Recognizable iso-triangular picture languages

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Abstract. In modern civilization, the art of tiling has become a Prerogative pattern. Tiling patterns have been used to decorate and cover floors and walls. Motivated by the problem of tiling, H. Wang introduced a tile in 1961, called by his name, Wang tile and in which the edges are coloured. A finite set of Wang tiles admits a valid tiling of the plane if copies of tiles can be arranged one by one without rotation or reflection to fill the plane so that gluable tiles which have the same colour. Wang tiling patterns motivate Kalyani et.al to introduce an iso-triangular labelled Wang tiles. A finite automaton is one of the machines that scan the pictures and recognizes the pictures. In this paper, we define iso-triangular Wang automaton and iso-triangular Wang P system. It is showed that an iso-triangular Wang automaton stimulates the computational power of the P system. The iso-triangular Wang P system recognizes the iso-triangular pictures. The computational complexity of iso-triangular Wang automaton and iso-triangular Wang P system is examined with the computational powers of iso-triangular Wang system, iso-triangular tiling system and hexagonal tiling system.

Keywords: iso-triangular tiles, iso-triangular Wang tiles, finite automata, Wang automata, P system.

1. Introduction

Tiling patterns have been introduced in modern civilization. The interest on pattern recognition and image processing motivates the researcher to introduce more types of tiling patterns on two-dimensional picture languages. A two-dimensional pattern generating model called tile pasting P system that generates tiling patterns by pasting square tiles at the edges has been introduced and studied in [9, 10]. Iso-Triangular tiles and iso-triangular tile pasting system are defined to generate the iso-triangular picture languages [6], in which the labelled edges of iso-triangular tiles are pasted. H. Wang [11] generated a tiling of the plane using the set of square tiles with marked edges with a condition that the

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edges of the tiles should be of the same colour. An aperiodic tile set containing 13 tiles over five tiles is defined in [7] by Karel Culik II et. al. Automata theory is a scanning strategy to recognize the strings. The finite symbols or words are taken as input and the symbols are read by the transition rules [5, 12]. The working rule in automaton is, the input string or alphabet in the initial state can be read by the machine and the read alphabet can be transmitted to the next state. Likewise the transition is processed. The automaton stop reading the symbol after the output string is collected in the final state of the automata.

In [4] P automaton is introduced for the multisets of objects. In traditional automata the order or the position of the words or symbols is considered. But for multisets of objects concern the order is not important to recognize the picture.

Membrane computing is an area of theory of computation and it focusing on computing models from the structure and functioning of biological living cells. P system is one among the areas of membrane computing models introduced in [8]. It contains a finite number of membranes and finite regions. The multisets of objects are placed in the regions of the membranes. In each region the pasting rules are applied to the existing iso-arrays. The pasting rules governing the modification of the objects in time and transfers the pictures to another membrane or retained in the same membrane with the presence of target symbol. Using the concepts of iso-triangular tile pasting, a membrane computing model called Triangular tile pasting P system is defined and examined its generative powers in [2]. In membrane computing, p automaton is a special branch which has not been considered so for the iso-triangular wang tiles. This survey is motivated to define the iso-triangular Wang automaton for scanning the iso-triangular pictures. Generating tessellations and tiling patterns by tile pasting system is studied in [6]. Computation of iso picture languages by rules of context-free iso-array grammar by re-writing iso-arrays has been explained in [1]. In [3] triangular picture languages are generated by triangular tile pasting P system and triangular array token Petri net are introduced. It is noted that triangular tile pasting p system is not comparable with triangular array token Petri net.

In this paper, a theoretical model called an iso-triangular Wang P System (ITWPS) is introduced to recognize or generate the iso triangular pictures. The computational complexity of the system is examined with the computational powers of HTS and ITWS. A non-deterministic machine model called iso-triangular Wang automaton is introduced to recognize the two-dimensional picture languages. It simulates the computational power of P system. This paper introduces Wang P system to check the acceptability of the iso-picture languages. We noticed that the iso-triangular Wang automata stimulate the computational power of iso-triangular Wang P system.

