Fuzzy congestion in data envelopment analysis

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Abstract. Congestion is an important aspect of economic concepts and data envelopment analysis (DEA) such that in recent years, special attention has been for mathematicians. Several models and methods have been proposed for measuring congestion in non-deterministic environment which cannot show inexact congestion. More important, the congestion is less been investigated with imprecise data which a comprehensive model has not been presented yet. In this paper, our aim is to introduce extension of measuring congestion in DEA with common set of weights and fuzzy inputs and outputs such that, comparisons in models are fuzzy. Finally the proposed models for fuzzy congestion will be shown by examples.

Keywords: data envelopment analysis, fuzzy congestion, common weights.

1. Introduction

Congestion is said to occur when an increase in one or more inputs can be associated with a decrease in one or more outputs, without improving any other inputs or outputs [2]. First, the research on congestion began by Fare and Svensson [5]. Then, it was completed in 1983 and 1985 by the Fare and Grosskopf [6, 7]. They presented a model according to the concept of data envelopment analysis. Another approach was presented by Cooper et al. [3]. Brockett et al. [1] and Cooper et al. [2] developed a new DEA-based approach to measure input congestion. While a significant literature exists on the subject, the two latter methodologies are considered to be fundamental congestion consideration. Another notable method for measuring congestion is Noura et al. [13]. First

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in this paper, we introduce a new methodology for congestion by common set of weights [11, 16] then the congestion of DEA model will be considered with input and output fuzzy number in which fuzzy congestion is proposed by using fuzzy definitions and fuzzy comparisons.

The paper is arranged as follows: in section 2, measuring congestion with common set of weights (CSW) is introduced. In section 3, the background of fuzzy numbers is presented. Congestion with fuzzy numbers is contributed in section 4. Numerical example and conclusion are drawn in section 5 and 6 respectively.

2. Measuring congestion with CSW

Based on Noura et al. methodology [14], we proposed the following multi objective linear programming (MOLP) with common set of weights (CSW).

$$\begin{aligned} Min \quad & v^t x_j + v_0 - u^t y_j, \qquad j = 1, \dots, n \\ s.t. \quad & v^t x_j + v_0 - u^t y_j \geq 0, \qquad j = 1, \dots, n \\ & u^t \geq 1_s \epsilon, \qquad v^t \geq 1_m \epsilon. \end{aligned} \tag{1}$$

Equal weights method is applied to solve the above MOLP as follows:

$$\begin{aligned} Min \quad & \sum_{j=1}^{n} (v^t x_j + v_0 - u^t y_j) \\ s.t. \quad & v^t x_j + v_0 - u^t y_j \geq 0, \qquad j = 1, \dots, n \\ & u^t \geq 1_s \epsilon, \qquad v^t \geq 1_m \epsilon. \end{aligned} \tag{2}$$

$\Rightarrow$

$$\begin{aligned} Min \quad & \sum_{j=1}^{n} (v^t x_j + v_0 - u^t y_j) \\ s.t. \quad & v^t x_j + v_0 - u^t y_j - \Delta_j = 0, \qquad j = 1, \dots, n \\ & u^t \geq 1_s \epsilon, \qquad v^t \geq 1_m \epsilon \\ & \Delta_j \geq 0, \qquad j = 1, \dots, n. \end{aligned} \tag{3}$$

From $v^t x_j + v_0 - u^t y_j - \Delta_j = 0$ we get $v^t x_j + v_0 - u^t y_j = \Delta_j$ so

$$\begin{aligned} Min \quad & \sum_{j=1}^{n} \Delta_j \\ s.t. \quad & v^t x_j + v_0 - u^t y_j - \Delta_j = 0, \qquad j = 1, \dots, n \\ & u^t \geq 1_s \epsilon, \qquad v^t \geq 1_m \epsilon \\ & \Delta_j \geq 0, \qquad j = 1, \dots, n. \end{aligned} \tag{4}$$

Above model is similar to Liu Peng's model [12] and Noura Hoseini's model [15]. Now we suppose $(u^*, v^*)^t$ will be the optimal solution of problem (4), which is called common set of weights (CSW) for ranking and comparing DMUs. According to achieved CSW, the efficiency score of $DMU_j (j = 1, \dots, n)$ will be $\varphi_j^* = \frac{v^{*t} x_j + v_0^*}{u^{*t} y_j}$. If φ_j^* is equal to one then the DMU under evaluation is efficient.

