New types of soft ordered mappings via soft α-open sets

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Abstract. The concept of soft topological ordered spaces is an extension of the soft topological spaces notion. The motivation of this paper is twofold: One is to generalize and extend the existing ordered maps and other is to contribute on making a general framework for studying soft topological ordered spaces. In this study, we utilize soft α-open sets to introduce new ordered maps, which generalize existing comparable notions, namely soft $x\alpha$-continuous, soft $x\alpha$-open, soft $x\alpha$-closed and soft $x\alpha$-homeomorphism maps, for $x \in \{I, D, B\}$, via soft topological ordered spaces. We show the relationships among these concepts and discuss the equivalent conditions for each one of them. Also, we derive that an extended soft topologies notion guarantees the equivalent between the soft maps initiated herein and their counterparts of maps on topological ordered spaces. For illustration and comparison, various examples are provided.

Keywords: soft $I(D, B)\alpha$-continuous map, Soft $I(D, B)\alpha$-open map and soft $I(D, B)\alpha$-homeomorphism map.

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1. Introduction and preliminaries

By combining a topological structure τ with a partial order relation $\preceq$ on a non-empty set X, Nachbin [42] named the triple $(X, \tau, \preceq)$ a topological ordered space and established some topological ordered notions about it which depend on topological structure and partial order relation. In his work, he introduced the concepts of normally ordered, regularly ordered and completely ordered spaces. McCartan [37] renamed some ordered separation axioms and characterised T_i-ordered spaces ($i = 0, 1, 2, 3, 4$). Also, he presented interesting examples to show that T_i-spaces are proper generalization of T_i-ordered spaces, for $i = 0, 1, 2$, and derived some results which connected T_i-ordered axioms and some topological notions like one-point compactification and local compactness. Some scholars investigated another types of ordered spaces by replacing a partial order relation by preorder relation or any binary relation (see, for example, [29, 38, 39, 45]). As a generalization of topological ordered spaces, Das [20] introduced and studied ordered spaces. For more investigation on ordered spaces, we refer the interested readers to [3, 7, 10, 13, 17, 22, 23, 24, 28, 29, 32, 33, 34, 49].

Molotdsov [41], in 1999, came up with an idea, namely soft sets, in order to approach uncertainties and vagues of data on some reality situations and phenomena. He discussed the strengths of soft set theory in compared with fuzzy theory and probability theory and investigated its applications on different fields. As a result of advantages of soft sets in overcoming incomplete data, many researchers began introducing some soft operations between soft sets and applying it in several situations arising in information science, decision making, mathematics and other related disciplines (see, for example, [1, 2, 19, 27, 44]).

To insert soft sets on topology studies, Shabir and Naz [48] formulated the soft topological spaces notion by analogy with the definition of topology. They fixed a parameters set to avoid anomaly arising in appearing various non-null soft sets. They gave elementary ideas on soft topological spaces and studied soft separation axioms. Min [40] observed some mistakes on Shabir and Naz's work and corrected them. In this regard, we think the existence of these mistakes was due to the lack of knowledge of shape of soft open sets in soft regular spaces. Later on, the desire of obtaining a deeper understanding of soft topology prompted interested researchers to carry out many studies on soft topological notions and their features (see, for example, [18, 30, 31, 43]). [8, 9, 12, 25] investigated and corrected some errors which appeared in some previous studies. In [4], the authors gave the definition of soft α-open sets and examined some related properties. They [5] also defined soft α-separation axioms and characterised them. In the regard of studying generalized soft open sets, Al-shami [11] originated a concept of somewhere dense sets and investigated many of its properties.

Recently, the authors of [14] introduced a concept of soft topological ordered spaces and established the notions of increasing (decreasing) soft sets and increasing (decreasing) soft maps. Also, they defined p-soft T_i-ordered spaces

$(i = 0, 1, 2, 3, 4)$ depending on totally non belong relations, which defined in [26], and monotone soft neighborhoods. Based on soft β-open sets, some of soft ordered maps were introduced and studied in [16]. In the end of this literature, we indicate to that the authors of [1, 2, 46] defined new types of soft subsets, soft equality, soft union and intersection by relaxing the conditions on the parameters sets.

The purpose of this study is to introduce and study the concepts of soft $x\alpha$-continuous, soft $x\alpha$-open, soft $x\alpha$-closed and soft $x\alpha$-homeomorphism maps, for $x \in \{I, D, B\}$, via soft topological ordered spaces and to obtain a deeper understanding of them. Also, we give various examples to show the relationships among these maps and illustrate that soft $x\alpha$-continuous, soft $x\alpha$-open, soft $x\alpha$-closed and soft $x\alpha$-homeomorphism maps are strictly stronger than soft α-continuous, soft α-open, soft α-closed and soft α-homeomorphism maps, respectively, for $x \in \{I, D, B\}$. Furthermore, we characterize each one of the initiated soft maps and clarify a significant role of extended soft topologies on studying the interrelations between these soft maps and their counterparts of maps in topological ordered spaces.

We allocate the rest of this section to present some fundamental definitions and findings that will be needed in the sequels.

Definition 1.1 ([41]). *An ordered pair (G, E) is said to be a soft set over X if G is a map of a set of parameters E into 2^X.*

Remark 1.2. (i) For short, we use the notation G_E instead of (G, E).

(ii) A soft set G_E can be written as a set of ordered pairs $G_E = \{(e, G(e)) : e \in E \text{ and } G(e) \in 2^X\}$.

Definition 1.3 ([36]). *A soft set G_E over X is called a null soft set, denoting by $\widetilde{\Phi}$, if $G(e) = \emptyset$, for each $e \in E$ and is called an absolute soft set, denoting by $\widetilde{X}$, if $G(e) = X$, for each $e \in E$.*

Definition 1.4 ([6]). *The relative complement of a soft set G_E is denoted by G^c_E, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$, for each $e \in E$.*

In this connection, it is worth noting that $x \notin G_E$ does not imply that $x \in G^c_E$.

