Some fixed soft point theorems on a new soft topology related to a self soft mapping

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Abstract. In the present paper, we introduce a new soft topology related to a self soft mapping and study its some basic properties. Also, we give the notion of a soft orbit and support it with examples. Moreover, we present the notion of a soft b-metric space, which is a soft version of b-metric space of Czerwik [7]. Next, by using these notions, we establish some fixed soft point theorems, which are the main results of our paper.

Keywords: soft set, fixed soft point, soft orbit, soft b-metric space.

1. Introduction

In 1999, Molodtsov [22] initiated the concept of a soft set theory as a new approach for coping with uncertainties and also presented the basic results of the new theory. In [22], Molodtsov successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration and theory of measurement. After presentation of the operations of soft sets [20], the properties and applications of this theory have been studied increasingly [2, 23, 27].

Maji et al. [21] gave the first practical application of soft sets in decision making problems. Pei and Miao [27] showed that soft sets are a class of special information systems. Ali et al. [2] presented some new algebraic operations on soft sets. Das and Samanta [9] introduced the notions of soft real sets and soft real numbers and studied their properties. Shabir and Naz [31] initiated the study of soft topological spaces. Recently, many papers concerning the soft set theory have been published [5, 10, 11, 15, 18, 19, 24, 25, 29, 34].

Fixed point theory plays a fundamental role in mathematics and applied sciences, such as optimization, mathematical models and economic theories. Also, this theory have been applied to show the existence and uniqueness of the solutions of diferential equations, integral equations and many other branches of mathematics [13, 26, 28]. A basic result in fixed point theory is the Banach contraction principle. Since the appearance of this principle, there has been a lot of activity in this area. Czerwik [7, 8] introduced the notion of a b-metric space and generalized the Banach contraction principle in the context of complete b-metric spaces. After that, some authors such as Boriceanu [6], Khamsi [17], Aydi [4], Roshan et al. [30], Arab [3] and Kamran et al. [16] studied the fixed point theory for single-valued or multivalued operators in b-metric spaces.

Extensions of fixed point theorems to the soft set theory have been studied by some authors. Wardowski [32] defined a new notion of soft element of a soft set and established some fixed point results. Later, Abbas et al. [1] studied soft metric versions of several important fixed point theorems for metric spaces. Afterwards, many authors obtained various existence theorems of fixed points using this theory [12, 14, 33].

In this work, we define a new soft topology with the help of a self soft mapping and investigate its related properties. Also, we prove some fixed soft point theorems on this soft topology. Then, we introduce the notion of a soft b-metric space and study its relation with a soft metric space. Finally, we give the concept of a soft orbit and obtain a fixed soft point theorem using this concept.

2. Preliminaries

In this section, we recollect some basic notions regarding soft sets. Throughout this work, let X be an initial universe, $P(X)$ be the power set of X and E be a set of parameters for X.

Definition 2.1 ([22]). *A soft set F on the universe X with the set of parameters E is defined by the set of ordered pairs*

$$F = \{(e, F(e)) : e \in E, F(e) \in P(X)\}$$

where F is a mapping given by $F : E \to P(X)$.

Throughout this paper, the family of all soft sets over X is denoted by $S(X, E)$ [5].

Definition 2.2 ([2, 20, 27]). *Let $F, G \in S(X, E)$. Then:*

(i) *The soft set F is called a null soft set, denoted by $\widetilde{\emptyset}$, if $F(e) = \emptyset$ for every $e \in E$.*

(ii) *The soft set F is called an absolute soft set, denoted by $\widetilde{X}$, if $F(e) = X$ for every $e \in E$.*

(iii) *F is a soft subset of G if $F(e) \subseteq G(e)$ for every $e \in E$. It is denoted by $F \sqsubseteq G$.*

(iv) *The complement of F is denoted by F^c, where $F^c : E \to P(X)$ is a mapping defined by $F^c(e) = X - F(e)$ for every $e \in E$. Clearly, $(F^c)^c = F$.*

(v) *The union of F and G is a soft set H defined by $H(e) = F(e) \cup G(e)$ for every $e \in E$. H is denoted by $F \sqcup G$.*

(vi) The intersection of F and G is a soft set H defined by $H(e) = F(e) \cap G(e)$ for every $e \in E$. H is denoted by $F \sqcap G$.

Definition 2.3 ([10, 19, 24]). *A soft set P over X is said to be a soft point if there exists an $e \in E$ such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \emptyset$ for every $e' \in E \backslash \{e\}$. This soft point is denoted as x^e.*

A soft point x^e is said to belongs to a soft set F, denoted by $x^e \widetilde{\in} F$, if $x \in F(e)$.

From now on, let $SP(X)$ be the family of all soft points over X.

Definition 2.4 ([18]). *Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings. Then, the mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi : S(X,E) \to S(Y,K)$.*

(i) *Let $F \in S(X,E)$. Then, $\varphi_\psi(F)$ is the soft set over Y defined as follows:*

$$\varphi_\psi(F)(k) = \begin{cases} \bigcup_{e \in \psi^{-1}(k)} \varphi(F(e)), & \text{if } \psi^{-1}(k) \neq \emptyset; \\ \emptyset, & \text{otherwise.} \end{cases},$$

for all $k \in K$. $\varphi_\psi(F)$ is called a soft image of a soft set F.