Now as Noura et al. [13], the efficient set of DMUs (E) is defined as follows:

$$E = \{j : \varphi_j^* = 1\}. \tag{5}$$

The highest value in each input among DMUs of E for all components is introduced with x_i^*.

$$x_i^* = \max\{x_{ij} : j \in E\}, \quad i = 1, \dots, m. \tag{6}$$

So the following revised definition is suggested to identifying congestion.

Definition 2.1. *Congestion in DMUo eventually occurs if for the optimal solution of DMUo (φ_o^*), the following condition is satisfied:*
$\varphi_o^* > 1$, and there is at least one $x_{io} > x_i^*, i = 1, \dots, m$.

The amount of congestion in the ith input of DMUo is shown with s_{io}^c as follow:

$$s_{io}^c = x_{io} - x_i^*. \tag{7}$$

The sum of all s_{io}^c is the amount of congestion in DMUo.

$$s_o^c = \sum_{i=1}^{m} s_{io}^c. \tag{8}$$

Congestion does not present in DMUo when $x_{io} \leq x_i^*$ or $s_{io}^c = 0$ for all $i = 1, \dots, m$.

3. Fuzzy background

In this section some definition have been given for better understanding of contains of the next section. Several definitions of LR-fuzzy numbers have been published. All of them are variations on the original definition by Dubios and Parade [4]. In this paper, we will use the following definition.

LR-fuzzy number: A fuzzy number ($\tilde{A}$) is called LR-fuzzy number if and only if it has the following membership function.

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m, \\ R\left(\frac{x-m}{\beta}\right), & x > m. \end{cases} \tag{9}$$

Where L and R are non-increasing functions from R^+ to [0,1] such that L(0)=R(0)=1 and it is shown to $(m, \alpha, \beta)_{LR}$.

Triangular fuzzy number. LR-fuzzy number is called triangular fuzzy number if and only if $L(x) = R(x) = 1 - x$.

Addition and subtraction of triangular fuzzy numbers. Assume $\tilde{A}_1 = (m_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, \alpha_2, \beta_2)$ then

$$\begin{aligned} \tilde{A}_1 + \tilde{A}_2 &= (m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2), \\ \tilde{A}_1 - \tilde{A}_2 &= (m_1 - m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2). \end{aligned} \tag{10}$$

In what follow, we give some results for triangular fuzzy numbers that belong to the same family and have different shapes.

Comparison of fuzzy numbers by dubois and paraden [4]. Let $\tilde{A}_1$ and $\tilde{A}_2$ be two fuzzy numbers. Then, $\tilde{A}_1 > \tilde{A}_2 \Leftrightarrow \tilde{A}_1 \cup \tilde{A}_2 = \tilde{A}_1$, where $\tilde{A}_1 \cup \tilde{A}_2$ represents the maximum of those fuzzy numbers.

Comparison of fuzzy numbers by ramik and rimanek [17]. Let $\tilde{A}_1$ and $\tilde{A}_2$ be two triangular fuzzy numbers. Then

$$\tilde{A}_1 \leq \tilde{A}_2 \Leftrightarrow m_1 \leq m_2, m_1 - \alpha_1 \leq m_2 - \alpha_2, m_1 + \beta_1 \leq m_2 + \beta_2. \tag{11}$$

Proposition ([17]). Let $\tilde{A}_1$ and $\tilde{A}_2$ be two fuzzy numbers. Then, $\tilde{A}_1 \cup \tilde{A}_2 = \tilde{A}_1$ if and only if $inf\{s : \mu_{\tilde{A}_1}(s) \geq w\} \geq inf\{t : \mu_{\tilde{A}_2}(t) \geq w\}$ and $sup\{s : \mu_{\tilde{A}_1}(s) \geq w\} \geq sup\{t : \mu_{\tilde{A}_2}(t) \geq w\}$ hold for all grades of membership w in [0, 1].