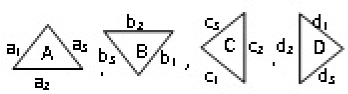
This paper is organized as follows. Section two reveals the preliminary definitions and section three introduces the iso-triangular Wang p system, in this section the generating power of iso-triangular Wang P system is compared with the generating power of hexagonal, iso-triangular tiling systems and iso-triangular

Wang system. In section four, the iso-triangular Wang automaton is introduced with suitable example and noticed that Wang automaton stimulates the Wang P automata when generating the iso-triangular picture languages. Section five gives the conclusion part of the paper.

2. Preliminaries

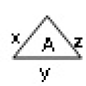
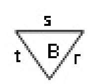
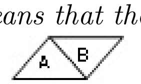
The art of tiling is a well-known theory in the applications of pattern generation. In this section, we recollect the notion of triangular tiles and tile pasting system. A tile is a topological disc with closed boundary in the XOY plane, whose edges are gluable. A tiling is a family of countable and gluable tiles with no gaps or overlaps that cover the Euclidean plane.

Definition 2.1. *Consider the labelled triangular tiles*



whose horizontal (Vertical) and other side edges are of length 1 unit and $1/\sqrt{2}$ unit respectively.

Definition 2.2. *A pasting rule is a pair (x, y) of labelled tiles with distinct edges.*

For example, the triangular tile  *and tile*  *are joined by the edges (z, t), which means that the edge z of tile A is glued with the edge t of tile B, we get the pattern* 

Note that the edges are of the same length.

The set of all edge labels is called an edge set denoted by E. Tile pasting rules of the tiles A, B, C, D are given below:

1. *Tile A can be glued with tile B by the pasting rules $\{(a_1, b_1), (a_2, b_2), (a_3, b_3)\}$ with tile C by the rule $\{(a_3, c_1)\}$ and with tile D by the rule $\{(a_1, d_3)\}$.*

2. *Tile B can be glued with tile A by the pasting rules $\{(b_1, a_1), (b_2, a_2), (b_3, a_3)\}$ with tile C by the rule $\{(b_1, c_3)\}$ and with tile D by the rule $\{(b_3, d_1)\}$.*

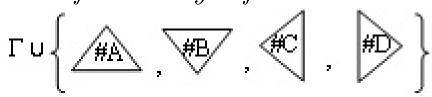
3. *Tile C can be glued with tile A by the pasting rule $\{(c_1, a_3)\}$ with tile B by $\{(c_3, b_1)\}$ and with the tile D by the pasting rules $\{(c_1, d_1), (c_2, d_2), (c_3, d_3)\}$.*

4. *Tile D can be glued with tile A by the pasting rule $\{(d_3, a_1)\}$ with tile B by $\{(d_1, b_3)\}$ and with the tile C by the pasting rules $\{(d_1, c_1), (d_2, c_2), (d_3, c_3)\}$.*

Definition 2.3. $S = (\Sigma, P, t_0)$ *is a construct of triangular tile pasting system (TTPS), where Σ is a finite set of labelled iso-triangular tiles, P is a finite set of pasting rules and t_0 is the an axiom of the pattern. A pattern p_2 is generated from a pattern p_1 by applying the pasting rules in a parallel manner to the edges of the pattern p_1, where pasting is possible. Note that the labels of pasted edges in* *a pattern are ignored once the tiles are pasted. The set of all patterns generated from the axiom t_0 constitutes the triangular picture language $T(S)$ of S.*

Definition 2.4. *Let $p \in \Sigma_I^{**}$ be an iso-triangular picture. Let Γ and Σ be two finite sets of iso triangular tiles and $\pi : \Gamma^T \rightarrow \Sigma^T$ is a mapping which we call, a projection. The projection by mapping π of the picture p is the picture $p' \in \Sigma_I^{**}$ such that $p'(i,j,k) = \pi(p(i,j,k))$ for all $1 \leq i \leq n$, $1 \leq j \leq m$, $1 \leq k \leq 2j-1$, where (n,m) is the size of the iso-triangular picture p. We denote $p' = \pi(p)$. By extension, we note $\pi(L)$ is in Σ_I^{**}. The projection by mapping π of L over Σ^T is the language $L' = \{p'/p' = \pi(p),\ \forall\ p \in L\} \subseteq \Sigma_I^{**}$.*