(ii) *Let $G \in S(Y,K)$. Then, $\varphi_\psi^{-1}(G)$ is the soft set over X defined as follows:*

$$\varphi_\psi^{-1}(G)(e) = \varphi^{-1}(G(\psi(e)))$$

for all $e \in E$. $\varphi_\psi^{-1}(G)$ is called a soft inverse image of a soft set G.

Theorem 2.5 ([18]). *Let $F, F_i \in S(X,E)$ and $G, G_i \in S(Y,K)$ for all $i \in J$, where J is an index set. Then, for a soft mapping $\varphi_\psi : S(X,E) \to S(Y,K)$, the following conditions are satisfied.*

(1) *If $F_1 \sqsubseteq F_2$, then $\varphi_\psi(F_1) \sqsubseteq \varphi_\psi(F_2)$.*

(2) *If $G_1 \sqsubseteq G_2$, then $\varphi_\psi^{-1}(G_1) \sqsubseteq \varphi_\psi^{-1}(G_2)$.*

(3) *$F \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(F))$ and $\varphi_\psi(\varphi_\psi^{-1}(G)) \sqsubseteq G$.*

(4) *$\varphi_\psi\big(\bigsqcup_{i\in J} F_i\big) = \bigsqcup_{i\in J} \varphi_\psi(F_i)$ and $\varphi_\psi\big(\bigsqcap_{i\in J} F_i\big) \sqsubseteq \bigsqcap_{i\in J} \varphi_\psi(F_i)$.*

(5) *$\varphi_\psi^{-1}\big(\bigsqcup_{i\in J} G_i\big) = \bigsqcup_{i\in J} \varphi_\psi^{-1}(G_i)$ and $\varphi_\psi^{-1}\big(\bigsqcap_{i\in J} G_i\big) = \bigsqcap_{i\in J} \varphi_\psi^{-1}(G_i)$.*

Proposition 2.6 ([25]). *Let $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping and $x^e \in SP(X)$. Then, we have $\varphi_\psi(x^e) = \varphi(x)^{\psi(e)} \in SP(Y)$.*

Definition 2.7 ([31]). *Let τ be a collection of soft sets over X. Then, τ is said to be a soft topology on X if*

(st_1) *$\widetilde{\emptyset}, \widetilde{X}$ belong to τ.*

(st_2) *the union of any number of soft sets in τ belongs to τ.*

(st_3) *the intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) *is called a soft topological space. The members of* τ *are called soft open sets in* X. *A soft set* F *over* X *is called a soft closed in* X *if* $F^c \in \tau$.

Definition 2.8 ([31]). *Let* (X, τ, E) *be a soft topological space and* $F \in S(X, E)$. *The soft closure of* F *is the soft set* $\overline{F} = \bigsqcap \{G : G \text{ is soft closed set and } F \sqsubseteq G\}$.

Definition 2.9 ([24]). *A collection* $\mathcal{B}$ *of soft open sets of a soft topological space* (X, τ, E) *is called a soft base for* τ *if for any* $F \in \tau$ *and every soft point* x^e *contained in* F, *there exists a* $G \in \mathcal{B}$ *such that* $x^e \widetilde{\in} G \sqsubseteq F$.

Definition 2.10 ([24]). *A soft set* F *in a soft topological space* (X, τ, E) *is called a soft neighborhood of the soft point* x^e *if there exists a soft open set* G *such that* $x^e \widetilde{\in} G \sqsubseteq F$.

Definition 2.11 ([31]). *Let* (X, τ, E) *be a soft topological space and* $F \in S(X, E)$. *Then,* $\tau_F = \{F \sqcap G : G \in \tau\}$ *is called the soft relative topology on* F *and* (F, τ_F, E) *is called a soft subspace of* (X, τ, E).

Definition 2.12 ([25]). *Let* (X, τ_1, E) *and* (Y, τ_2, K) *be two soft topological spaces. A soft mapping* $\varphi_\psi : (X, \tau_1, E) \to (Y, \tau_2, K)$ *is called a soft continuous mapping at* $x^e \widetilde{\in} \widetilde{X}$ *if for every soft neighborhood* G *of* $\varphi_\psi(x^e)$ *in* Y, *there exists a soft neighborhood* F *of* x^e *in* X *such that* $\varphi_\psi(F) \sqsubseteq G$.

A soft mapping φ_ψ *is called soft continuous on* X *if it is soft continuous at each* $x^e \widetilde{\in} \widetilde{X}$.

Theorem 2.13 ([25]). *Let* (X, τ_1, E) *and* (Y, τ_2, K) *be two soft topological spaces and* $\varphi_\psi : (X, \tau_1, E) \to (Y, \tau_2, K)$ *be a soft mapping. Then, the following conditions are equivalent:*

(1) φ_ψ *is a soft continuous mapping on* X.
(2) *For every soft open set* G *over* Y, $\varphi_\psi^{-1}(G)$ *is a soft open set over* X.
(3) *For every soft closed set* F *over* Y, $\varphi_\psi^{-1}(F)$ *is a soft closed set over* X.
(4) *For every* $F \in S(X, E)$, $\varphi_\psi(\overline{F}) \sqsubseteq \overline{\varphi_\psi(F)}$.

Theorem 2.14 ([19]). *Let* (X, τ, E) *be a soft topological space and* $F \in S(X, E)$. *A soft point* $x^e \widetilde{\in} \overline{F}$ *if and only if every soft neighborhood of* x^e *intersects* F.

