On connectedness of the Hausdorff fuzzy metric spaces

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Abstract. Several properties of the Hausdorff fuzzy metric, as completeness, precompactness and completion were discussed by Rodríguez-López and Romaguera [The Hausdorff fuzzy metric on compact sets, Fuzzy Sets and Systems 147 (2004) 273–283]. It is necessary to seek other more properties of the Hausdorff fuzzy metric. In the paper, we show that a fuzzy metric space is connected if and only if the corresponding Hausdorff fuzzy metric space on compact (finite) sets is connected.
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1. Introduction

The notion of fuzzy metric space has been defined by many authors in different ways [4, 6, 17, 18]. In particular, by extending the notion of Menger space to the fuzzy setting, Kramosil and Michalek [18] obtained the notion of fuzzy metric space with the help of continuous t-norms. In order to make the topology generated by a fuzzy metric to be Hausdorff, George and Veeramani [6] modified in a slight but appealing way the notion given by Kramosil and Michalek. In [15], Gregori and Romaguera proved that the topological space generated by a modified fuzzy metric is metrizable and then, some classical theorems on metric properties are adapted to the realm of the modified version of fuzzy metric. In view of them, some authors became interested in the new version of fuzzy metric. Dinarvand [2] gave Some fixed point results for admissible Geraghty contraction type mappings in fuzzy metric spaces. Romaguera and Sanchis [25] introduced a notion of fuzzy metric group and investigated properties of the quotient subgroups of a fuzzy metric group. In [1], an arclength notion

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of continuous curves in fuzzy metric spaces was proposed, and some arclength properties , including invariance, additive, continuity and boundedness were studied by Chen et al. In recent years, Gregori et al [9, 10, 11, 12, 13, 14, 15, 16] gave much progress to the study of fuzzy metric spaces. We can also find other more studies of fuzzy metric spaces in [7, 8, 19, 20, 21, 23, 26, 27, 28].

To explore hyperspaces in given fuzzy metric spaces, Rodríguez-López and Romaguera [24] introduced the Hausdorff fuzzy metric on the collection of nonempty compact sets and studied completeness, precompactness and compactness of the Hausdorff fuzzy metric spaces. In [5], an identification theorem for the completion of the Hausdorff fuzzy metric was explored. Recently, we gave several equivalent conditions for the Hausdorff fuzzy metric spaces on the family of nonempty compact sets to be complete in [22]. It is nature to seek other more properties of the Hausdorff fuzzy metric. In the paper we do it. Here, we obtain that connectedness of a fuzzy metric space and connectedness of the corresponding Hausdorff fuzzy metric space on compact (finite) sets coincide.

2. Preliminaries

Throughout the paper the letter $\mathbb{N}$ shall denote the set of all positive integer numbers. Our basic reference for general topology is [3].

Definition 2.1 ([6]). *A binary operation* $* : [0,1]\times[0,1] \to [0,1]$ *is a continuous t-norm if it satisfies the following conditions:*

(i) $*$ *is associative and commutative;*

(ii) $*$ *is continuous;*

(iii) $a * 1 = a$ *for all* $a \in [0,1]$*;*

(iv) $a * b \le c * d$ *whenever* $a \le c$ *and* $b \le d$*, and* $a, b, c, d \in [0,1]$*.*

Observe that $a*b = \min\{a,b\}$*,* $a*b = a\cdot b$ *and* $a*b = \max\{a+b-1, 0\}$ *are three common examples of continuous t-norms.*

Clearly, Definition 2.1 shows that, if $1 \ge r > s \ge 0$, then there exists a $\delta \in (s,1)$ such that $r * \delta \ge s$.

Definition 2.2 ([6]). *An ordered triple* $(X, M, *)$ *is said to be a fuzzy metric space if* X *is a nonempty set,* $*$ *is a continuous t-norm and* M *is a fuzzy set on* $X \times X \times (0,\infty)$ *satisfying the following conditions for all* $x, y, z \in X$ *and* $s, t \in (0,\infty)$*:*

(i) $M(x,y,t) > 0$*;*

(ii) $M(x,y,t) = 1$ *if and only if* $x = y$*;*

(iii) $M(x,y,t) = M(y,x,t)$*;*

(iv) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$*;*

(v) the function $M(x,y,\cdot) : (0,\infty) \to [0,1]$ *is continuous.*

If $(X,M,*)$ *is a fuzzy metric space, then we shall call* $(M,*)$ *a fuzzy metric on* X*.*