Comparison of fuzzy numbers by tanaka et al. [8] : Let $\tilde{A}_1$ and $\tilde{A}_2$ be two fuzzy numbers and h a real number, h∈[0, 1], Then $\tilde{A}_1 \leq^h \tilde{A}_2$ if and only if $infs : \mu_{\tilde{A}_1}(s) \geq w \leq inf\{t : \mu_{\tilde{A}_2}(t) \geq w\}$ and $sup\{s : \mu_{\tilde{A}_1}(s) \geq w\} \leq sup\{t : \mu_{\tilde{A}_2}(t) \geq w\}$. Hence, for two LR-fuzzy numbers $\tilde{A}_1 = (m_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, \alpha_2, \beta_2)$ at a given possibility level h; $\tilde{A}_1 \leq^h \tilde{A}_2$, then it's necessary to check $m_1 - \alpha_1 L^{-1}(w) \leq m_2 - \alpha_2 L^{-1}(w)$ and $m_1 + \beta_1 R^{-1}(w) \leq m_2 + \beta_2 R^{-1}(w), w \in [h, 1]$.

4. Fuzzy congestion

In this section, we assume that inputs and outputs are triangular fuzzy number in model 2. So suppose $\tilde{x}_{ij} = (\alpha^L_{ij}, x_{ij}, \alpha^U_{ij}), \tilde{y}_{rj} = (\beta^L_{rj}, y_{rj}, \beta^U_{rj})$ in the possibility level of h; $0 \leq$ h ≤ 1 therefore

$$\begin{aligned} Min \quad & \sum_{j=1}^{n} (v^t \tilde{x}_j + v_0 - u^t \tilde{y}_j), \\ s.t. \quad & v^t \tilde{x}_j + v_0 - u^t \tilde{y}_j \geq 0, \qquad j = 1, \ldots, n \\ & u^t \geq 1_s \epsilon, \qquad v^t \geq 1_m \epsilon. \end{aligned} \tag{12}$$

According to the Guo et al. [8, 9, 10] the following model obtains.

$$
\begin{aligned}
& Min \quad \sum_{j=1}^{n}(v^t x_j - (1-h)v^t\alpha_j^L + v_0 - u^t y_j + (1-h)u^t\beta_j^L), \\
(13) \qquad & s.t. \quad v^t x_j - (1-h)v^t\alpha_j^L + v_0 - u^t y_j + (1-h)u^t\beta_j^L \geq 0, j = 1,\ldots,n \\
& \qquad v^t x_j + (1-h)v^t\alpha_j^U + v_0 - u^t y_j - (1-h)u^t\beta_j^U \geq 0, j = 1,\ldots,n \\
& \qquad u^t \geq 1_s\epsilon, \qquad v^t \geq 1_m\epsilon.
\end{aligned}
$$

The fuzzy efficiency of an evaluated DMU_j with the triangular fuzzy input vector $\tilde{x}_{ij} = (\alpha_{ij}^L, x_{ij}, \alpha_{ij}^U)$ and output vector $\tilde{y}_{rj} = (\beta_{rj}^L, y_{rj}, \beta_{rj}^U)$ is defined a triangular fuzzy number $\tilde{e}_j = (L_j, \theta_j, R_j)$ as follows:

$$(14) \qquad \phi_j = \frac{v^{*t}x_j + v_0^*}{u^{*t}y_j} \quad \theta_j = \frac{1}{\phi_j},$$

$$(15) \qquad L_j = \theta_j - \frac{u^{*t}y_j - (1-h)u^{*t}\beta_j^L}{v^{*t}x_j + (1-h)v^{*t}\alpha_j^U + v_0^*},$$

$$(16) \qquad R_j = \frac{u^{*t}y_j + (1-h)u^{*t}\beta_j^U}{v^{*t}x_j - (1-h)v^{*t}\alpha_j^L + v_0^*} - \theta_j,$$

where v^* and u^* are obtained coefficient vectors from (13). The DMU_j with $\theta_j + R_j > 1$ for the h possibility level is called an h-possibilistic efficient DMU. On the contrary, the DMU with $\theta_j + R_j \leq 1$ for the h possibility level is called an h-possibilistic inefficient DMU.