Definition 2.15 ([9]). *Let* $\mathbb{R}$ *be the set of real numbers,* $\mathfrak{B}(\mathbb{R})$ *be the collection of all non-empty bounded subsets of* $\mathbb{R}$ *and* E *be a set of parameters. Then,*

$$F = \{(e, F(e)) : e \in E, F(e) \in \mathfrak{B}(\mathbb{R})\}$$

is called a soft real set.

If specifically F *is a singleton soft set, then after identifying* F *with the corresponding soft element, it will be called a soft real number.*

Definition 2.16 ([9]). *Let F, G be soft real numbers. Then:*

(i) *The sum is defined by* $(F+G)(e) = F(e)+G(e)$ *for every* $e \in E$.

(ii) *The difference is defined by* $(F-G)(e) = F(e)-G(e)$ *for every* $e \in E$.

(iii) *The product is defined by* $(F.G)(e) = F(e).G(e)$ *for every* $e \in E$.

From the above definition of soft real numbers it follows that $F+G, F-G$ *and* $F.G$ *are soft real numbers.*

Definition 2.17 ([9]). *Let F be a soft real number. Then, F is said to be a non-negative soft real number if $F(e)$ is a non-negative real number for every $e \in E$.*

Let $\mathbb{R}(E)^*$ denote the set of all non-negative soft real numbers.

We use notations $\tilde{r}$, $\tilde{s}$, $\tilde{k}$ to denote soft real numbers whereas $\bar{r}$, $\bar{s}$, $\bar{k}$ will denote a particular type of soft real numbers such that $\bar{r}(e) = r$ for every $e \in E$ etc. For example, $\overline{0}$ is the soft real number, where $\overline{0}(e) = 0$ for every $e \in E$ [10].

Definition 2.18 ([10]). *Let $\tilde{r}$ and $\tilde{s}$ be two soft real numbers. Then:*

(i) $\tilde{r} \mathrel{\widetilde{\leq}} \tilde{s}$ *if* $\tilde{r}(e) \leq \tilde{s}(e)$ *for every* $e \in E$.

(ii) $\tilde{r} \mathrel{\widetilde{\geq}} \tilde{s}$ *if* $\tilde{r}(e) \geq \tilde{s}(e)$ *for every* $e \in E$.

(iii) $\tilde{r} \mathrel{\widetilde{<}} \tilde{s}$ *if* $\tilde{r}(e) < \tilde{s}(e)$ *for every* $e \in E$.

(iv) $\tilde{r} \mathrel{\widetilde{>}} \tilde{s}$ *if* $\tilde{r}(e) > \tilde{s}(e)$ *for every* $e \in E$.

Definition 2.19 ([10]). *A mapping $d : SP(X) \times SP(X) \to \mathbb{R}(E)^*$ is called a soft metric on X if it satisfies the following axioms:*

(sm_1) $d(x_1^{e_1}, x_2^{e_2}) = \overline{0}$ *if and only if* $x_1^{e_1} = x_2^{e_2}$ *for every* $x_1^{e_1}, x_2^{e_2} \mathrel{\widetilde{\in}} \widetilde{X}$.

(sm_2) $d(x_1^{e_1}, x_2^{e_2}) = d(x_2^{e_2}, x_1^{e_1})$ *for every* $x_1^{e_1}, x_2^{e_2} \mathrel{\widetilde{\in}} \widetilde{X}$.

(sm_3) $d(x_1^{e_1}, x_2^{e_2}) \mathrel{\widetilde{\leq}} d(x_1^{e_1}, x_3^{e_3}) + d(x_3^{e_3}, x_2^{e_2})$ *for every* $x_1^{e_1}, x_2^{e_2}, x_3^{e_3} \mathrel{\widetilde{\in}} \widetilde{X}$.

The triplet (X, d, E) is called a soft metric space on X.

Definition 2.20 ([11]). *A soft topological space (X, τ, E) is called a soft Hausdorff space or a soft T_2-space if for any two distinct soft points $x_1^{e_1}$, $x_2^{e_2} \in SP(X)$ there exist soft open sets F and G such that $x_1^{e_1} \mathrel{\widetilde{\in}} F, x_2^{e_2} \mathrel{\widetilde{\in}} G$ and $F \sqcap G = \widetilde{\emptyset}$.*

Definition 2.21 ([5, 34]). *Let (X, τ, E) be a soft topological space and $F \in S(X, E)$.*

(i) *A family of soft sets $\mathcal{C} = \{F_i : i \in I\}$ over X is called a cover of F if it satisfies $F \sqsubseteq \bigsqcup_{i \in I} F_i$. It is called a soft open cover if each member of $\mathcal{C}$ is a soft open set. A subfamily of $\mathcal{C}$ is called a subcover of $\mathcal{C}$ if it is also a cover of F.*

(ii) *(X, τ, E) is called a soft compact space if every soft open cover of $\widetilde{X}$ has a finite subcover.*

Theorem 2.22 ([29]). *Let (X, τ, E) be a soft topological space and $F \in S(X, E)$. Then, (F, τ_F, E) is a soft compact space if and only if every cover of F by soft open sets over X contains a finite subcover.*

Theorem 2.23 ([29]). *Every soft compact subspace of a soft Hausdorff space is soft closed.*

3. A new soft topology related to a self soft mapping

In this section, using a self soft mapping, we introduce a new soft topology and prove some fixed soft point theorems in this soft topology. Also, we give the concept of a soft b-metric space and establish its connection with a soft metric space.