It is well known that $M(x,y,\cdot)$ *is a non-decreasing function on* $(0,\infty)$ *for all* $x,y \in X$*.*

Definition 2.3 ([6])**.** *Let* $(X,M,*)$ *be a fuzzy metric space and let* $r \in (0,1), t > 0$ *and* $x \in X$*. The set*

$$B_M(x,r,t) = \{y \in X | M(x,y,t) > 1-r\}$$

is called the open ball with center x *and radius* r *with respect to* t*.*

In [6], George and Veeramani proved that $\{B_M(x,r,t) | x \in X, t > 0, r \in (0,1)\}$ forms a base of a topology τ_M in X and $\{B_M(x,\frac{1}{n},\frac{1}{n}) | n \in \mathbb{N}\}$ is a neighborhood base at x for the topology τ_M for every $x \in X$.

Definition 2.4 ([15])**.** *A fuzzy metric space* $(X,M,*)$ *is said to be compact if* (X,τ_M) *is compact.*

3. Connectedness of the Hausdorff fuzzy metric spaces

Given a fuzzy metric space $(X,M,*)$, we shall denote by $\mathcal{P}_0(X)$, $\mathcal{K}_0(X)$ and $\mathcal{F}_0(X)$, the collection of nonempty subsets, the collection of nonempty compact subsets and the collection of nonempty finite subsets of (X,τ_M), respectively. Put $M(x,A,t) := \sup_{a\in A} M(x,a,t)$, $M(A,x,t) := \sup_{a\in A} M(a,x,t)$ for all $x \in X$, $A \in \mathcal{P}_0(X)$ and $t > 0$ (see Definition 2.4 of [28]). It is obvious that $M(x,A,t) = M(A,x,t)$. In the following, for any $B \in \mathcal{P}_0(X)$, the cardinality of B will be denote by $|B|$.

Definition 3.1 ([24])**.** *Let* $(X,M,*)$ *be a fuzzy metric space. For every* $A,B \in \mathcal{K}_0(X)$ *and* $t > 0$*, define* $H_M : \mathcal{K}_0(X) \times \mathcal{K}_0(X) \times (0,\infty) \to [0,1]$ *by*

$$H_M(A,B,t) = \min\{\inf_{a\in A} M(a,B,t), \inf_{b\in B} M(A,b,t)\}.$$

Then $(\mathcal{K}_0(X),H_M,*)$ *is a fuzzy metric space. We call* $(H_M,*)$ *the Hausdorff fuzzy metric on* $\mathcal{K}_0(X)$*.*

Lemma 3.1 ([19])**.** *Let* $(X,M,*)$ *be a fuzzy metric space. Then* $H_M(A,B,t) = 1 - \inf\{r | A \subseteq B_M(B,r,t), B \subseteq B_M(A,r,t)\}$ *for all* $A,B \in \mathcal{K}_0(X)$ *and* $t > 0$*, where* $B_M(A,r,t) = \bigcup_{a\in A} B_M(a,r,t)$*.*

Lemma 3.2 ([24])**.** *Let* $(X,M,*)$ *be a fuzzy metric space. Then* M *is a continuous function on* $X \times X \times (0,\infty)$*.*

Definition 3.2. *Let $(X, M, *)$ be a fuzzy metric space. For every $A \in \mathcal{K}_0(X)$ and $t > 0$, define $diam_t(A)$ by*

$$\mathrm{diam}_t(A) = \inf\{M(a, b, t)|a, b \in A\}$$

and call it the fuzzy diameter of A with t.

Proposition 3.1. *Let $(X, M, *)$ be a fuzzy metric space, $A \in \mathcal{K}_0(X)$ and $t > 0$. Then $diam_t(A) > 0$.*

Proof. By Lemma 3.2, we conclude that $\{M(a, b, t)|a, b \in A\}$ is a closed subset of [0,1]. Hence

$$\inf\{M(a, b, t)|a, b \in A\} \in \{M(a, b, t)|a, b \in A\}.$$

Thus, there exist $a_0, b_0 \in A$ such that

$$\inf\{M(a, b, t)|a, b \in A\} = M(a_0, b_0, t) > 0.$$

We complete the proof. □

Lemma 3.3. *Let $(Y, M, *)$ be a compact subspace of a fuzzy metric space $(X, M, *)$, and let $\mathcal{U}$ be a family of open subsets of X which covers Y. Then there exist $r \in (0, 1)$ and $t > 0$ with the property that every $A \in \mathcal{K}_0(X)$ with $diam_t(A) > 1 - r$ and $A \cap Y \neq \emptyset$ is contained in an element $U \in \mathcal{U}$.*