Now, the fuzzy efficient set (E) is defined as follows:

$$(17) \qquad E = \{j : \theta_j + R_j > 1\}.$$

Also, we obtain the highest value in each input for all components among the DMUs in set E based on fuzzy comparison as $\tilde{x}_i^*$.

Moreover, in the crisp mode Congestion occurs when an increase in one or more inputs can be associated with a decrease in one or more outputs, without improving any other inputs or outputs. So, with extended this definition of congestion in the fuzzy mode for DMUo, we have:

$$
(18) \qquad \begin{aligned}
&\text{If} \quad \tilde{X}_o \to \tilde{X}_o + \tilde{\Delta X}; \tilde{\Delta X} \geq 0, \tilde{\Delta X} \neq 0, \\
&\text{Then} \quad \tilde{Y}_o \to \tilde{Y}_o - \tilde{\Delta Y}; \tilde{\Delta Y} \geq 0, \tilde{\Delta Y} \neq 0.
\end{aligned}
$$

or

$$
(19) \qquad \begin{aligned}
&\text{If} \quad \tilde{X}_o \to \tilde{X}_o - \tilde{\Delta X}; \tilde{\Delta X} \geq 0, \tilde{\Delta X} \neq 0, \\
&\text{Then} \quad \tilde{Y}_o \to \tilde{Y}_o + \tilde{\Delta Y}; \tilde{\Delta Y} \geq 0, \tilde{\Delta Y} \neq 0.
\end{aligned}
$$

Therefore, with respect to fuzzy increasing and fuzzy decreasing (based on fuzzy comparison), definition of fuzzy congestion is exactly suggested as follows.

Definition 4.1. *Fuzzy congestion eventually occurs if for every optimal solution of DMUj, the following condition is satisfied:* $\theta_j + R_j \leq 1$*, and there is at least one* $\tilde{x}_{ij} > \tilde{x}_i^*, i = 1, \ldots, m$.

The amount of fuzzy congestion in the ith input of DMUj is shown by $\tilde{S}_{ij}^c$ as follow:

$$\tilde{S}_{ij}^c = \tilde{x}_{ij} - \tilde{x}_i^*. \tag{20}$$

The sum of all $\tilde{S}_{ij}^c$ is the amount of fuzzy congestion in DMUj.

$$\tilde{S}_j^c = \sum_{i=1}^{m} \tilde{S}_{ij}^c. \tag{21}$$

Fuzzy congestion does not present when $\tilde{x}_{ij} \leq \tilde{x}_i^*$ or $\tilde{S}_{ij}^c = 0$ for all $i = 1, \ldots, m$.

5. Numerical examples

Example 5.1. Table 1 shows five DMUs where input and output are symmetrical triangular fuzzy numbers. The fuzzy efficiencies of DMUs were obtained by model 13 for the different h values (table 2).

With considering table 2, we have $h = 0$, $E = \{DMU_1, DMU_2, DMU_3, DMU_4\}$, $\forall DMU \in E : \tilde{x}_1^* = \max Input1 = (1, 5, 1)$. For $DMU_5, \tilde{x}_{15} = (0.5, 5, 0.5) \leq \tilde{x}_1^*$ so there is no congestion.

$h = 0.25$, $E = \{DMU_1, DMU_2, DMU_4\}$, $\forall DMU \in E : \tilde{x}_1^* = \max Input1 = (1, 5, 1)$. For DMU_3, $\tilde{x}_{13} = (0.6, 3, 0.6) \leq \tilde{x}_1^*$ so there is no congestion. For DMU_5, $\tilde{x}_{15} = (0.5, 5, 0.5) \leq \tilde{x}_1^*$ so there is no congestion.

$h = 0.5, 0.75, 1, E = \{DMU_2\}, \forall DMU \in E : \tilde{x}_1^* = \max Input1 = (0.5, 3, 0.5)$.

For $DMU_1, \tilde{x}_{11} = (0.5, 2, 0.5) \leq \tilde{x}_1^*$ so there is no congestion.

For $DMU_3, \tilde{x}_{13} = (0.6, 3, 0.6) \leq \tilde{x}_1^*$ so there is no congestion.

