde{\tilde{\in}} g_A$, then for $e \in A$, $f(e) \subseteq g(e)$. This implies $f(e) \nsubseteq g^c(e)$. Therefore, we have $e_f \tilde{\tilde{\notin}} g_A^c \tilde{\tilde{=}} (g_A)^c$. This completes the proof.

Remark 4.5. The following example shows that the converse of the above proposition is not true.

Example 4.6. Let $X = \{x_1, x_2\}$, $A = \{e_1, e_2\}$ and let us consider the IF soft point as: $e_{1f} \tilde{\tilde{=}} (e_1, \{x_{(0.2,0.8)}, x_{(0.6,0.3)}\})$ and an IF soft set g_A defined by:
$g(e_1)(x_1) = (0.1, 0.8)$, $g(e_1)(x_2) = (0.6, 0.1)$,
$g(e_2)(x_1) = (0.2, 0.8)$, $g(e_2)(x_2) = (0.8, 0.1)$.

Then $g_A \tilde{\tilde{=}} (\{x_{(0.1,0.8)}, x_{(0.6,0.1)}\}, \{x_{(0.2,0.8)}, x_{(0.8,0.1)}\})$. It is clear that $e_{1f} \tilde{\tilde{\notin}} g_A$. Moreover, $e_{1f} \tilde{\tilde{\notin}} (g_A)^c \tilde{\tilde{=}} (\{x_{(0.1,0.8)}, x_{(0.6,0.1)}\}, \{x_{(0.2,0.8)}, x_{(0.8,0.1)}\})$.

Definition 4.7. Let e_h be an IF soft point in an IF soft topological space (X, τ, A) over X. If there is an IF soft semi-open set g_A such that $e_h \tilde{\tilde{\in}} g_A$, then g_A is called an IF soft semi-open neighborhood (in short: nbd) of e_h. The set of all IF soft semi-open nbds of e_h, denoted $\widetilde{\widetilde{snbd}}(e_h)$, is called an IF soft semi-open nbd systems of e_h. That is, $\widetilde{\widetilde{snbd}}(e_h) \tilde{\tilde{=}} \{g_A : g_A$ is an IF soft semi-open set and $e_h \tilde{\tilde{\in}} g_A\}$.
The following theorem gives us important properties of an IF soft open nbd system:

Proposition 4.8. *Let f_A, g_A and h_A are IF soft sets in an IF soft topological spaces (X, τ, A) over X such that $g_A, h_A \tilde{\subseteq} f_A$. Then the collection of an IF soft semi-open nbd $\widetilde{\widetilde{snbd}}(e_k)$ at e_k in (X, τ, A) has the following properties:*

(1) *If $g_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, then $e_k \tilde{\tilde{\in}} g_A$.*

(2) *If $g_A, h_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, then $g_A \tilde{\tilde{\cap}} h_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$.*

(3) *If $g_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$ and $g_A \tilde{\subseteq} h_A$, then $h_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$.*

(4) *If $g_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, then there is an $h_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$ such that $g_A \tilde{\tilde{\in}} \widetilde{\widetilde{snbd}}(e_l)$, for each $e_l \tilde{\tilde{\in}} h_A$.*

(5) *$g_A \tilde{\subseteq} f_A$ is an IF soft semi-open if and only if g_A contains an IF soft semi-open nbd of each of its IF soft points.*

Proof. (1) is obvious, since g_A is an IF soft semi-open nbd of $e_k \widetilde{\widetilde{\in}} f_A$. So g_A is an IF soft semi-open set such that $e_k \widetilde{\widetilde{\in}} g_A$.

(2) If $g_A, h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, then there exist an IF soft semi-open sets m_A and n_A such that $e_k \widetilde{\widetilde{\in}} m_A \widetilde{\widetilde{\subseteq}} g_A$ and $e_k \widetilde{\widetilde{\in}} n_A \widetilde{\widetilde{\subseteq}} h_A$. Therefore $e_k \widetilde{\widetilde{\in}} m_A \widetilde{\widetilde{\cap}} n_A \widetilde{\widetilde{\subseteq}} g_A \widetilde{\widetilde{\cap}} h_A$ and hence $g_A \widetilde{\widetilde{\cap}} h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$.

(3) Since $g_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, then there exists an IF soft semi-open set m_A such that $e_k \widetilde{\widetilde{\in}} m_A \widetilde{\widetilde{\subseteq}} g_A$. Then $e_k \widetilde{\widetilde{\in}} m_A \widetilde{\widetilde{\subseteq}} g_A \widetilde{\widetilde{\subseteq}} h_A$ or $e_k \widetilde{\widetilde{\in}} m_A \widetilde{\widetilde{\subseteq}} h_A$. Hence $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$.