Proof. Assume that such r and t do not exist. Then, for every $n \in \mathbb{N}$, we can choose an $A_n \in \mathcal{K}_0(X)$ such that

(1) $\mathrm{diam}_{\frac{1}{n}}(A_n) > 1 - \frac{1}{n}$,

(2) $A_n \cap Y \neq \emptyset$,

(3) A_n is not contained in any element of $\mathcal{U}$.

Take $y_n \in A_n \cap Y$ for every $n \in \mathbb{N}$. Since Y is compact, there is a subsequence of $\{y_n\}_{n\in\mathbb{N}}$ such that the subsequence converges to a point $y \in Y$. Without loss of generality, we can assume that

$$y = \lim_{n\to\infty} y_n.$$

Since $\mathcal{U}$ covers Y, we can find a $U \in \mathcal{U}$ such that $y \in U$. Note that U is open, there exists $l \in \mathbb{N}$ such that $B_M(y, \frac{1}{l}, \frac{1}{l}) \subseteq U$. In addition, there exists $N \in \mathbb{N}$ such that $y_m \in B_M(y, \frac{1}{l+1}, \frac{1}{l+1})$ for all $m \geq N$. Now choose $m \geq N$ so large that $\frac{1}{m} \leq \frac{1}{l(l+1)}$ and $(1 - \frac{1}{m}) * (1 - \frac{1}{l+1}) \geq [1 - \frac{1}{2}(\frac{1}{l} + \frac{1}{l+1})] > 1 - \frac{1}{l}$. Since

$$\mathrm{diam}_{\frac{1}{m}}(A_m) = \inf\{M(a, b, \frac{1}{m})|a, b \in A_m\} > 1 - \frac{1}{m},$$

we get that $M(a, b, \frac{1}{m}) > 1 - \frac{1}{m}$ whenever $a, b \in A_m$. Let $m \geq N$ so large. Then, for every $x \in A_m$, we have that $M(x, y, \frac{1}{l}) \geq M(x, y, \frac{1}{m} + \frac{1}{l+1}) \geq M(x, y_m, \frac{1}{m}) * M(y_m, y, \frac{1}{l+1}) \geq (1 - \frac{1}{m}) * (1 - \frac{1}{l+1}) > 1 - \frac{1}{l}$. Hence $x \in B_M(y, \frac{1}{l}, \frac{1}{l})$. It follows that $A_m \subseteq B_M(y, \frac{1}{l}, \frac{1}{l}) \subseteq U$, which contradicts (3). This concludes the proof. □

Lemma 3.4. *Let $(Y, M, *)$ be a compact subspace of a fuzzy metric space $(X, M, *)$, and let U be an open neighborhood of Y in X. Then there exist $r \in (0,1)$ and $t > 0$ such that $B_M(Y, r, t) \subseteq U$.*

Proof. Observe that $\{U\}$ covers Y. According to Lemma 3.3, we can find $r \in (0,1)$ and $t > 0$ with the property that every $A \in \mathcal{K}_0(X)$ with $\mathrm{diam}_t(A) > 1-r$ and $A \cap Y \neq \emptyset$ is contained in $U \in \{U\}$. Let $x \in B_M(Y, r, t) = \bigcup_{y \in Y} B_M(y, r, t)$. Then there exists a $y_0 \in Y$ such that $x \in B_M(y_0, r, t)$, which means that $M(x, y_0, t) > 1 - r$. It follows that $\mathrm{diam}_t(\{x, y_0\}) = M(x, y_0, t) > 1 - r$. Note that $\{x, y_0\} \in \mathcal{K}_0(X)$ and $\{x, y_0\} \cap Y \neq \emptyset$. Hence $\{x, y_0\} \subseteq U$, which implies that $x \in U$. Thus $B_M(Y, r, t) \subseteq U$. □

The proof of the next lemma is straightforward.