Definition 1.5 ([48]). *For $x \in X$ and a soft set G_E over X, we say that $x \in G_E$ if $x \in G(e)$, for each $e \in E$ and $x \notin G_E$ if $x \notin G(e)$, for some $e \in E$.*

Definition 1.6 ([48]). *A soft topology on a non-empty set X is a collection τ of soft sets over X under a parameters set E satisfying the following axioms:*

(i) *$\widetilde{X}$ and $\widetilde{\emptyset}$ belong to τ.*

(ii) *The soft intersection of finite members in τ belongs to τ.*

(iii) The soft union of any members in τ belongs to τ.

The triple (X, τ, E) is called a soft topological space. Every member of τ is called soft open and its relative complement is called soft closed.

Proposition 1.7 ([48]). *Let (X, τ, E) be a soft topological space. Then $\tau_e = \{G(e) : G_E \in \tau\}$ defines a topology on X, for each $e \in E$.*

Definition 1.8 ([4]). *A soft subset H_E of (X, τ, E) is said to be soft α-open if $H_E \widetilde{\subseteq} int(cl(int(H_E)))$. Its complement is said to be soft α-closed.*

Definition 1.9 ([4], [48]). *For a soft subset H_E of (X, τ, E), we define the following four operators:*

(i) $int(H_E)$ is the largest soft open set contained in H_E.

(ii) $cl(H_E)$ is the smallest soft closed set containing H_E.

(iii) $int_\alpha(H_E)$ is the largest soft α-open set contained in H_E.

(iv) $cl_\alpha(H_E)$ is the smallest soft α-closed set containing H_E.

Definition 1.10 ([43]). *Consider (X, τ, E) is a soft topological space and τ_e is a topology on X as in the above proposition. Then $\tau^\star = \{G_E : G(e) \in \tau_e$, for each $e \in E\}$ is a soft topology on X finer than τ.*
In this work, we term $\tau^\star$ an extended soft topology.

Definition 1.11 ([50]). *Consider $f : X \to Y$ and $\phi : A \to B$ are two maps and let $f_\phi : S(X_A) \to S(Y_B)$ be a soft map. Let G_K and H_L be soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then*

(i) $f_\phi(G_K) = (f_\phi(G))_B$ is a soft subset of $S(Y_B)$ such that

$$f_\phi(G)(b) = \begin{cases} \bigcup_{a \in \phi^{-1}(b) \bigcap K} f(G(a)), & \phi^{-1}(b) \bigcap K \neq \emptyset \\ \emptyset, & \phi^{-1}(b) \bigcap K = \emptyset \end{cases}$$

for each $b \in B$.

(ii) $f_\phi^{-1}(H_L) = (f_\phi^{-1}(H))_A$ is a soft subset of $S(X_A)$ such that

$$f_\phi^{-1}(H)(a) = \begin{cases} f^{-1}(H(\phi(a))), & \phi(a) \in L \\ \emptyset, & \phi(a) \notin L \end{cases}$$

for each $a \in A$.

Remark 1.12. Henceforth, a soft map $f_\phi : S(X_A) \to S(Y_B)$ implies that a map f of the universe set X into the universe set Y and a map ϕ of a set of parameters A into a set of parameters B

Definition 1.13 ([50]). *A soft map $f_\phi : S(X_A) \to S(Y_B)$ is said to be injective (resp. surjective, bijective) if f and ϕ are injective (resp. surjective, bijective).*

Proposition 1.14 ([43]). *Consider $f_\phi : S(X_A) \to S(Y_B)$ is a soft map and let G_A and H_B be two soft subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then we have the following results:*

(i) $G_A \widetilde{\subseteq} f_\phi^{-1} f_\phi(G_A)$ *and the equality relation holds if f_ϕ is injective.*

(ii) $f_\phi f_\phi^{-1}(H_B) \widetilde{\subseteq} H_B$ *and the equality relation holds if f_ϕ is surjective.*

Definition 1.15 ([4]). *A soft map $f_\phi : (X, \tau, A) \to (Y, \theta, B)$ is said to be:*

(i) *Soft α-continuous if the inverse image of each soft open subset of (Y, θ, B) is a soft α-open subset of (X, τ, A).*

(ii) *Soft α-open (resp. soft α-closed) if the image of each soft open (resp. soft closed) subset of (X, τ, A) is a soft α-open (resp. soft α-closed) subset of (Y, θ, B).*

(iii) *Soft α-homeomorphism if it is bijective, soft α-continuous and soft α-open.*

Definition 1.16 ([21], [43]). *A soft subset P_E over X is called soft point if there exists $e \in E$ and there exists $x \in X$ such that $P(e) = \{x\}$ and $P(a) = \emptyset$, for each $a \in E \setminus \{e\}$. A soft point will be shortly denoted by P_e^x and we say that $P_e^x \in G_E$, if $x \in G(e)$.*

Definition 1.17 ([14]). *Let $\preceq$ be a partial order relation on a non-empty set X and let E be a set of parameters. A triple $(X, E, \preceq)$ is said to be a partially ordered soft set.*

Definition 1.18 ([14]). *We define an increasing soft operator $i : (SS(X)_E, \preceq\ ) \to (SS(X)_E, \preceq)$ and a decreasing soft operator $d : (SS(X)_E, \preceq\ ) \to (SS(X)_E, \preceq)$ as follows, for each soft subset G_E of $SS(X_E)$*

(i) *$i(G_E) = (iG)_E$, where iG is a mapping of E into X given by $iG(e) = i(G(e)) = \{x \in X : y \preceq x$, for some $y \in G(e)\}$.*