Let φ_ψ be a soft mapping from $S(X,E)$ to itself, which is called a self soft mapping. We define

$$\tau_{\varphi_\psi} = \{F \in S(X,E) : \varphi_\psi(F) \sqsubseteq F\}.$$

Then, we get the following theorem.

Theorem 3.1. *The family τ_{φ_ψ} is a soft topology on X.*

Proof. It is clear from Theorem 2.5 (4).

Example 3.2. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. Let us define a soft mapping $\varphi_\psi : S(X,E) \to S(X,E)$ by

$$\varphi(x_1) = x_2, \varphi(x_2) = x_1 \quad \textit{and} \quad \psi(e_1) = e_2, \psi(e_2) = e_2.$$

Then, it is easy to see that $\tau_{\varphi_\psi} = \{\widetilde{X}, \widetilde{\emptyset}, F_1, F_2, F_3\}$ is a soft topology on X since $\varphi_\psi(F_1) \sqsubseteq F_1$, $\varphi_\psi(F_2) \sqsubseteq F_2$, $\varphi_\psi(F_3) \sqsubseteq F_3$, $\varphi_\psi(\widetilde{X}) \sqsubseteq \widetilde{X}$, $\varphi_\psi(\widetilde{\emptyset}) \sqsubseteq \widetilde{\emptyset}$, where $F_1 = \{(e_1, \{x_1\}), (e_2, X)\}$, $F_2 = \{(e_1, \{x_2\}), (e_2, X)\}$ and $F_3 = \{(e_1, \emptyset), (e_2, X)\}$.

Lemma 3.3. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft topological space. Then:*

(1) *If $F \in \tau_{\varphi_\psi}$, then $F \sqsubseteq \varphi_\psi^{-1}(F)$.*

(2) *If $F^c \in \tau_{\varphi_\psi}$, then $\varphi_\psi^{-1}(F) \sqsubseteq F$.*

Proof. It follows immediately from Theorem 2.5 (2) and (3).

Theorem 3.4. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft topological space, $F \in S(X,E)$ and $x^e \in SP(X)$. If $x^e \widetilde{\in} \overline{F}$, then there exists a soft point $y^t \widetilde{\in} F$ such that $\varphi_\psi(y^t) \widetilde{\in} G$ for every $G \in \tau_{\varphi_\psi}$ containing x^e.*

Proof. Let $x^e \widetilde{\in} \overline{F}$ and take a soft open set G such that $x^e \widetilde{\in} G$. Suppose that $\varphi_\psi(y^t) \widetilde{\notin} G$ for every soft point $y^t \widetilde{\in} F$. Then, we have $\varphi_\psi(y^t) \widetilde{\in} G^c$. By Lemma 3.3(2), we obtain $y^t \widetilde{\in} \varphi_\psi^{-1}(G^c) \sqsubseteq G^c$. Thus, from the fact that $\overline{F} \sqsubseteq G^c$ it follows that $x^e \widetilde{\in} G^c$, which is a contradiction.

The following example shows that the converse of the above theorem does not hold in general.

Example 3.5. Consider Example 3.2. Let $F = x_2^{e_1} \in S(X, E)$ and let us take a soft point $x_1^{e_1}$. Then, we have soft open sets F_1 and $\widetilde{X}$ containing $x_1^{e_1}$. One can readily verify that $\varphi_\psi(x_2^{e_1}) = x_1^{e_2} \widetilde{\in} F_1$. The same also holds for the soft set $\widetilde{X}$. But, $x_1^{e_1} \widetilde{\notin} \overline{F}$.

Definition 3.6. *Let $\varphi_\psi : S(X, E) \to S(X, E)$ be a soft mapping. A soft point x^e is called a fixed soft point of φ_ψ if $\varphi_\psi(x^e) = x^e$.*

Remark 3.7. The converse of Theorem 3.4 is true if for a soft set F, each soft point in F is a fixed soft point of φ_ψ.

Proof. Suppose that $x^e \widetilde{\notin} \overline{F}$. Then, there exits a soft closed set H such that $F \sqsubseteq H$ and $x^e \widetilde{\notin} H$. Therefore, we have $H^c \in \tau_{\varphi_\psi}$ with $x^e \widetilde{\in} H^c$. On the other hand, since $F \sqcap H^c = \widetilde{\emptyset}$, we obtain $\varphi_\psi(y^t) = y^t \widetilde{\notin} H^c$ for every $y^t \widetilde{\in} F$, which yields a contradiction.

Lemma 3.8. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft topological space. Then, a soft mapping $\varphi_\psi : (X, \tau_{\varphi_\psi}, E) \to (X, \tau_{\varphi_\psi}, E)$ is a soft continuous mapping.*

Proof. We can easily prove it by using the definition of τ_{φ_ψ}.

Theorem 3.9. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft Hausdorff space. Then, the soft mapping $\varphi_\psi : (X, \tau_{\varphi_\psi}, E) \to (X, \tau_{\varphi_\psi}, E)$ has a fixed soft point.*

Proof. Suppose that $\varphi_\psi(x^e) \neq x^e$ for every $x^e \widetilde{\in} \widetilde{X}$. By hypothesis, there exist soft open sets F and G such that $x^e \widetilde{\in} F$, $\varphi_\psi(x^e) \widetilde{\in} G$ and $F \sqcap G = \widetilde{\emptyset}$. From Lemma 3.8, there exists a soft open set H such that $x^e \widetilde{\in} H$ and $\varphi_\psi(H) \sqsubseteq G$. Therefore, since

$$\varphi_\psi(H \sqcap F) \sqsubseteq \varphi_\psi(H) \sqcap \varphi_\psi(F) \sqsubseteq G \sqcap F = \widetilde{\emptyset}$$

we have $\varphi_\psi(H \sqcap F) = \widetilde{\emptyset}$, so $H \sqcap F = \widetilde{\emptyset}$. This is a contradiction. Thus, there exists a fixed soft point of φ_ψ.