For DMU_4, $\tilde{x}_{14} = (1, 5, 1) > \tilde{x}_1^* = (0.5, 3, 0.5)$, so DMU_4 has congestion and we have, $\tilde{S}_{14}^c = (0.5, 5, 0.5) - (0.5, 3, 0.5) = (1.5, 2, 1.5) = \tilde{S}_4^c$.

For DMU_5, $\tilde{x}_{15} = (0.5, 5, 0.5) > \tilde{x}_1^* = (0.5, 3, 0.5)$, so DMU_5 has congestion and we have, $\tilde{S}_{15}^c = (1, 5, 1) - (0.5, 3, 0.5) = (1, 2, 1) = \tilde{S}_5^c$.

Example 5.2. Table 3 shows eight DMUs where inputs and output are non-symmetrical triangular fuzzy numbers. The fuzzy efficiencies of DMUs were obtained by model 13 for the different h values which have listed in table 4.

With considering table 4, we have

For $h = 0, 0.25, 0.5, 0.75$, $E = \{DMU_3, DMU_4, DMU_8, \forall\ DMU \in E : \tilde{x}_1^* =$ max Input $1 = (3, 6, 1)$.

Table 1: DMUs with single fuzzy input and single fuzzy output. Source: [10]

DMUs	Input s	Outputs
DMU1	(0.5,2.0,0.5)	(0.3,1.0,0.3)
DMU2	(0.5,3.0,0.5)	(0.7,3.0,0.7)
DMU3	(0.6,3.0,0.6)	(0.6,2.0,0.4)
DMU4	(1.0,5.0,1.0)	(1.0,4.0,1.0)
DMU5	(0.5,5.0,0.5)	(0.2,2.0,0.2)

Table 2: Fuzzy efficiency ($\tilde{e}_j = (L_j, \theta_j, R_j)$) of DMUs with different h values for example 1

h	DMU1	DMU2	DMU3	DMU4	DMU5
0	(0.33, 0.63, 0.81)	(0.38, 1, 0.61)	(0.25, 0.67, 0.44)	(0.27, 0.69, 0.45)	(0.06, 0.34, 0.09)
0.25	(0.27, 0.63, 0.51)	(0.30, 1, 0.42)	(0.20, 0.67, 0.30)	(0.22, 0.69, 0.31)	(0.05, 0.34, 0.07)
0.5	(0.12, 0.48, 0.14)	(0.18, 0.97, 0.21)	(0.12, 0.65, 0.14)	(0.16, 0.78, 0.20)	(0.03, 0.39, 0.04)
0.75	(0.06, 0.49, 0.07)	(0.09, 0.98, 0.11)	(0.07, 0.66, 0.06)	(0.08, 0.79, 0.10)	(0.02, 0.40, 0.02)
1	(0.00, 0.50, 0.00)	(0.00, 1.00, 0.00)	(0.00, 0.67, 0.00)	(0.00, 0.80, 0.00)	(0.00, 0.40, 0.00)

For DMU_1, $\tilde{x}_{11} = (0.5, 0.5, 0.5) \leq \tilde{x}_1^*$, so there is no congestion in Input 1.

For DMU_2, $\tilde{x}_{12} = (2, 4, 2) \leq \tilde{x}_1^*$, so there is no congestion in Input 1.

For DMU_5, $\tilde{x}_{15} = (4, 8, 4) > \tilde{x}_1^* = (3, 6, 1)$, so DMU_5 has congestion in Input 1 and we have $\tilde{S}_{15}^c = (4, 8, 4) - (3, 6, 1) = (5, 2, 7)$.

For DMU_6, $\tilde{x}_{16} = (3, 6, 1) \leq \tilde{x}_1^*$, so there is no congestion in Input 1.

For DMU_7, $\tilde{x}_{17} = (4, 7, 2) > \tilde{x}_1^* = (3, 6, 1)$, so DMU_7 has congestion in Input 1 and we have $\tilde{S}_{17}^c = (4, 7, 2) - (3, 6, 1) = (5, 1, 5)$, $\forall\, DMU \in E : \tilde{x}_2^* = \max$ Input $2 = (3, 6, 1)$.

For DMU_1, $\tilde{x}_{21} = (2, 4, 2) \leq \tilde{x}_2^*$, so there is no congestion in Input 2.