Definition 2.5. *An iso-triangular tiling system ITTS is a 4-tuple $(\Sigma, \Gamma, \pi, \theta)$ where Σ and Γ are two finite sets of symbols, $\pi : \Gamma \rightarrow \Sigma$ is a projection and θ is a set of iso-arrays of size 2 over the alphabet*



Definition 2.6. *A labelled iso-triangular Wang tile is a 4-tuple consisting of 3 colours, chosen in a finite set of colours. There is particular colour b (black) used only to delimit the boundary of picture. Two iso-triangular Wang tiles are adjacent if the edges of the tiles are of the same colour, whereas, two iso-triangular labelled Wang tiles are adjacent if the gluable edges are of the same colour.*

Definition 2.7. *An iso-triangular Wang system is a four tuple $ITWS = (\Sigma, T_w, C, P_r)$, where Σ is a finite set of alphabets, T_w is a finite set of iso-triangular Wang tiles, C is a finite set of colours and P_r is the set of pasting rules. We denote by $L(IW)$, the language of iso-pictures generated by an iso-triangular Wang system. An iso-picture language $L \subseteq \Sigma_I^{**}$ is iso-triangular Wang recognizable if there exists an iso-triangular Wang system ITWS such that $L = L(ITWS)$.*

Now we recall the notions of Hexagonal picture languages.

Definition 2.8. Hexagonal Tiles *Let I be an alphabet, a finite non-empty set of symbols. The set of all hexagonal tiles over I (including the empty array #) is denoted by I^{**H} and $I^{++H} = I^{**H} - \{\#\}$.*

The size of the hexagonal tile is defined by parameters LU (Left up), LD (Left down), RU (Right up), RD (Right down), U (Up), D (Down) as shown in Figure 1 For $X \in V^{**H}$ the length of left upper side of X is denoted by $|X|_{LU}$; similarly we define $|X|_{LD}$, $|X|_{RU}$, $|X|_{RD}$, $|X|_U$ and $|X|_L$. Here it is enlighten that $|X|_{LD} = |X|_{RD} = |X|_{LU} = |X|$, $|X|_L = |X|_U$.

Definition 2.9. *The projection of the hexagonal picture is defined as $\pi : \Gamma^H \rightarrow \Sigma^H$, where Γ^H and Σ^H be two finite sets of hexagonal tiles. Let $L' \subset \Gamma^{++H}$ be a hexagonal picture language. The projection by mapping π of the hexagonal picture language L is $L^H = \{p'/p' = \pi(p),\ \forall\ p \in L\} \subseteq \Sigma^{**H}$.*

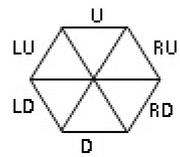


Figure 1: hexagonal picture

Definition 2.10. *A hexagonal tiling system T is a 4-tuple $(\Sigma', \Gamma', \pi, \theta')$ where Σ' and Γ' are two finite sets of symbols, $\pi : \Gamma' \to \Sigma'$ is a projection and θ', is a set of hexagonal tiles over the alpha $\Gamma \cup \{\#\}$.*

The hexagonal picture language $L' \subset \Gamma^{++H}$, if there exists a tiling system $T = (\Sigma', \Gamma', \pi, \theta')$ such that $L' = \pi(\theta')$.

3. Iso-triangular Wang P system

In this section we introduce an iso-triangular Wang tile pasting P system with suitable example and computational complexity of the system is examined by compared with other introduced systems.

Definition 3.1. *Iso-Triangular Wang tiling P system is a construct $\Pi = (\Sigma, \mu, C, T_w, F_1, F_2, \ldots, F_m, R_1, R_2, \ldots, R_m, i_0)$, where*

Σ - is a finite set of labels,

C - Finite set of distinct colours,

μ - Membrane structure, it contains m membranes labelled in a one-to-one manner with labels $1, 2, 3 \ldots m$.