Lemma 3.5. *Let $(X, M, *)$ be a fuzzy metric space and $A \in \mathcal{P}_0(X)$. If $0 < r_1 < r_2 < 1$ and $0 < t_1 < t_2$, then $B_M(A, r_1, t_1) \subseteq B_M(A, r_2, t_2)$.*

Let $(X, M, *)$ be a fuzzy metric space. For every finite family $\mathcal{V}$ of subsets of X, put

$$\langle \mathcal{V} \rangle = \{A \in \mathcal{K}_0(X) | A \subseteq \bigcup \mathcal{V} \text{ and for each } V \in \mathcal{V}, V \cap A \neq \emptyset\}.$$

Lemma 3.6. *Let $(X, M, *)$ be a fuzzy metric space, and let $\mathcal{V}$ be a finite family of open subsets of X. Then $\langle \mathcal{V} \rangle$ is an open subset of $\mathcal{K}_0(X)$.*

Proof. Let $A \in \langle \mathcal{V} \rangle$. For each $V \in \mathcal{V}$ we choose a point $x_V \in V \cap A$. Notice that $\langle \mathcal{V} \rangle$ is finite, V is open and $A \in \mathcal{K}_0(X)$ with $A \subseteq \bigcup \mathcal{V}$. Then, according to Lemma 3.4 and Lemma 3.5, we can find $r \in (0,1)$ and $t > 0$ such that

(1) $B_M(x_V, r, t) \subset V$ for every $V \in \mathcal{V}$,

(2) $B_M(A, r, t) \subseteq \bigcup \mathcal{V}$.

Let $B \in B_{H_M}(A, r, t)$. Then, due to Lemma 3.1, we obtain that $B \subseteq B_M(A, r, t)$. It follows from (2) that $B \subseteq \bigcup \mathcal{V}$. Since $A \subseteq B_M(B, r, t)$, we can find a $y_V \in B$ such that $M(x_V, y_V, t) > 1 - r$ for every $V \in \mathcal{V}$. According to (1), we conclude that $y_V \in V$ for every $V \in \mathcal{V}$. Thus, for each $V \in \mathcal{V}$, $y_V \in V \cap B$, i.e., $V \cap B \neq \emptyset$. Hence $B \in \langle \mathcal{V} \rangle$. We immediately deduce that $B_{H_M}(A, r, t) \subseteq \langle \mathcal{V} \rangle$. The proof is finished. □

Definition 3.3. *A fuzzy metric space $(X, M, *)$ is said to be connected if (X, τ_M) is connected.*

Theorem 3.1. *Let $(X, M, *)$ be a fuzzy metric space. If $(\mathcal{K}_0(X), H_M, *)$ is connected, then so is $(X, M, *)$.*

Proof. Let $(\mathcal{K}_0(X), H_M, *)$ be a connected fuzzy metric space. Suppose that $(X, M, *)$ is not connected. Then we can find two disjoint nonempty open subsets U and V of X such that $X = U \cup V$. Notice that

$$\mathcal{K}_0(X) = \langle \{U\} \rangle \cup \langle \{V\} \rangle \cup \langle \{U, V\} \rangle.$$

Obviously, by Lemma 3.6, $\langle\{U\}\rangle$, $\langle\{V\}\rangle$ and $\langle\{U, V\}\rangle$ are pairwise disjoint, nonempty and open in $\mathcal{K}_0(X)$. It follows that $(\mathcal{K}_0(X), H_M, *)$ is not connected, which is a contradiction. We are done. $\square$

Lemma 3.7 ([24])**.** *Let Y be a dense subset of a fuzzy metric space $(X, M, *)$. Then $\mathcal{F}_0(Y)$ is dense in $(\mathcal{K}_0(X), H_M, *)$.*

Lemma 3.8 ([3])**.** *Let A be a connected subspace of a topological space (X, τ_X). Then the closure $\overline{A}$ of A is also connected.*

Theorem 3.2. *Let $(X, M, *)$ be a fuzzy metric space. If $(\mathcal{F}_0(X), H_M, *)$ is connected, then so is $(X, M, *)$.*

Proof. Suppose that $(\mathcal{F}_0(X), H_M, *)$ is connected. According to Lemma 3.7 and Lemma 3.8, we get that $(\mathcal{K}_0(X), H_M, *)$ is connected. It follows from Theorem 3.1 that $(X, M, *)$ is connected. $\square$

Lemma 3.9 ([3])**.** *A finite cartesian product of connected spaces is connected.*

Lemma 3.10 ([3])**.** *The image of a connected space under a continuous mapping is connected.*

Lemma 3.11 ([3])**.** *The union of collection of connected subspaces of a topological space (X, τ_X) that have a point in common is connected.*

Let $(X, M, *)$ be a fuzzy metric space. For each $n \in \mathbb{N}$, put $\mathcal{F}_0^n(X) = \{A \subseteq X | 1 \leq |A| \leq n\}$, which we regard as a subspace of $\mathcal{K}_0(X)$.