For DMU_2, $\tilde{x}_{22} = (0.5, 0.5, 0.5) \leq \tilde{x}_2^*$, so there is no congestion in Input 2.

For DMU_5, $\tilde{x}_{25} = (3, 6, 1) \leq \tilde{x}_2^*$, so there is no congestion in Input 2.

For DMU_6, $\tilde{x}_{26} = (4, 8, 4) > \tilde{x}_2^*$, so DMU_6 has congestion in Input 2 and we have $\tilde{S}_{26}^c = (4, 8, 4) - (3, 6, 1) = (5, 2, 7)$.

For DMU_7, $\tilde{x}_{27} = (4, 7, 2) > \tilde{x}_2^*$, so DMU_7 has congestion in Input 2 and we have $\tilde{S}_{27}^c = (4, 7, 2) - (3, 6, 1) = (5, 1, 5)$

$\Rightarrow \tilde{S}_5^c = (5, 2, 7), \tilde{S}_6^c = (5, 2, 7), \tilde{S}_7^c = (5, 1, 5) + (5, 1, 5) = (10, 2, 10)$.

For $h = 1, E = \{DMU_3, DMU_8\}$, $\forall DMU \in E : \tilde{x}_1^* = \max$ Input $1 = (2, 4, 2)$.

For DMU_1, $\tilde{x}_{11} = (0.5, 0.5, 0.5) \leq \tilde{x}_1^*$, so there is no congestion in Input 1.

For DMU_2, $\tilde{x}_{12} = (2, 4, 2) \leq \tilde{x}_1^*$, so there is no congestion in Input 1.

For DMU_4, $\tilde{x}_{14} = (3, 6, 1) > \tilde{x}_1^*$, so DMU_4 has congestion in Input 1 and we have $\tilde{S}_{14}^c = (3, 6, 1) - (2, 4, 2) = (3, 2, 5)$.

For DMU_5, $\tilde{x}_{15} = (4, 8, 4) > \tilde{x}_1^*$, so DMU_5 has congestion in Input 1 and we have $\tilde{S}_{15}^c = (4, 8, 4) - (2, 4, 2) = (6, 4, 6)$.

Table 3: DMUs with fuzzy inputs and single fuzzy output. Source: [18]

DMUs	Input 1	Input 2	Output
DMU1	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	(2, 4, 2)	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
DMU2	(2, 4, 2)	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
DMU3	(2, 4, 2)	(2, 4, 2)	(3, 5, 5)
DMU4	(3, 6, 1)	(3, 6, 1)	(4, 7, 2)
DMU5	(4, 8, 4)	(3, 6, 1)	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
DMU6	(3, 6, 1)	(4, 8, 4)	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
DMU7	(4, 7, 2)	(4, 7, 2)	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$
DMU8	(1, 1, 1)	(1, 1, 1)	$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

Table 4: Fuzzy efficiency ($\tilde{e}_j = (L_j, \theta_j, R_j)$) of DMUs with different h values for example 2

DMUs	h=0	h=0.25	h=0.5	h=0.75	h=1
DMU1	(0.13,0.13,0.47)	(0.12,0.15,0.33)	(0.10,0.17,0.21)	(0.07,0.19,0.11)	(0.00,0.22,0.00)
DMU2	(0.13,0.13,0.47)	(0.12,0.15,0.33)	(0.10,0.17,0.21)	(0.07,0.19,0.11)	(0.00,0.22,0.00)
DMU3	(0.55,0.75,2.25)	(0.48,0.78,1.51)	(0.39,0.85,0.92)	(0.24,0.92,0.43)	(0.00,1.00,0.00)
DMU4	(0.44,0.70,1.10)	(0.36,0.73,0.72)	(0.26,0.77,0.43)	(0.15,0.81,0.20)	(0.00,0.86,0.00)
DMU5	(0.04,0.04,0.13)	(0.36,0.04,0.08)	(0.03,0.05,0.05)	(0.02,0.05,0.02)	(0.00,0.05,0.00)
DMU6	(0.04,0.04,0.13)	(0.36,0.04,0.08)	(0.03,0.05,0.05)	(0.02,0.05,0.02)	(0.00,0.05,0.00)
DMU7	(0.04,0.04,0.16)	(0.36,0.04,0.10)	(0.03,0.05,0.05)	(0.02,0.05,0.02)	(0.00,0.05,0.00)
DMU8	(0.15,0.15,∞)	(0.17,0.19,6.81)	(0.19,0.26,2.74)	(0.23,0.42,1.24)	(0.00,1.00,0.00)

For DMU_6, $\tilde{x}_{16} = (3,6,1) > \tilde{x}_1^*$, so DMU_6 has congestion in Input 1 and we have $\tilde{S}_{16}^c = (3,6,1) - (2,4,2) = (3,2,5)$.