F_i - initial pictures inside the regions of the membranes $(1 \leq i \leq m)$.

T_w - The finite set of iso-triangular Wang tiles.

R_i - Finite set of pasting rules associated with m regions of the membranes.

i_0 - Out put membrane.

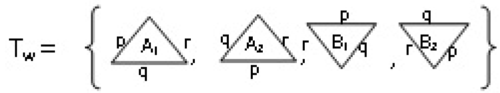
The pasting rules in R_i, is defined by $f(P_i, (T_w \times T_w)/(C \times C))_{Tar} \to P_j$.

The computation starts with axiom iso-triangular Wang tile. In every computational steps, the rules in R_i $(1 \leq i \leq m)$ is applied to all possible edges and the target symbol associated with the pasting rules decides that whether the resultant picture will be communicated to the next immediate membrane or retained in the same membrane. It depends the target symbol from $Tar = \{here, in_j, out\}$ associated with the rules in the region. It is assumed that the tiles defined in T_w can have more number of copies during the computation. The selection of the pasting rules in R_i will be considered non-deterministically. The process of evaluation continues in this way and finally it stops, if no rule can be further applicable. It means the computation stops and then the resultant picture pattern is collected in the output membrane. The set of all languages collected in this manner by Π is denoted by $L(\Pi) = \{p/p \in i_0\}$. The family of all language by Π is denoted by $L(ITWPS)_m$

Example 3.1. A class of hexagonal picture language L_H can be generated by an iso-triangular Wang system consists of 4 iso-triangular Wang tiles.

Consider the iso-triangular Wang system $ITWS = (\Sigma, T_w, C, P_r)$, where Σ is a set of all labels, $C = \{p, q, r\}$ and

$p_r = \{((A_1, B_1)/(r, r)), ((B_2, A_1)/(p, p)), ((A_2, B_1)/(p, p)), ((A_2, B_1)/(r, r)),$
$(B_2, A_1)/(r, r)), ((B_1, A_2)/(p, p)), ((A_2, B_1)/(q, q)), ((A_1, B_2)/(q, q)),$
$((B_1, A_2)/(q, q)), ((A_1, B_2)/(p, p)), ((A_2, B_2)/(r, r))\}.$



Here the rules are considered in the non-deterministic order. The edges are glued with the condition that if the edges of iso-triangular Wang tiles have the same colour. The language L' is generated and which is recognized by the iso-triangular Wang system.

The hexagonal picture language is shown in figure 2

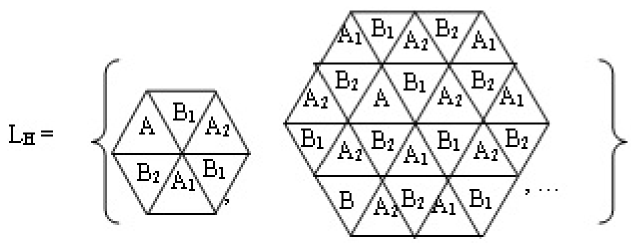
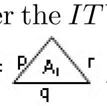


Figure 2: A class of hexagonal picture language

Example 3.2. An iso-triangular Wang P system with two membrane structure generates a class of hexagonal picture language.

Consider the $ITWPS\ \Pi_1 = (\Sigma, [_1[_2]_2]_1, \{p, q, r\}, \{A_1, A_2, B_1, B_2\}, F_1, F_2, R_1, R_2, 1)$, where $F_1 =$  , $F_2 = \phi$ and A_1, A_2, B_1, B_2 are the iso-triangular Wang tiles which are defined in Example 3.1 and the rules are

$R_1 = \{((A_1, B_1)/(r, r))_{in2}, ((B_2, A_1)/(p, p))_{in2}, ((A_2, B_1)/(p, p))_{here},$
$((A_2, B_1)/(r, r))_{in2}, ((B_2, A_1)/(r, r))_{in2}, ((B_1, A_2)/(p, p))_{in2},$
$((A_2, B_1)/(q, q))_{here}\},$
$R_2 = \{((A_1, B_2)/(q, q))_{out}, ((B_1, A_2)/(q, q))_{out}, ((A_1, B_2)/(p, p))_{here},$
$((B_2, A_1)/(q, q))_{out}, (((B_2, A_1)/(p, p))_{out}, ((A_2, B_2)/(r, r))_{out}\}.$