(ii) *$d(G_E) = (dG)_E$, where dG is a mapping of E into X given by $dG(e) = d(G(e)) = \{x \in X : x \preceq y$, for some $y \in G(e)\}$.*

Definition 1.19 ([14]). *A soft subset G_E of a partially ordered soft set $(X, E, \preceq)$ is said to be increasing (resp. decreasing) if $G_E = i(G_E)$(resp. $G_E = d(G_E)$).*

Theorem 1.20 ([14]). *If a soft map $f_\phi : (S(X_A), \preceq_1) \to (S(Y_B), \preceq_2)$ is increasing, then the inverse image of each increasing (resp. decreasing) soft subset of $\widetilde{Y}$ is an increasing (resp. a decreasing) soft subset of $\widetilde{X}$.*

Definition 1.21 ([14]). *A quadrable system $(X, \tau, E, \preceq)$ is said to be a soft topological ordered space, where (X, τ, E) is a soft topological space and $(X, E, \preceq)$ is a partially ordered soft set. Henceforth, the two notations $(X, \tau, E, \preceq_1)$ and $(Y, \theta, F, \preceq_2)$ stand for soft topological ordered spaces.*

Definition 1.22 ([47]). *A map $(X, \tau, \preceq_1) \to (Y, \theta, \preceq_2)$ is said to be:*

(i) *I (resp. D, B) α-continuous if the inverse image of each open set is I (resp. D, B) α-open.*

(ii) *I (resp. D, B) α-open if the image of each open set is I (resp. D, B) α-open.*

(iii) *I (resp. D, B) α-closed if the image of each open set is I (resp. D, B) α-closed.*

(iv) *I (resp. D, B) α-homeomorphism if it is bijective, I (resp. D, B) α-continuous and I (resp. D, B) α-open.*

Definition 1.23 ([15]). *The composition of two soft maps $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2 \ )$ and $g_\lambda : (Y, \theta, F, \preceq_2 \ ) \to (Z, \upsilon, K, \preceq_3)$ is a soft map $f_\phi \circ g_\lambda : (X, \tau, E, \preceq_1) \to (Z, \upsilon, K, \preceq_3)$ and is given by $(f_\phi \circ g_\lambda)(P_e^x) = f_\phi(g_\lambda(P_e^x))$.*

2. Soft $I(D, B)\alpha$-continuous maps

The purpose of this section is to define soft $I(D, B)\alpha$-continuity at soft point, ordinary point and on the universe set and to elucidate the relationships among them with the help of examples. The characterisations of of the given soft maps are studied and the interrelations between these soft maps and their counterparts of maps on topological ordered spaces are discussed.

Definition 2.1. *A soft subset H_E of $(X, \tau, E, \preceq_1)$ is said to be:*

(i) *Soft I (resp. Soft D, Soft B) α-open if it is soft α-open and increasing (resp. decreasing, balancing).*

(ii) *Soft I (resp. Soft D, Soft B) α-closed if it is soft α-closed and increasing (resp. decreasing, balancing).*

Definition 2.2. *A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is called:*

(i) *Soft I (resp. Soft D, Soft B) α-continuous at $P_e^x \in \widetilde{X}$ if for each soft open set H_F containing $f_\phi(P_e^x)$, there exists a soft I (resp. soft D, soft B) α-open set G_E containing P_e^x such that $f_\phi(G_E) \widetilde{\subseteq} H_F$.*

(ii) *Soft I (resp. Soft D, Soft B) α-continuous at $x \in X$ if it is soft I (resp. soft D, soft B) α-continuous at each P_e^x.*

(*iii*) *Soft I (resp. Soft D, Soft B) α-continuous if it is soft I (resp. soft D, soft B) α-continuous at each $x \in X$.*

Theorem 2.3. *A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) α-continuous if and only if the inverse image of each soft open subset of $\widetilde{Y}$ is a soft I (resp. soft D, soft B) α-open subset of $\widetilde{X}$.*

Proof. We only present theorem's proof in case of f_ϕ is soft Iα-continuous and the cases between parenthesis can be made similarly.
To prove the necessary part, let G_F be a soft open subset of $\widetilde{Y}$, Then we have the following two cases:

(i) Either $f_\phi^{-1}(G_F) = \widetilde{\emptyset}$.

(ii) Or $f^{-1}(G_F) \neq \widetilde{\emptyset}$. By choosing $P_e^x \in X$ such that $P_e^x \in f_\phi^{-1}(G_F)$, we obtain that $f_\phi(P_e^x) \in G_F$. So there exists a soft Iα-open set H_E containing P_e^x such that $f_\phi(H_E) \widetilde{\subseteq} G_F$. Since P_e^x is chosen arbitrary, then $f_\phi^{-1}(G_F) = \widetilde{\bigcup}_{P_e^x \in f_\phi^{-1}(G_F)} H_E$.

From the two cases above, we conclude that $f_\phi^{-1}(G_F)$ is a soft Iα-open subset of $\widetilde{X}$. To prove the sufficient part, let G_F be a soft open subset of $\widetilde{Y}$ containing $f_\phi(P_e^x)$. Then $P_e^x \in f_\phi^{-1}(G_F)$. By hypothesis, $f_\phi^{-1}(G_F)$ is a soft Iα-open set. Since $f_\phi(f_\phi^{-1}(G_F)) \widetilde{\subseteq} G_F$, then f_ϕ is a soft Iα-continuous map at $P_e^x \in X$ and since P_e^x is chosen arbitrary, then f_ϕ is a soft Iα-continuous map. □

Remark 2.4. From Definition (2.2), we can note the following:

(i) Every soft I (D, B) α-continuous map is always soft α-continuous.

(ii) Every soft Bα-continuous map is soft Iα-continuous or soft Dα-continuous.

To elucidate that the converse of the two results of the remark above need not be true, we construct the following two examples.