Lemma 3.10. *A soft topological space $(X, \tau_{\varphi_\psi}, E)$ is soft compact space if and only if every family of soft closed sets over X satisfying the finite intersection property has non-empty intersection.*

Proof. We can easily prove it by using the definition of soft compact space.

Lemma 3.11. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft compact space. Then, $\varphi_\psi(\widetilde{X})$ is a soft compact set.*

Proof. Let $\{F_i : i \in I\}$ be a family of soft open sets over X such that $\varphi_\psi(\widetilde{X}) \sqsubseteq \bigsqcup_{i\in I} F_i$. By Lemma 3.8, the family $\{\varphi_\psi^{-1}(F_i) : i \in I\}$ is a soft open cover of $\widetilde{X}$. Then, there exists a finite set $\{i_1, i_2, ..., i_n\} \subseteq I$ such that

$$\widetilde{X} = \varphi_\psi^{-1}(F_{i_1}) \sqcup \varphi_\psi^{-1}(F_{i_2}) \sqcup ... \sqcup \varphi_\psi^{-1}(F_{i_n})$$

and this implies that $\varphi_\psi(\widetilde{X}) \sqsubseteq F_{i_1} \sqcup F_{i_2} \sqcup ... \sqcup F_{i_n}$. Thus, $\varphi_\psi(\widetilde{X})$ is a soft compact set.

Theorem 3.12. *Let (F, τ_F, E) be a soft open compact Hausdorff subspace of a soft topological space $(X, \tau_{\varphi_\psi}, E)$ with $\varphi_\psi(F) \neq F$. If every non-empty soft set $H \neq F$ being both soft open and soft closed over F contains only one soft point of F, then there exists a soft point $x^e \widetilde{\in} F$ such that $\varphi_\psi(x^e) = x^e$.*

Proof. Let us consider a family of soft sets of the form

$$G_1 = \varphi_\psi(F),\ G_2 = \varphi_\psi(G_1) = \varphi_\psi^2(F),\ ... , G_n = \varphi_\psi(G_{n-1}) = \varphi_\psi^n(F),\ ...$$

where $n \in \mathbb{N}$. Since $F \in \tau_{\varphi_\psi}$, we have $G_n \sqsubseteq G_{n-1}$ for all $n \in \mathbb{N}$. By Lemma 3.11 and Theorem 2.23, G_n is a soft closed set over F for all $n \in \mathbb{N}$. Then, from Lemma 3.10 it follows that $H = \bigsqcap_{n\in\mathbb{N}} G_n$ is a non-empty soft closed set over F. Also, since

$$\varphi_\psi(H) \sqsubseteq \bigsqcap_{n\in\mathbb{N}} \varphi_\psi^{n+1}(F) \sqsubseteq \bigsqcap_{n\in\mathbb{N}} \varphi_\psi^{n}(F) = H$$

H is a soft open set over F. Therefore, by hypothesis, the soft set H contains only one soft point of F. Let us choose such a soft point x^e. Thus, from the fact that

$$\varphi_\psi(x^e) \sqsubseteq \varphi_\psi(H) \sqsubseteq \bigsqcap_{n\in\mathbb{N}} \varphi_\psi^{n}(F) = x^e$$

it follows that $\varphi_\psi(x^e) = x^e$.

Czerwik [7] defined the notion of a soft b-metric. We introduce this notion in soft setting as follows:

Definition 3.13. *Let X be a non-empty set, E be a non-empty parameter set and $\tilde{s} \widetilde{\geq} \overline{1}$. A mapping $d_b : SP(X) \times SP(X) \to \mathbb{R}(E)^*$ is called a soft b-metric on X if it satisfies the following axioms:*

(sbm_1) $d_b(x_1^{e_1}, x_2^{e_2}) = \overline{0}$ *if and only if* $x_1^{e_1} = x_2^{e_2}$ *for all* $x_1^{e_1}, x_2^{e_2} \widetilde{\in} \widetilde{X}$.

(sbm_2) $d_b(x_1^{e_1}, x_2^{e_2}) = d_b(x_2^{e_2}, x_1^{e_1})$ *for all* $x_1^{e_1}, x_2^{e_2} \widetilde{\in} \widetilde{X}$.

(sbm_3) $d_b(x_1^{e_1}, x_2^{e_2}) \widetilde{\leq} \tilde{s}\left(d_b(x_1^{e_1}, x_3^{e_3}) + d_b(x_3^{e_3}, x_2^{e_2})\right)$ *for all* $x_1^{e_1}, x_2^{e_2}, x_3^{e_3} \widetilde{\in} \widetilde{X}$.

The triplet (X, d_b, E) is called a soft b-metric space on X.

It is seen that the above definition coincides with that of the soft metric when $\tilde{s} = \overline{1}$. Thus, the class of the soft b-metric spaces is larger than that of the soft metric spaces, that is, every soft metric space is a soft b-metric space. But the converse is not true in general as seen in the following example.