In region one, the axiom of pattern, tile A_1 is considered and the rule of R_1 is applied, the picture is grown in right up direction and then the existing pattern is entered into the membrane two. In region two, the rule of R_2 is applied. Wang tile B_2 is pasted with the resultant picture that is the coloured edge of the resultant picture is pasted with the same coloured edge of tile B_2, the pattern is developed its shape in left down direction and it is communicated to the region one of the membrane one by the target symbol associated with the rule. Again in membrane one, the rule (p, p) of the tile A_1 is pasted with the tile B_2. The picture is sent to the region two, in which the tile B_1 is pasted with the tile A_1 and the pattern halts in the region one. Thus the computation gives the first member of the language, is collected in output membrane. The computational process starts again to generate the second member of the language and it is collected finally. During the computation the copies of iso-triangular Wang tiles are considered and the pasting rules are applied non-deterministically.

The members of hexagonal picture language are derived in the derivation steps. The first and second members are shown in fig. 3 and 4.

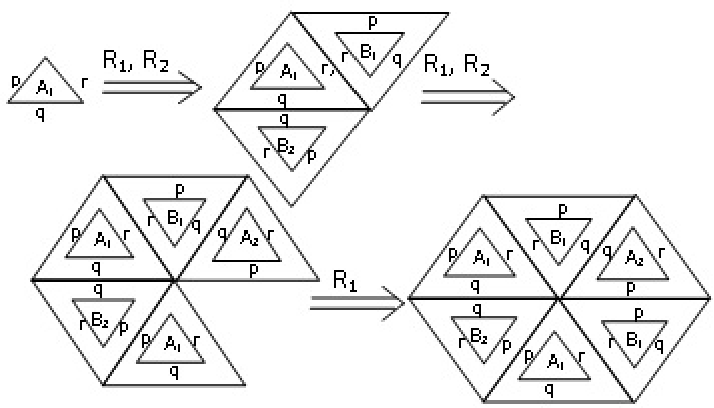
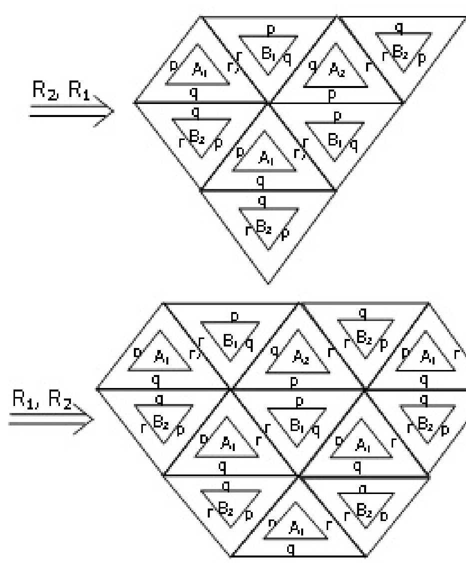