Example 2.5. Let the two universe sets $X = \{2, 3, 5\}$, $Y = \{7, 11, 13\}$ and the two parameters sets $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$. Consider a map $\phi : A \to B$ is defined as, $\phi(a_1) = \phi(a_2) = b_1$ and a map $f : X \to Y$ is defined as, $f(2) = 7$ and $f(3) = f(5) = 11$. We define a partial order relation on X as $\preceq = \triangle \bigcup \{(3, 2), (2, 5), (3, 5)\}$ and we define two soft topologies τ and θ on X and Y, respectively, as $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_A\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, H_B\}$, where $F_A = \{(a_1, \{2\}), (a_2, \emptyset)\}$ and $H_B = \{(b_1, \{7\}), (b_2, \emptyset)\}$. Since $f_\phi^{-1}(H_B) = \{(a_1, \{2\}), (a_2, \{2\})\}$ is a soft α-open set, then $f_\phi : S(X_A) \to S(Y_B)$ is a soft α-continuous map. On the other hand, $f_\phi^{-1}(H_B)$ is neither a soft Dα-open nor a soft Iα-open set. Hence f_ϕ is not soft I (soft D, soft B) α-continuous.

Example 2.6. In Example above, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(2, 5)\}$(resp. $\preceq = \triangle \bigcup \{(3, 2)\}$), then the soft map f_ϕ is soft D-continuous (resp. soft I-continuous), but is not soft B-continuous.

Definition 2.7. *For a soft subset H_E of $(X,\tau,E,\preceq)$, we define the following six operators:*

(i) $H_E^{i\alpha o}$ *(resp.* $H_E^{d\alpha o}, H_E^{b\alpha o}$*) is the largest soft I (resp. soft D, soft B) α-open set contained in H_E.*

(ii) $H_E^{i\alpha cl}$ *(resp.* $H_E^{d\alpha cl}, H_E^{b\alpha cl}$*) is the smallest soft I (resp. soft D, soft B) α-closed set containing H_E.*

Lemma 2.8. *We have the following three properties for a soft subset H_E of $(X,\tau,E,\preceq)$:*

(i) $(H_E^{d\alpha cl})^c = (H_E^c)^{i\alpha o}$.

(ii) $(H_E^{i\alpha cl})^c = (H_E^c)^{d\alpha o}$.

(iii) $(H_E^{b\alpha cl})^c = (H_E^c)^{b\alpha o}$.

Proof. (i) $(H_E^{d\alpha cl})^c = \{\widetilde{\bigcap} F_E$: F_E is a soft Dα-closed set containing $H_E\}^c$ $=\widetilde{\bigcup}\{F_E^c : F_E^c$ is a soft Iα-open set contained in $H_E^c\} = (H_E^c)^{i\alpha o}$.

By analogy with (i), one can prove (ii) and (iii). □

Theorem 2.9. *The following five properties of a soft map $f_\phi : (X,\tau,E \preceq_1) \to (Y,\theta,F,\preceq_2)$ are equivalent:*

(i) f_ϕ *is soft Iα-continuous;*

(ii) $f_\phi^{-1}(L_F)$ *is a soft Dα-closed subset of $\widetilde{X}$, for each soft closed subset L_F of $\widetilde{Y}$;*

(iii) $(f_\phi^{-1}(M_F))^{d\alpha cl}\widetilde{\subseteq} f_\phi^{-1}(cl(M_F))$, *for every* $M_F\widetilde{\subseteq}\widetilde{Y}$;

(iv) $f_\phi(N_E^{d\alpha cl})\widetilde{\subseteq} cl(f_\phi(N_E))$, *for every* $N_E\widetilde{\subseteq}\widetilde{X}$;

(v) $f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{i\alpha o}$, *for every* $M_F\widetilde{\subseteq}\widetilde{Y}$.

Proof. $(i) \Rightarrow (ii)$: Consider L_F is a soft closed subset of $\widetilde{Y}$. By hypothesis, $f_\phi^{-1}(L_F^c)$ is a soft Iα-open subset of $\widetilde{X}$ and by the fact that $f_\phi^{-1}(L_F^c) = (f_\phi^{-1}(L_F))^c$, we obtain that $f_\phi^{-1}(L_F)$ is soft Dα-closed as required.

$(ii) \Rightarrow (iii)$: It follows from statement (ii) that $f_\phi^{-1}(cl(M_E))$ is a soft Dα-closed subset of $\widetilde{X}$, for every $M_F\widetilde{\subseteq}\widetilde{Y}$. So $(f_\phi^{-1}(M_F))^{d\alpha cl}\widetilde{\subseteq}(f_\phi^{-1}(cl(M_F))^{d\alpha cl} = f_\phi^{-1}(cl(M_F))$.

$(iii) \Rightarrow (iv)$: From the fact that $N_E^{d\alpha cl}\widetilde{\subseteq}(f_\phi^{-1}(f_\phi(N_E))^{d\alpha cl}$ and from (iii), we have $(f_\phi^{-1}(f_\phi(N_E))^{d\alpha cl} \widetilde{\subseteq} f_\phi^{-1}(cl(f_\phi(N_E))$.

This implies that $f_\phi(N_E^{d\alpha cl})\widetilde{\subseteq} cl(f_\phi(N_E))$.