(4) Since $g_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, then $e_k \widetilde{\widetilde{\in}} h_A \widetilde{\widetilde{\subseteq}} g_A$, for h_A an IF soft semi-open set in f_A. Since $e_k \widetilde{\widetilde{\in}} h_A \widetilde{\widetilde{\subseteq}} h_A$ then $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$. If $e_k \widetilde{\widetilde{\in}} h_A$, then by (3) $h_A \widetilde{\widetilde{\subseteq}} g_A$ implies $g_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$, for each $e_k \widetilde{\widetilde{\in}} h_A$.

(5) (i) Suppose g_A is an IF soft semi-open in f_A, then $e_k \widetilde{\widetilde{\in}} g_A \widetilde{\widetilde{\subseteq}} f_A$ implies g_A is an IF soft semi-open nbd of each $e_k \widetilde{\widetilde{\in}} g_A$.

(ii) If each $e_k \widetilde{\widetilde{\in}} g_A$ has an IF soft semi-open nbd $(h_A)_{e_k} \widetilde{\widetilde{\subseteq}} g_A$, then $g_A = \{e_k : e_k \widetilde{\widetilde{\in}} g_A\} \widetilde{\widetilde{\subseteq}} \widetilde{\widetilde{\bigcup}}_{e_k \widetilde{\widetilde{\in}} g_A} (h_A)_{e_k} \widetilde{\widetilde{\subseteq}} g_A$ or $g_A = \widetilde{\widetilde{\bigcup}}_{e_k \widetilde{\widetilde{\in}} g_A} (h_A)_{e_k}$. This gives g_A is an IF soft semi-open in f_A. Hence the proof.

Definition 4.9. Let (X, τ, A) be an IF soft topological space over X. An IF soft semi-nbd base at $e_k \widetilde{\widetilde{\in}} f_A$ is a subcollection $\widetilde{\widetilde{snbd}}^s(e_k)$ of IF soft semi-nbd $\widetilde{\widetilde{snbd}}(e_k)$ having the property that each $g_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}(e_k)$ contains some $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$. That is, $\widetilde{\widetilde{snbd}}(e_k)$ must be determined by $\widetilde{\widetilde{snbd}}^s(e_k)$ as follows:

$\widetilde{\widetilde{snbd}}(e_k) = \{g_A \widetilde{\widetilde{\subseteq}} f_A : h_A \widetilde{\widetilde{\subseteq}} g_A$, for some $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)\}$.

Each $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$ is called a basic IF soft semi-open neighborhood of e_k.

For an IF soft basic semi-nbd system, the following properties directly follows by the corresponding properties of Proposition 4.8.:

Proposition 4.10. *Let f_A be an IF soft set in an IF soft topological space (X, τ, A) over X and for each $e_k \widetilde{\widetilde{\in}} f_A$, let $\widetilde{\widetilde{snbd}}^s(e_k)$ be an IF soft semi-nbd base at e_k. Then we have the following properties.*

(1) *If $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$, then $e_k \widetilde{\widetilde{\in}} h_A$.*

(2) *If g_A, $m_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$, then there is some $n_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$ such that $n_A \widetilde{\widetilde{\subseteq}} g_A \widetilde{\cap} m_A$.*

(3) *If $h_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$, then there is some $l_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_k)$ such that if $e_p \widetilde{\widetilde{\in}} l_A$, then there is some $m_A \widetilde{\widetilde{\in}} \widetilde{\widetilde{snbd}}^s(e_p)$ with $m_A \widetilde{\widetilde{\subseteq}} h_A$.*

(4) *$g_A \widetilde{\widetilde{\subseteq}} f_A$ is an IF soft semi-open if and only if g_A contains an IF soft semi-basic nbd of each of its IF soft points.*

Conclusion. The parameters may not always be crisp, rather they may be intuitionistic fuzzy in nature in real life situations. The problems of object recognition have received paramount importance in recent years. The recognition problem may be viewed as multiobserver decision making problem, where the final identification of the object is based on the set of inputs from different observers who provide the overall object characterization in terms of diverse set of parameters. Intuitionistic fuzzy set is fruitful and more realistic model of uncertainty than the fuzzy sets. Extending this approach further, intuitionistic fuzzy soft sets were introduced, as a generalization of fuzzy soft sets and standard soft sets. Intuitionistic fuzzy soft sets used to handle the problem of muti-criteria decision making. Here, we defined and established the topological structures of intuitionistic fuzzy soft semi-open sets and intuitionistic fuzzy soft semi-closed sets. We also defined and explored the properties of intuitionistic fuzzy soft semi-interior, intuitionistic fuzzy soft semi-closure and discussed the relationship between them. Moreover, we established and discussed intuitionistic fuzzy soft semi-open neighborhood systems. We hope that the findings in this paper will enhance and promote the further study proceed towards practical life application.

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