Figure 3: 1^{st} member of hexagonal picture language



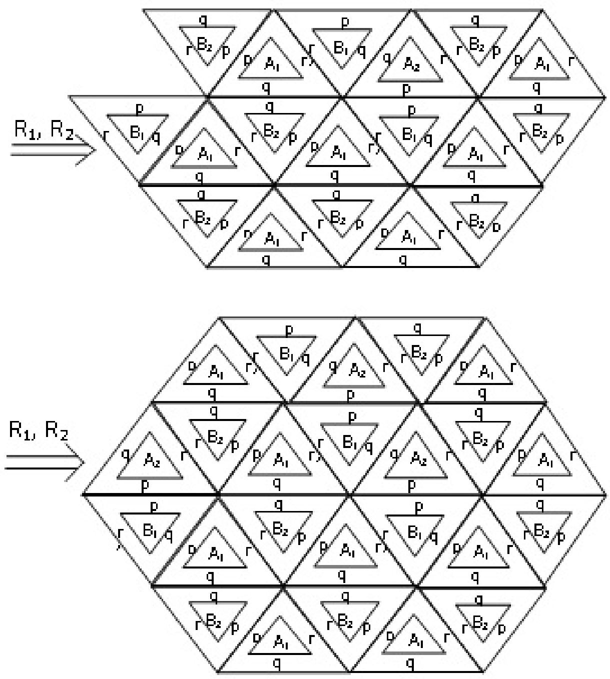


Figure 4: 2^{nd} member of hexagonal picture language

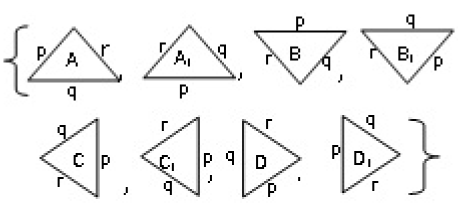
Theorem 3.1. *ITWPS generating a sub class of hexagonal picture languages*

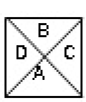
Proof. It is clear from the example 3.2.

Theorem 3.2. $L(ITWPS) - L(HTS) \neq \phi$

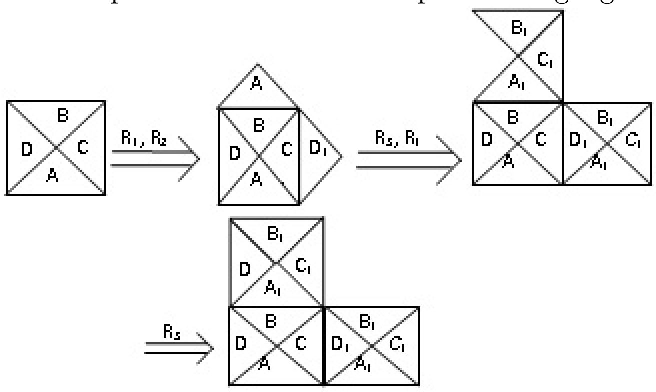
Proof. In example 3.2, a class of hexagonal picture language is generated by iso-triangular Wang tilie Pasting p system. This hexagonal language is recogonized by the HTS. It shows that HTS can generate the hexagonal picture language, but not all the triangular picture languages which are generated by IWTPPS. In order To explain that we considered the triangular picture language of stair case model, which can be generated by ITWTPPS and it cannot be generated by any HTS.

Consider the ITWPS $\Pi_2 = (\Sigma, [_1[_2]_2[_3]_3]_1, \{p, q, r\}, T'_w, F_1, F_2, F_3, R_1, R_2, R_3, 1)$, where Σ is the finite set of all labels



$F_1 =$ , $F_2 = \phi$, $F_3 = \phi$,

$R_1 = \{((A,C)/(r,r))_{in2}, ((B,A_1)/(p,p))_{in2}, ((A_1,C_1)/(q,q))_{in2},$
$((B_1,A)/(q,q))_{in2}, ((B_1,D)/(r,r))_{here}\},$
$R_2 = \{((C,B)/q,q))_{in3}, ((C,D_1)/(p,p))_{in3}, ((C_1,B_1)/(p,p))_{out},$
$((C_1,D_1)/(p,p))_{in3}\},$
$R_3 = \{((D_1,A_1)/(r,r))_{out}, ((D_1,A)/(p,p))_{out}, ((B,D)/(r,r))_{out}\}.$
The derivation step of one member of the picture language is shown below.



□

Theorem 3.3. $L(ITWS) \subseteq L(ITWPS)$

Proof. It is clear from the definitions 2.7 and 3.1. □

Theorem 3.4. $L(ITTS) \subseteq L(ITWPS)$

Proof. It is true from definitions 2.5 and 3.1. □

4. Iso-triangular Wang automaton and Wang P system

As every non-deterministic automaton is converted to deterministic automaton and we do not concentrate on explicit form. In this section iso-triangular, Wang automaton is defined non-deterministically to recognize the iso-triangular pictures.