$(iv) \Rightarrow (v)$: For any soft subset M_F of $\widetilde{Y}$, we obtain from Lemma (2.8) that $f_\phi(\widetilde{X} - (f_\phi^{-1}(N_E))^{i\alpha o}) = f_\phi(((f_\phi^{-1}(N_E))^c)^{d\alpha cl})$. It follows from statement (iv), that $f_\phi(((f_\phi^{-1}(N_E))^c)^{d\alpha cl})\,\widetilde{\subseteq}cl(f_\phi(f_\phi^{-1}(N_E))^c) = cl(f_\phi(f_\phi^{-1}(N_E^c)))\widetilde{\subseteq}cl(\widetilde{Y} - N_E) = \widetilde{Y} - int(N_E)$. Therefore $(\widetilde{X} - (f_\phi^{-1}(N_E))^{i\alpha o})\,\widetilde{\subseteq}f_\phi^{-1}(\widetilde{Y} - int(N_E)) = \widetilde{X} - f_\phi^{-1}(int(N_E))$. Thus $f_\phi^{-1}(int(N_E))\widetilde{\subseteq}(f_\phi^{-1}(N_E))^{i\alpha o}$.

$(v) \Rightarrow (i)$: Consider M_F is a soft open subset of $\widetilde{Y}$. Then $f_\phi^{-1}(M_F) = f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{i\alpha o}$. So $(f_\phi^{-1}(M_F))^{i\alpha o} = f_\phi^{-1}(M_F)$ and this means that $f_\phi^{-1}(M_F)$ is a soft Iα-open subset of $\widetilde{X}$. Hence the desired result is proved. □

Theorem 2.10. *The following five properties of a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:*

- (i) *f_ϕ is soft Dα-continuous (resp. soft Bα-continuous);*
- (ii) *$f_\phi^{-1}(L_F)$ is a soft Iα-closed (resp. soft Bα-closed) subset of $\widetilde{X}$, for each soft closed subset L_F of $\widetilde{Y}$;*
- (iii) *$(f_\phi^{-1}(M_F))^{i\alpha cl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_F))$(resp. $(f_\phi^{-1}(M_F))^{b\alpha cl}\widetilde{\subseteq}f_\phi^{-1}(cl(M_F))$, for every $M_F\widetilde{\subseteq}\widetilde{Y}$;*
- *(iv) $f_\phi(N_E^{i\alpha cl})\widetilde{\subseteq}cl(f_\phi(N_E))$(resp. $f_\phi(N_E^{b\alpha cl})\widetilde{\subseteq}cl(f_\phi(N_E))$, for every $N_E\widetilde{\subseteq}\widetilde{X}$;*
- (v) *$f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{d\alpha o}$(resp. $f_\phi^{-1}(int(M_F))\widetilde{\subseteq}(f_\phi^{-1}(M_F))^{b\alpha o}$, for every $M_F\widetilde{\subseteq}\widetilde{Y}$.*

Proof. The proof is similar to that of Theorem (2.9). □

Theorem 2.11. *Let $\tau^\star$ be an extended soft topology on X. Then a soft map $g_\phi : (X, \tau^\star, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft I (resp. soft D, soft B) α-continuous if and only if a map $g : (X, \tau_e^\star, \preceq_1) \to (Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) α-continuous.*

Proof. Necessity: Let U be an open subset of $(Y, \theta_{\phi(e)}, \preceq_2)$. Then there exists a soft open subset G_F of $(Y, \theta, F, \preceq_2)$ such that $G(\phi(e)) = U$. Since g_ϕ is a soft I (resp. soft D, soft B) α-continuous map, then $g_\phi^{-1}(G_F)$ is a soft I (resp. soft D, soft B) α-open set. From Definition (1.11), it follows that a soft subset $g_\phi^{-1}(G_F) = (g_\phi^{-1}(G))_E$ of $(X, \tau, E, \preceq_1)$ is given by $g_\phi^{-1}(G)(e) = g^{-1}(G(\phi(e)))$, for each $e \in E$. By hypothesis, $\tau^\star$ is an extended soft topology on X, we obtain that a subset $g^{-1}(G(\phi(e))) = g^{-1}(U)$ of $(X, \tau_e, \preceq_1)$ is I (resp. D, B) α-open. Hence a map g is I (resp. D, B) α-continuous.
Sufficiency: Let G_F be a soft open subset of $(Y, \theta, F, \preceq_2)$. Then from Definition (1.11), it follows that a soft subset $g_\phi^{-1}(G_F) = (g_\phi^{-1}(G))_E$ of $(X, \tau^\star, E, \preceq_1)$ is given by $g_\phi^{-1}(G)(e) = g^{-1}(G(\phi(e)))$, for each $e \in E$. Since a map g is I (resp. D, B) α-continuous, then a subset $g^{-1}(G(\phi(e)))$ of $(X, \tau_e^\star, \preceq_1)$ is I (resp. D, B) α-open. By hypothesis, $\tau^\star$ is an extended soft topology on X, we obtain that $g_\phi^{-1}(G_F)$ is a soft I (resp. soft D, soft B) α-open subset of $(X, \tau^\star, E, \preceq_1)$. Hence a soft map g_ϕ is soft I (resp. soft D, soft B) α-continuous. □

Proposition 2.12. *Let $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ be a surjective soft $B\alpha$-continuous. If $\preceq_1$ is linearly ordered, then θ is the soft indiscrete topology.*

3. Soft $I(D, B)\alpha$-open and soft $I(D, B)\alpha$-closed maps

In this part, we establish the notions of soft $I(D, B)\alpha$-open and soft $I(D, B)\alpha$-closed maps and show the relationships among them. Also, we give the equivalent conditions for each one of these soft maps and investigate the interrelations between these soft maps and their counterparts of maps on topological ordered spaces.

Definition 3.1. *A soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \tau, F, \preceq_2)$ is called:*

(i) *Soft I (resp. Soft D, Soft B) α-open if the image of every soft open subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) α-open subset of $\widetilde{Y}$.*

(ii) *Soft I (resp. Soft D, Soft B) α-closed if the image of every soft closed subset of $\widetilde{X}$ is a soft I (resp. soft D, soft B) α-closed subset of $\widetilde{Y}$.*

Remark 3.2. From Definition (3.1), we can note the following:

(i) Every soft I (D, B) α-open map is soft α-open.