Example 3.14. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and a mapping $d_b : SP(X) \times SP(X) \to \mathbb{R}(E)^*$ be defined by:

$$\begin{array}{ll} d_b(x_1^{e_1}, x_1^{e_1}) = d_b(x_1^{e_2}, x_1^{e_2}) = \overline{0}, & d_b(x_2^{e_1}, x_2^{e_1}) = d_b(x_2^{e_2}, x_2^{e_2}) = \overline{0}, \\ d_b(x_1^{e_1}, x_1^{e_2}) = d_b(x_1^{e_2}, x_1^{e_1}) = \overline{3}, & d_b(x_1^{e_1}, x_2^{e_1}) = d_b(x_2^{e_1}, x_1^{e_1}) = \overline{1}, \\ d_b(x_1^{e_1}, x_2^{e_2}) = d_b(x_2^{e_2}, x_1^{e_1}) = \overline{\frac{5}{2}}, & d_b(x_1^{e_2}, x_2^{e_1}) = d_b(x_2^{e_1}, x_1^{e_2}) = \overline{\frac{1}{2}}, \\ d_b(x_1^{e_2}, x_2^{e_2}) = d_b(x_2^{e_2}, x_1^{e_2}) = \overline{\frac{3}{2}}, & d_b(x_2^{e_1}, x_2^{e_2}) = d_b(x_2^{e_2}, x_2^{e_1}) = \overline{\frac{1}{8}}. \end{array}$$

Then, (X, d_b, E) is a soft b-metric space with constant $\tilde{s} = \overline{2}$. However, since

$$d_b(x_1^{e_1}, x_2^{e_1}) + d_b(x_2^{e_1}, x_1^{e_2}) = \overline{\frac{3}{2}} \mathrel{\tilde{<}} d_b(x_1^{e_1}, x_1^{e_2}) = \overline{3},$$

it is not a soft metric space.

Definition 3.15. *Let (X, d_b, E) be a soft b-metric space, $\tilde{r}$ be a non-negative soft real number and $x^e \mathrel{\tilde{\in}} \widetilde{X}$. A soft b-open ball with centre x^e and radius $\tilde{r}$ is defined by*

$$B_b(x^e, \tilde{r}) = \bigsqcup \{y^t \mathrel{\tilde{\in}} \widetilde{X} : d_b(x^e, y^t) \mathrel{\tilde{<}} \tilde{r}\}.$$

Example 3.16. Let (X, d_b, E) be a soft b-metric space which is defined in Example 3.14. Then, for $x_1^{e_1} \mathrel{\tilde{\in}} \widetilde{X}$

$$B_b(x_1^{e_1}, \overline{2}) = x_1^{e_1} \sqcup x_2^{e_1} = \{(e_1, X), (e_2, \emptyset)\}.$$

Definition 3.17. *Let (X, d_b, E) be a soft b-metric space. A soft mapping $\varphi_\psi : S(X, E) \to S(X, E)$ is called a soft contraction if there exists a non-negative soft real number $\tilde{k} \mathrel{\tilde{<}} \overline{1}$ such that $d_b\big(\varphi_\psi(x^e), \varphi_\psi(y^t)\big) \mathrel{\tilde{\leq}} \tilde{k}\, d_b(x^e, y^t)$ for every $x^e, y^t \mathrel{\tilde{\in}} \widetilde{X}$.*

Theorem 3.18. *Let (X, d_b, E) be a soft b-metric space and $\varphi_\psi : S(X, E) \to S(X, E)$ be a soft contraction mapping such that $\tilde{s}.\tilde{k} \mathrel{\tilde{<}} \overline{1}$. If τ_{φ_ψ} is a soft topology on X, then, for every $x^e \mathrel{\tilde{\in}} \widetilde{X}$, there exists a soft real number $\tilde{r}$ such that $B_b(x^e, \tilde{r}) \in \tau_{\varphi_\psi}$.*

Proof. Let $x^e \mathrel{\tilde{\in}} \widetilde{X}$. Let us take a soft real number $\tilde{r}$ satisfying the following condition:

$$\frac{\tilde{s}\, d_b(x^e, \varphi_\psi(x^e))}{\overline{1} - \tilde{s}.\tilde{k}} \mathrel{\tilde{<}} \tilde{r}.$$

Then, we have $B_b(x^e, \tilde{r}) \in \tau_{\varphi_\psi}$. Indeed, let $y^t \mathrel{\tilde{\in}} \varphi_\psi\big(B_b(x^e, \tilde{r})\big)$. Then, there exists a soft point $x_1^{e_1} \mathrel{\tilde{\in}} B_b(x^e, \tilde{r})$ such that $\varphi_\psi(x_1^{e_1}) = y^t$. Therefore, since

$$\begin{aligned} d_b(x^e, \varphi_\psi(x_1^{e_1})) &\mathrel{\tilde{\leq}} \tilde{s}\left(d_b(x^e, \varphi_\psi(x^e)) + d_b(\varphi_\psi(x^e), \varphi_\psi(x_1^{e_1}))\right) \\ &\mathrel{\tilde{\leq}} \tilde{s}\left(d_b(x^e, \varphi_\psi(x^e)) + \tilde{k}\, d_b(x^e, x_1^{e_1})\right) \\ &\mathrel{\tilde{<}} \tilde{s}\left(d_b(x^e, \varphi_\psi(x^e)) + \tilde{k}.\tilde{r}\right) \mathrel{\tilde{<}} \tilde{r} \end{aligned}$$

we obtain $\varphi_\psi(x_1^{e_1}) = y^t \mathrel{\tilde{\in}} B_b(x^e, \tilde{r})$. Thus, from the fact that $\varphi_\psi\big(B_b(x^e, \tilde{r})\big) \sqsubseteq B_b(x^e, \tilde{r})$ it follows that $B_b(x^e, \tilde{r}) \in \tau_{\varphi_\psi}$, which completes the proof.