Lemma 3.12. *Let $(X, M, *)$ be a fuzzy metric space. For each $n \in \mathbb{N}$, define the function $g_n : X^n \to \mathcal{F}_0^n(X)$ by*

$$g_n(x_1, x_2, \cdots, x_n) = \{x_1, x_2, \cdots, x_n\}.$$

Then g_n is a continuous surjection for every $n \in \mathbb{N}$.

Proof. Obviously, g_n is a surjection for every $n \in \mathbb{N}$. We will now prove that g_n is continuous for every $n \in \mathbb{N}$. Fix $n \in \mathbb{N}$. Let $(a_1, a_2, \cdots, a_n) \in X^n$. Then $A = g_n(a_1, a_2, \cdots, a_n) = \{a_1, a_2, \cdots, a_n\} \in \mathcal{F}_0^n(X)$. To complete our proof, it suffices to show that, for $r \in (0, 1)$ and $t > 0$,

$$g_n(B_M(a_1, r, t) \times B_M(a_2, r, t) \times \cdots \times B_M(a_1, r, t)) \subseteq B_{H_M}(A, r, t) \cap \mathcal{F}_0^n(X).$$

Let $(b_1, b_2, \cdots, b_n) \in B_M(a_1, r, t) \times B_M(a_2, r, t) \times \cdots \times B_M(a_n, r, t)$. Then $b_i \in B_M(a_i, r, t)$ $(1 \leq i \leq n)$, which means that $M(a_i, b_i, t) > 1 - r$ $(1 \leq i \leq n)$. It follows that $A \subseteq B_M(B, r, t)$ and $B \subseteq B_M(A, r, t)$, where $B = \{b_1, b_2, \cdots, b_n\}$. Due to Lemma 3.1, we obtain that

$$B \in B_{H_M}(A, r, t) \cap \mathcal{F}_0^n(X),$$

i.e., $g_n(b_1, b_2, \cdots, b_n) \in B_{H_M}(A, r, t) \cap \mathcal{F}_0^n(X)$. This concludes the proof. $\square$

Theorem 3.3. *Let* $(X, M, *)$ *be a connected fuzzy metric space. Then* $(\mathcal{F}_0(X), H_M, *)$*, and hence* $(\mathcal{K}_0(X), H_M, *)$*, is connected.*

Proof. Assume that (X, τ_M) is connected. According to Lemma 3.9, we get that X^n is connected. For each $n \in \mathbb{N}$, define the function $g_n : X^n \to \mathcal{F}_0^n(X)$ by

$$g_n(x_1, x_2, \cdots, x_n) = \{x_1, x_2, \cdots, x_n\}.$$

Then, by Lemma 3.10 and Lemma 3.12, we obtain that $(\mathcal{F}_0^n(X), H_M, *)$ is connected for all $n \in \mathbb{N}$. Observe that $\bigcup_{n=1}^{\infty} \mathcal{F}_0^n(X) = \mathcal{F}_0(X)$ and $\bigcap_{n=1}^{\infty} \mathcal{F}_0^n(X) = \mathcal{F}_0^1(X) \neq \emptyset$. It follows from Lemma 3.11 that $\mathcal{F}_0(X)$ is connected. This concludes the proof. □

As a consequence of Theorem 3.1, Theorem 3.2 and Theorem 3.3, we immediately deduce the next corollary.

Corollary 3.1. *Let* $(X, M, *)$ *be a fuzzy metric space. Then the following are equivalent.*

(i) $(X, M, *)$ *is connected.*

(ii) $(\mathcal{F}_0(X), H_M, *)$ *is connected.*

(iii) $(\mathcal{K}_0(X), H_M, *)$ *is connected.*

4. Conclusion

In this work, we have proven that a fuzzy metric space is connected if and only if the corresponding Hausdorff fuzzy metric space on compact (finite) sets is connected.

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