(ii) Every soft I (D, B) α-closed map is soft α-closed.

(iii) Every soft Bα-open (resp. soft Bα-closed) map is soft Iα-open or soft Dα-open (resp. soft Iα-closed or soft Dα-closed).

In the following two examples, we show that the converse of the three statements of remark above fails.

Example 3.3. Let the two universe sets X, Y, the two parameters sets A, B and a map $f : X \to Y$ be the same as in Example (2.5). Consider a map $\phi : A \to B$ is defined as, $\phi(a_m) = b_m$, for $m \in \{1, 2\}$. We define a partial order relation on Y as $\preceq = \triangle \bigcup \{(7, 11), (11, 13), (7, 13)\}$ and we define two soft topologies τ and θ on X and Y, respectively, as $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_A\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, H_B\}$, where $F_A = \{(a_1, \{2, 3\}), (a_2, \{5\})\}$ and $H_B = \{(b_1, \{11\}), (b_2, \{11\})\}$. Since $f_\phi(F_A) = \{(b_1, \{7, 11\}), (b_2, \{11\})\}$ is a soft α-open set, then $f_\phi : S(X_A) \to S(Y_B)$ is a soft α-open map and since $f_\phi(F_A^c) = \{(b_1, \{11\}), (b_2, \{7, 11\})\}$ is a soft α-closed set, then $f_\phi : S(X_A) \to S(Y_B)$ is a soft α-closed map. On the other hand, $f_\phi(F_A)$ is neither a soft Dα-open nor a soft Iα-open set and $f_\phi(F_A^c)$ is neither a soft Dα-closed nor a soft Iα-closed set. Hence f_ϕ is not a soft I (soft D, soft B) α-open map and not a soft I (soft D, soft B) α-closed map.

Example 3.4. In Example above, if we replace only the partial order relation by $\preceq = \triangle \bigcup \{(7, 11)\}$(resp. $\preceq = \triangle \bigcup \{(11, 13)\}$), then the soft map f_ϕ is soft Iα-open and soft Iα-closed (resp. soft Dα-open and soft Dα-closed), but is not soft Bα-open and soft Bα-closed.

Theorem 3.5. *The following three properties of a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:*

(i) f_ϕ *is soft Iα-open;*

(ii) $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{i\alpha o})$, *for every* $M_F \widetilde{\subseteq} \widetilde{Y}$;

(iii) $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{i\alpha o}$, *for every* $N_E \widetilde{\subseteq} \widetilde{X}$.

Proof. $(i) \Rightarrow (ii)$: Given a soft subset M_F of $\widetilde{Y}$, it is obvious that $int(f_\phi^{-1}(M_F))$ is a soft open subset of $\widetilde{X}$. Then, by hypothesis, it follows that $f_\phi(int(f_\phi^{-1}(M_F)))$ is a soft Iα-open subset of $\widetilde{Y}$. Since $f_\phi(int(f_\phi^{-1}(M_F))) \widetilde{\subseteq} f_\phi(f_\phi^{-1}(M_F)) \widetilde{\subseteq} M_F$, then $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{i\alpha o})$.

$(ii) \Rightarrow (iii)$: Given a soft subset N_E of $\widetilde{X}$, from (ii), we obtain that $int(f_\phi^{-1}(f_\phi(N_E))) \widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_E))^{i\alpha o})$.

Since $int(N_E) \widetilde{\subseteq} f_\phi^{-1}(f_\phi(int(f_\phi^{-1}(f_\phi(N_E))))) \widetilde{\subseteq} f_\phi^{-1}((f_\phi(N_E))^{i\alpha o})$, then $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{i\alpha o}$ as required.

$(iii) \Rightarrow (i)$: Let G_E be a soft open subset of $\widetilde{X}$. Then $f_\phi(int(G_E)) = f_\phi(G_E) \widetilde{\subseteq} (f_\phi(G_E))^{i\alpha o}$. Hence f_ϕ is a soft Iα-open map. □

The following theorem can be proved in a similar manner.

Theorem 3.6. *The following three properties of a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ are equivalent:*

(i) f_ϕ *is soft Dα-open (resp. soft Bα-open);*

(ii) $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{d\alpha o})$(*resp.* $int(f_\phi^{-1}(M_F)) \widetilde{\subseteq} f_\phi^{-1}(M_F^{b\alpha o})$), *for every* $M_F \widetilde{\subseteq} \widetilde{Y}$;

(iii) $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{d\alpha o}$(*resp.* $f_\phi(int(N_E)) \widetilde{\subseteq} (f_\phi(N_E))^{b\alpha o}$), *for every* $N_E \widetilde{\subseteq} \widetilde{X}$.

Theorem 3.7. *The following three statements hold for a soft map $f_\phi:(X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$:*

(i) f_ϕ *is soft Iα-closed if and only if* $(f_\phi(G_E))^{i\alpha cl} \widetilde{\subseteq} f_\phi(cl(G_E))$, *for every* $G_E \widetilde{\subseteq} \widetilde{X}$.

(ii) f_ϕ *is soft Dα-closed if and only if* $(f_\phi(G_E))^{d\alpha cl} \widetilde{\subseteq} f_\phi(cl(G_E))$, *for every* $G_E \widetilde{\subseteq} \widetilde{X}$.

(iii) f_ϕ *is soft Bα-closed if and only if* $(f_\phi(G_E))^{b\alpha cl} \widetilde{\subseteq} f_\phi(cl(G_E))$, *for every* $G_E \widetilde{\subseteq} \widetilde{X}$.

Proof. We only give a proof for the first statement and the others follow similar lines.