A non-deterministic iso-triangular Wang automata is a six tuple (ITWA) $A_w = (T_w, Q, C, q_0, F, \Delta)$ where T_w-The finite set of iso-triangular wang tiles, Q-The finite set of states, C-The finite set of colors, q_0-The initial states, F-The set of final states, Δ-The set of transition rules, $\Delta \subseteq Q \times T_w \times C$, the relation in the transition consisting of transition rules, which is of the form $(q, T_w, p) \in \Delta$. The working rule is as follows;

1. A tile of T_w can be considered as an initial symbol in the initial state.
2. A Wang tile is glued with one another, only if the edges have the same colour.
3. After the glueing is occurred by the transition rules of Δ, the resultant picture transmit to the next state.

4. The process continued in this way until the iso-triangular picture is read by the scanner tape presented in the machine and the picture is transmitted to the final state.

5. The computation halts if the reading head of the automata reach the rightmost end and further no more rule can be applicable.

The automata A_w accepts the picture $p \in \Sigma_I^{**}$, if the picture p is collected in the set of final states F. Suppose, if the picture p does not in the state of final state set F, then the picture p is rejected by A_w.

The set of all iso-triangular pictures is a two dimensional picture language which is recognized by the automaton A_w and it is denoted as $L(A_w) = \{p \in \Sigma_I^{**}/A_w \text{ accepts } p\}$.

The strings are considered in automaton to stimulate the P systems. We now define iso-triangular Wang P system with iso-triangular Wang automaton to recognize the iso pictures. Iso-triangular Wang P system is defined as a structure with iso-triangular Wang automaton $A_w = (T_w, Q, C, q_0, F, \Delta)$. The P system is $\Pi_{ITW} = (V, \mu, F_i, \alpha_i, R_i, i_0)$ where $V = \Sigma \cup (T_w \times C) \cup Q$, μ-the membrane structure consists of m membranes which are labelled. F_i - initial pictures inside the membranes, $\alpha_i = (rep, rot)$, R_i - the set of transition rules Δ associated with the target indications and of the form $(p, T_w, \alpha_i, q)_{Tar}$, $Tar = (here, in_j, out\}$ $(1 \leq i \leq m, 2 \leq j \leq m)$.

The rules in R_i are applied in parallel manner. If a Wang tile goes to a point on running head of the automaton in the P system, the suitable transition rule is applied in that particular state and the edges of the tiles are glued when the edges have the same colour. In string case the automata can read the first symbol. But as for the iso-triangular Wang tiles only the suitable tile can be read and it is pasted with a iso-triangular tile, which has the same coloured edge. The resultant picture is scanned and transmitted from the present state to the next state. This mechanism takes place until the resultant picture goes to final state of F. Note that the final state is presented in the region of the output membrane. The computation is success if the picture halts in the output membrane.

The picture language is denoted by $L = \{p_i/p_i \in F_{Aw} \in i_0\}$.

Proposition 4.1. *The iso-triangular Wang automata recognize the iso-triangular picture language of size* (n, m). *The Wang automata* $A_w = (T_w, Q, C, q_0, F, \Delta)$, *with the three iso-triangular Wang tiles generates the iso-picture language, whose members are the iso-triangles an odd number of iso-triangular Wang tiles in the base* $(n \geq 3)$ *with* m *length of tiles. Here the initial state is* q_0 *and the final state is* F, *the three colours are labelled as* p, q *and* r *which are present in the colour set* C. Q*-the set of all states and* Δ *consists of the following transition rules,* $\{(q_0, A_1, q_1)$, (q_0, A_2, q_2), (q_0, B_1, q_0), (q_1, B_1, q_2), (q_2, A_2, q_3), (q_3, B_1, q_0), $(q_3, A_2, q_3)\}$. *In the transition rules all* $A's$ *and* $B's$ *are the labelled iso-triangular wang tiles. For space and time concern the labels of Wang tiles are considered* *in the rules. The transition graph of the language of iso-triangles is given in figure 5. Initial state is defined by enclosed circle and an arrow, the final state is represented by a concentric circle. The iso-triangular Wang tile is pasted with other Wang tile, only if the edges have the same colour. Initial state is q_0 and the initial iso-Wang tile is A_1. The transition rules are applied and the member of the language is collected in the final state of the automata. The language is recognizable since the language is accepted by the automata.*