4. Soft orbit

Now, we present the notion of a soft orbit and investigate some of its basic properties. Also, on the basis of this notion, we establish a fixed soft point theorem.

Definition 4.1. *A soft orbit of a soft point* $x^e \widetilde{\in} \widetilde{X}$ *under a soft mapping* $\varphi_\psi : S(X,E) \to S(X,E)$ *is defined as*

$$O_s(x^e) = \bigsqcup \left\{ \varphi_\psi^n(x^e) : n = 0, 1, 2, ... \right\}$$

where φ_ψ^n *is the nth iterate of* φ_ψ.

Then, we get the following properties.

Theorem 4.2. *Let* $(X, \tau_{\varphi_\psi}, E)$ *be a soft topological space.*

(1) *For every* $x^e \widetilde{\in} \widetilde{X}$, $O_s(x^e)$ *is the smallest soft open set containing* x^e.

(2) *A family* $\left\{ O_s(x^e) : x^e \widetilde{\in} \widetilde{X} \right\}$ *of soft orbits is a soft base for* τ_{φ_ψ}.

(3) *If a soft point* $x^e \widetilde{\in} \widetilde{X}$ *is a fixed soft point of* φ_ψ, *then we have* $O_s(x^e) = x^e$.

Proof. (1) Let $x^e \widetilde{\in} \widetilde{X}$. One can readily verify that $\varphi_\psi(O_s(x^e)) \sqsubseteq O_s(x^e)$. So, we have $O_s(x^e) \in \tau_{\varphi_\psi}$. Now, we shall show that $O_s(x^e) = \bigsqcap \{F \in S(X,E) : x^e \widetilde{\in} F \textit{ and } F \in \tau_{\varphi_\psi}\}$. Let $y^t \widetilde{\in} O_s(x^e)$. Then, there exists an $n \in \mathbb{N}$ such that $\varphi_\psi^n(x^e) = y^t$. Suppose that there exists an $F \in \tau_{\varphi_\psi}$ containing x^e such that $y^t \widetilde{\notin} F$. Therefore, we have $\varphi_\psi(x^e) \widetilde{\in} F$. Again, since $\varphi_\psi(F) \sqsubseteq F$, we obtain $\varphi_\psi^2(x^e) \widetilde{\in} F$. Continuing this process, we get $\varphi_\psi^n(x^e) = y^t \widetilde{\in} F$. But this result is in contradiction with the fact that $y^t \widetilde{\notin} F$. Similarly, the converse inclusion holds as well, since $x^e \widetilde{\in} O_s(x^e)$ and $O_s(x^e) \in \tau_{\varphi_\psi}$.

(2) and (3) are clear from (1) and the definition of soft orbit.

Now, we give the following example to show the validity of the properties in Theorem 4.2.

Example 4.3. Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$. Let us define a soft mapping $\varphi_\psi : S(X,E) \to S(X,E)$ by

$$\varphi(x_1) = x_2, \varphi(x_2) = x_2 \quad \textit{and} \quad \psi(e_1) = e_2, \psi(e_2) = e_2.$$

One can readily verify that $\tau_{\varphi_\psi} = \{\widetilde{X}, \widetilde{\emptyset}, F_1, F_2, F_3, F_4, F_5, F_6, F_7\}$, where $F_1 = \{(e_1, \{x_1\}), (e_2, \{x_2\})\}$, $F_2 = \{(e_1, \{x_2\}), (e_2, \{x_2\})\}$, $F_3 = \{(e_1, X), (e_2, \{x_2\})\}$, $F_4 = \{(e_1, \emptyset), (e_2, \{x_2\})\}$, $F_7 = \{(e_1, \emptyset), (e_2, X)\}$.

Therefore, we have $O_s(x_1^{e_1}) = F_1$, $O_s(x_1^{e_2}) = F_7$, $O_s(x_2^{e_1}) = F_2$ and $O_s(x_2^{e_2}) = F_4$ and so that $O_s(x_i^{e_j})$ is the smallest soft open set containing $x_i^{e_j}$ for all $i \in \{1, 2\}$ and all $j \in \{1, 2\}$.

Also, as it can be easily seen, $\{O_s(x_1^{e_1}), O_s(x_1^{e_2}), O_s(x_2^{e_1}), O_s(x_2^{e_2})\}$ is a soft base for τ_{φ_ψ}.

Moreover, the soft point $x_2^{e_2}$ is a fixed soft point of φ_ψ and we obtain $O_s(x_2^{e_2}) = x_2^{e_2}$.