Necessity: Since f_ϕ is soft Iα-closed, then $f_\phi(cl(G_E))$ is a soft Iα-closed subset of $\widetilde{Y}$ and since $f_\phi(G_E)\widetilde{\subseteq}f_\phi(cl(G_E))$, then $(f_\phi(G_E))^{i\alpha cl}\widetilde{\subseteq}f_\phi(cl(G_E))$.

Sufficiency: Consider H_E is a soft closed subset of $\widetilde{X}$.

Then $f_\phi(H_E)\widetilde{\subseteq}(f_\phi(H_E))^{i\alpha cl}\widetilde{\subseteq}f_\phi(cl(H_E)) = f_\phi(H_E)$. Therefore $f_\phi(H_E) = (f_\phi(H_E))^{i\alpha cl}$. This means that $f_\phi(H_E)$ is a soft Iα-closed set. Hence the proof is complete. □

Theorem 3.8. *The following three statements hold for a bijective soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$:*

(*i*) *f_ϕ is soft I (resp. soft D, soft B) α-open if and only if f_ϕ is soft D (resp. soft D, soft B) α-closed.*

(*ii*) *f_ϕ is soft I (resp. soft D, soft B) α-open if and only if f_ϕ^{-1} is soft I (resp. soft D, soft B) α-continuous.*

(*iii*) *f_ϕ is soft D (resp. soft I, soft B) α-closed if and only if f_ϕ^{-1} is soft I (resp. soft D, soft B) α-continuous.*

Proof. For the sake of economy, we only give proofs of cases outside the parenthesis for the three statements above and the cases between parenthesis can be made similarly.

(i) To prove the necessary condition, let H_E be a soft closed subset of $\widetilde{X}$ and consider f_ϕ is a soft Iα-open map. Then H_E^c is soft open and $f_\phi(H_E^c)$ is soft Iα-open. It follows from the bijectiveness of f_ϕ, that $f_\phi(H_E^c) = [f_\phi(H_E)]^c$. This automatically implies that $f_\phi(H_E)$ is soft Dα-closed. Thus f_ϕ is a soft Dα-closed map. In a similar manner, we can prove the sufficiency condition.

(ii) Necessity: Let G_E be a soft open subset of $\widetilde{X}$ and consider f_ϕ is a soft Iα-open map. Then $f_\phi(G_E)$ is soft Iα-open. It follows from the bijectiveness of f_ϕ, that $f_\phi(G_E) = (f_\phi^{-1})^{-1}(G_E)$. This automatically implies that $(f_\phi^{-1})^{-1}(G_E)$ is soft Iα-open. Thus f_ϕ^{-1} is a soft Iα-continuous map. In a similar manner, we can prove the sufficiency condition.

(iii) The proof of this statement comes immediately from (i) and (ii) above.

□

Theorem 3.9. *Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective map. Then a soft map $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) α-open if and only if a map $g : (X, \tau_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) α-open.*

Proof. Necessity: Let U be an open subset of $(X, \tau_e, \preceq_1)$ and $\phi(e) = f$. Then there exists a soft open subset G_E of $(X, \tau, E, \preceq_1)$ such that $G(e) = U$. Since g_ϕ is a soft I (resp. soft D, soft B) α-open map, then $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) α-open set. From Definition (1.11), it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta, F, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e\in\phi^{-1}(f)} g(G(e))$, for each $f \in F$. By hypothesis, $\theta^\star$ is an extended soft topology on Y, a subset $\bigcup_{e\in\phi^{-1}(f)} g(G(e)) = g(U)$ of $(Y, \theta_{\phi(e)}, \preceq_2)$ is I (resp. D, B) α-open. Hence a map g is I (resp. D, B) α-open.

Sufficiency: Let G_E be a soft open subset of $(X, \tau, E, \preceq_1)$. Then from Definition (1.11), it follows that a soft subset $g_\phi(G_E) = (g_\phi(G))_F$ of $(Y, \theta^\star, F, \preceq_2)$ is given by $g_\phi(G)(f) = \bigcup_{e\in\phi^{-1}(f)} g(G(e))$, for each $f \in F$. Since a map g is I (resp. D, B) α-open, then a subset $\bigcup_{e\in\phi^{-1}(f)} g(G(e))$ of $(Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) α-open. By hypothesis, $\theta^\star$ is an extended soft topology on Y, $g_\phi(G_E)$ is a soft I (resp. soft D, soft B) α-open subset of $(Y, \theta^\star, F, \preceq_2)$. Hence a soft map g_ϕ is soft I (resp. soft D, soft B) α-open. □

The result above is restated in case of a soft I (resp. soft D, soft B) α-closed map. One can prove them similarly and so the proof will be omitted.

Theorem 3.10. *Let $\theta^\star$ be an extended soft topology on Y and ϕ is an injective map. Then a soft map $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) α-closed if and only if a map $g : (X, \tau_e, \preceq_1) \to (Y, \theta^\star_{\phi(e)}, \preceq_2)$ is I (resp. D, B) α-closed.*

Proposition 3.11. *Consider τ is not the indiscrete topology on X. If an injective soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft Bα-open or soft Bα-closed, then $\preceq_2$ is not linearly ordered.*

Proposition 3.12. *Let $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ and $g_\lambda : (Y, \theta, F, \preceq_2) \to (Z, \upsilon, K, \preceq_3)$ be two soft maps. Then then following properties hold, for $x \in \{I, D, B\}$.*

(i) *If f_ϕ is a soft $x\alpha$-continuous map and g_λ is a soft continuous map, then $g_\lambda \circ f_\phi$ is a soft x-continuous map.*

(ii) *If f_ϕ is a soft open (resp. soft closed) map and g_λ is a soft $x\alpha$-open (resp. $x\alpha$-closed) map, then $g_\lambda \circ f_\phi$ is a soft x-open (resp. $x\alpha$-closed) map.*

(iii) *If $g_\lambda \circ f_\phi$ is a soft x-open map and f_ϕ is surjective soft continuous, then g_λ is a soft x-open map.*

(iv) *If $g_\lambda \circ f_\phi$ is a soft closed map and g_λ is an injective soft x-continuous map, then f_ϕ is a soft y-closed map, where $(x, y) \in \{(I, D), (D, I), (B, B)\}$.*

4. Soft $I(D,B)\alpha$-homeomorphism maps

The concepts of soft $I(D,B)\alpha$-homeomorphism maps are introduced and their main properties are discussed. Some examples are constructed to illustrate the relationships among the initiated soft maps.