For DMU_7, $\tilde{x}_{17} = (4,7,2) > \tilde{x}_1^*$, so DMU_7 has congestion in Input 1 and we have $\tilde{S}_{17}^c = (4,7,2) - (2,4,2) = (4,3,6)$, $\forall\ DMU \in E : \tilde{x}_2^* =$ max Input $2 = (2,4,2)$.

For DMU_1, $\tilde{x}_{21} = (2,4,2) \leq \tilde{x}_2^*$, so there is no congestion in Input 2.

For DMU_2, $\tilde{x}_{22} = (0.5,0.5,0.5) \leq \tilde{x}_2^*$, so there is no congestion in Input 2.

For DMU_4, $\tilde{x}_{24} = (3,6,1) > \tilde{x}_2^*$, so DMU_4 has congestion in Input 2 and we have $\tilde{S}_{24}^c = (3,6,1) - (2,4,2) = (3,2,5)$.

For DMU_5, $\tilde{x}_{25} = (3,6,1) > \tilde{x}_2^*$, so DMU_5 has congestion in Input 2 and we have $\tilde{S}_{25}^c = (3,6,1) - (2,4,2) = (3,2,5)$.

For DMU_6, $\tilde{x}_{26} = (4,8,4) > \tilde{x}_2^*$, so DMU_6 has congestion in Input 2 and we have $\tilde{S}_{26}^c = (4,8,4) - (2,4,2) = (6,4,6)$.

For DMU_7, $\tilde{x}_{27} = (4,7,2) > \tilde{x}_2^*$, so DMU_7 has congestion in Input 2 and we have $\tilde{S}_{27}^c = (4,7,2) - (2,4,2) = (4,3,6)$.

$\Rightarrow \tilde{S}_4^c = (3,2,5)+(3,2,5) = (6,4,10, \tilde{S}_5^c = (6,4,6)+(3,2,5) = (9,6,11), \tilde{S}_6^c = (3,2,5) + (6,4,6) = (9,6,11), \tilde{S}_7^c = (4,3,6) + (4,3,6) = (8,6,12)$.

Table 5: Results of example2 by Rostami et al.'s method [18]

DMUs	h'_{BCC}	Congestion
DMU1	1	No
DMU2	1	No
DMU3	1	No
DMU4	1	No
DMU5	10	Yes
DMU6	10	Yes
DMU7	10	Yes
DMU8	1	No

Table 6: Results of example2 by proposed method

DMUs	Measure of fuzzy congestion($\tilde{S}^c$	Congestion
DMU1	0	No
DMU2	0	No
DMU3	0	No
DMU4	0	No
DMU5	(5,2,7)	Yes
DMU6	(5,2,7)	Yes
DMU7	(10,2,10)	Yes
DMU8	1	No

Results of measuring congestion by proposed method and Rostami et al.'s (2011) method are shown in tables 5 and 6. As can be observed, two methods get similar results for identifying congestion but Rostami et al.'s (2011) method don't determine measure of congestion while the proposed method is suggested measure of congestion by a fuzzy number.

6. Conclusion

In this paper, we proposed a new method for calculating fuzzy congestion with applying fuzzy rectangular inputs and outputs based on Noura et al.'s method and Common weights. The main advantage of the proposed method is to solve one linear programming in order to measuring fuzzy congestion for all DMUs. Therefore, calculation of this method is very low and it provides an easy understandable knowledge of fuzzy congestion. Moreover, congestion is less been investigated with imprecise data while the proposed method suggests measure of congestion by a fuzzy number. This important was shown by examples and compared with other methods.

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