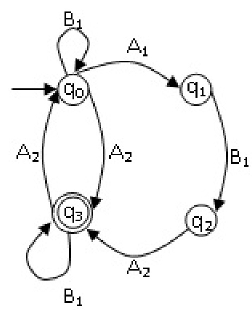


Figure 5: Transition graph of the iso-triangular picture language

Lemma 4.1. *Two pictures patterns p_1 and p_2 shown in figure 6 are equal in shape but not in size. Because the picture pattern p_1 is generated by the iso-triangular wang tiles with no geometrical operation the second one is generated by the equilateral-triangular wang tiles with rotation.*

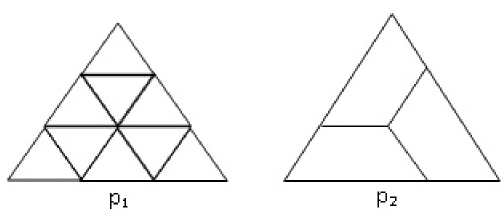
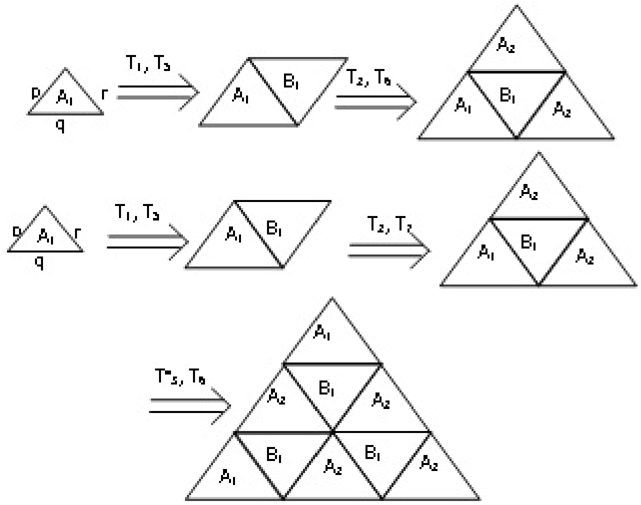


Figure 6: Triangular pictures

Theorem 4.1. *The iso-triangular Wang automata A_w stimulate the iso-triangular Wang P system.*

Consider The iso-triangular Wang automata $A_w = (T_w, Q, C, q_0, q_3, \Delta)$ and the Wang p system $(V, [_1[_2]_2[_3]_3]_1, F_1, F_2, F_3, \alpha_i, R_1, R_2, R_3, 3)$, where $v = T_w \cup Q$, $R_1 = \{T_1, T_2, T_3, T_4\}$, $R_2 = \{T_5\}$, $R_3 = \{T_6, T_7\}$, $F_2 = \phi$, $F_3 = \phi$ and $\alpha_i = \phi$. The states in Q are q_0, q_1, q_2, q_3 and the Wang tiles are $T_w = \{A_1, A_2, B_1\}$. The transition rules are $T_1 = (q_0, A_1, q_1)_{in2}$, $T_2 = (q_2, A_2, q_3)_{in3}$, $T_3 = (q_0, B_1, q_0)_{here}$, $T_4 = (q_0, A_2, q_3)_{in3}$, $T_5 = (q_1, B_1, q_2)_{out}$, $T_6 = (q_3, A_2, q_3)_{here}$, $T_7 = (q_3, A_2, q_0)_{out}$. The P system generates the iso-triangular picture language and recognized by the automata. The graph is shown fig. 5 and the two members are given below with two derivation steps.



5. Conclusion

This paper, introduced an iso-triangular Wang P system and iso-triangular Wang automata with suitable examples, which are generating iso triangular arrays. Also, the computational complexity of ITWPS is examined by comparing the generative powers of ITWS, ITTS. It is noticed that the ITWA stimulates the ITWPS to generate iso-picture languages.

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