Definition 4.1. *A bijective soft map $g_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is called soft I (resp. soft D, soft B) α-homeomorphism if it is soft Iα-continuous and soft Iα-open (resp. soft Dα-continuous and soft Dα-open, soft Bα-continuous and soft Bα-open).*

Remark 4.2. From Definition (4.1), we can note the following:

(i) Every soft I (soft D, soft B) α-homeomorphism map is soft α-homeomorphism.

(ii) Every soft Bα-homeomorphism map is soft Iα-homeomorphism or soft Dα-homeomorphism.

To elucidate that the two items of the remark above are not conversely, we give two examples below.

Example 4.3. Let $X = \{3, 5, 7, 11\}$ be a universe set and $A = \{a_1, a_2\}$ be a parameters set. Consider $\phi : A \to A$ and $f : X \to X$ are both identity maps. We define two partial order relations on X and Y, respectively, as $\preceq_1 = \triangle \bigcup \{(7, 5)\}$ and $\preceq_2 = \triangle \bigcup \{(3, 11)\}$ and we define two soft topologies τ and θ on X and Y, respectively, as $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F_A\}$ and $\theta = \{\widetilde{\emptyset}, \widetilde{Y}, F_A, H_A\}$, where $F_A = \{(a_1, \{3\}), (a_2, \{5, 7\})\}$ and $H_A = \{(a_1, \{3, 5\}), (a_2, \{5, 7\})\}$. Then one can readily check that a soft map $f_\phi : S(X_A) \to S(Y_B)$ is soft α-homeomorphism. On the other hand, $f_\phi(F_A) = F_A$ is not a soft Iα-open set and $f_\phi^{-1}(H_A) = H_A$ is not a soft Dα-open set. Hence f_ϕ is not soft I (soft D, soft B) α-homeomorphism.

Example 4.4. In Example above, if we replace only the partial order relation $\preceq_2$ by $\preceq = \triangle \bigcup \{(5, 11)\}$, then the soft map f_ϕ is soft D-homeomorphism, but is not soft B-homeomorphism. Also, if we replace only the partial order relation $\preceq_1$ by $\preceq = \triangle \bigcup \{(11, 3)\}$, then the soft map f_ϕ is soft I-homeomorphism, but is not soft B-homeomorphism.

Theorem 4.5. *If a bijective soft map $f_\phi : (X, \tau, E, \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft Iα-continuous (resp. soft Dα-continuous, soft Bα-continuous), Then the following three statements are equivalent:*

(i) f_ϕ is soft Iα-homeomorphism (resp. soft Dα-homeomorphism, soft Bα-homeomorphism);

(ii) f_ϕ^{-1} is soft Iα-continuous (resp. soft Dα-continuous, soft Bα-continuous);

(iii) f_ϕ is soft Dα-closed (resp. soft Iα-closed, soft Bα-closed).

Proof. $(i) \Rightarrow (ii)$ Since f_ϕ is a soft Iα-homeomorphism (resp. soft Dα-homeomorphism, soft Bα-homeomorphism) map, then f_ϕ is soft Iα-open (resp. soft Dα-open , soft Bα-open). It follows from item (ii) of Theorem (3.8), that f_ϕ^{-1} is soft Iα-continuous (resp. soft Dα-continuous, soft Bα-continuous).

$(ii) \Rightarrow (iii)$ The proof follows from item (iii) of Theorem (3.8).

$(iii) \Rightarrow (i)$ It sufficient to prove that f_ϕ is a soft Iα-open (resp. soft Dα-open, soft Bα-open) map. This follows from item (i) of Theorem (3.8). □

Theorem 4.6. *Let $\tau^\star$ and $\theta^\star$ be extended soft topologies on X and Y, respectively. Then a soft map $g_\phi : (X, \tau^\star, E, \preceq_1) \to (Y, \theta^\star, F, \preceq_2)$ is soft I (resp. soft D, soft B) α-homeomorphism if and only if a map $g : (X, \tau_e^\star, \preceq_1) \to (Y, \theta_{\phi(e)}^\star, \preceq_2)$ is I (resp. D, B) α-homeomorphism.*

Proof. The proof is obtained immediately from Theorem (2.11) and Theorem (3.9) □

Proposition 4.7. *Let the two soft topologies τ and θ on X and Y, respectively, do not belong to {soft discrete topology, soft indiscrete topology}. If a soft map $f_\phi : (X, \tau, E \preceq_1) \to (Y, \theta, F, \preceq_2)$ is soft Bα-homeomorphism, then $\preceq_1$ and $\preceq_2$ is not linearly ordered.*

Conclusion

Al-shami et al. [14] combined a soft partially ordered set with a soft topological space to constitute a soft topological ordered space concept. As a contribution of this, we have established the concepts of soft $x\alpha$-continuous, soft $x\alpha$-open, soft $x\alpha$-closed and soft $x\alpha$-homeomorphism maps, for $x \in \{I, D, B\}$. We have completely described these concepts and have deduced some results which connect the initiated soft maps with those maps via topological ordered spaces. To some extent there is a similarity between the results obtained herein and those presented in [15]. The newly ordered maps initiated herein will be an important basis for the further developments on soft topological ordered spaces. In our upcoming research, we intend to give more soft ordered topological concepts and investigate their properties.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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