Definition 4.4. *A soft orbit of a soft set $F \in S(X,E)$ under a soft mapping $\varphi_\psi : S(X,E) \to S(X,E)$ is defined as*

$$O_s(F) = \bigsqcup_{x^e \,\widetilde{\in}\, F} O_s(x^e).$$

Example 4.5. Let $X = \{x_1, x_2, x_3\}$ and $E = \{e_1, e_2\}$. Let us consider a soft mapping $\varphi_\psi : S(X,E) \to S(X,E)$ such that

$$\varphi(x_1) = x_3, \varphi(x_2) = x_1, \varphi(x_3) = x_3 \quad \textit{and} \quad \psi(e_1) = e_1, \psi(e_2) = e_2.$$

Then, the soft orbit of a soft set $F = \{(e_1, \{x_1\}), (e_2, \{x_1\})\}$ is defined as the following:

$$O_s(F) = O_s(x_1^{e_1}) \sqcup O_s(x_1^{e_2}) = \{(e_1, \{x_1, x_3\}), (e_2, \{x_1, x_3\})\}.$$

Theorem 4.6. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft topological space and $F, G \in S(X,E)$. Then, the following statements are satisfied.*

(1) $F \sqsubseteq O_s(F)$.
(2) $O_s(F) \in \tau_{\varphi_\psi}$.
(3) *If $F \sqsubseteq G$, then $O_s(F) \sqsubseteq O_s(G)$.*
(4) *If $F \in \tau_{\varphi_\psi}$, then $O_s(F) = F$.*
(5) $O_s(F) = \bigsqcap \{H : F \sqsubseteq H \textit{ and } H \in \tau_{\varphi_\psi}\}$.

Proof. (1), (2) and (3) are clear.

(4) Let $F \in \tau_{\varphi_\psi}$. Then, we have $O_s(x^e) \sqsubseteq F$ for every $x^e \,\widetilde{\in}\, F$. Therefore, we obtain

$$\bigsqcup_{x^e \,\widetilde{\in}\, F} O_s(x^e) = O_s(F) \sqsubseteq F.$$

From the fact that $F \sqsubseteq O_s(F)$ it follows that $O_s(F) = F$.

(5) By (1) and (2), we have $\bigsqcap \{H : F \sqsubseteq H \textit{ and } H \in \tau_{\varphi_\psi}\} \sqsubseteq O_s(F)$. For the converse, let $x^e \,\widetilde{\in}\, O_s(F)$. Then, there exists a soft point $y^t \,\widetilde{\in}\, F$ such that $x^e \,\widetilde{\in}\, O_s(y^t)$. Suppose that $x^e \,\widetilde{\notin}\, \bigsqcap \{H : F \sqsubseteq H \textit{ and } H \in \tau_{\varphi_\psi}\}$. Therefore, there exists a soft open set H such that $F \sqsubseteq H$ and $x^e \,\widetilde{\notin}\, H$. From $\varphi_\psi(F) \sqsubseteq \varphi_\psi(H) \sqsubseteq H$ it follows that $x^e \,\widetilde{\notin}\, O_s(y^t)$, which is a contradiction. Thus, the required equality is satisfied.

Lemma 4.7. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft topological space. Then, for every $x^e \,\widetilde{\in}\, \widetilde{X}$, the soft set $\overline{O_s(x^e)}$ is both soft open and soft closed on X.*

Proof. It is clear that $\overline{O_s(x^e)}$ is a soft closed set on X. We shall show that $\overline{O_s(x^e)}$ is a soft open set over X. Let $y^t \,\widetilde{\in}\, \varphi_\psi(\overline{O_s(x^e)})$. Then, there exists a soft point $x_1^{e_1} \widetilde{\in} \overline{O_s(x^e)}$ such that $y^t = \varphi_\psi(x_1^{e_1})$. Now, let us take a soft open set F_{y^t} containing $\varphi_\psi(x_1^{e_1})$. Since φ_ψ is a soft continuous mapping, there exists a soft open set $G_{x_1^{e_1}}$ containing $x_1^{e_1}$ such that $\varphi_\psi(G_{x_1^{e_1}}) \sqsubseteq F_{y^t}$. From Theorem 2.14 it follows that $G_{x_1^{e_1}} \sqcap O_s(x^e) \neq \widetilde{\emptyset}$ and therefore $F_{y^t} \sqcap O_s(x^e) \neq \widetilde{\emptyset}$. Hence, we have $y^t = \varphi_\psi(x_1^{e_1}) \widetilde{\in} \overline{O_s(x^e)}$. Thus, we obtain $\varphi_\psi\big(\overline{O_s(x^e)}\big) \sqsubseteq \overline{O_s(x^e)}$, which completes the proof.

Theorem 4.8. *Let $(X, \tau_{\varphi_\psi}, E)$ be a soft topological space and $\varphi_\psi : (X, \tau_{\varphi_\psi}, E) \to (X, \tau_{\varphi_\psi}, E)$ be a soft mapping such that if x^e is not a fixed soft point of φ_ψ, then $x^e \widetilde{\notin} \overline{O_s\big(\varphi_\psi^2(x^e)\big)}$. If $\overline{O_s(x_0^{e_0})}$ is a soft compact set for some $x_0^{e_0} \widetilde{\in} \widetilde{X}$, then there exists a soft point $y^t \widetilde{\in} \overline{O_s(x_0^{e_0})}$ such that $\varphi_\psi(y^t) = y^t$.*

Proof. Let us consider a family $\mathcal{U} = \big\{F \sqsubseteq \overline{O_s(x_0^{e_0})} : F \neq \widetilde{\emptyset} \text{ and } F, F^c \in \tau_{\varphi_\psi}\big\}$. By Lemma 4.7, $\mathcal{U}$ is a non-empty family. Let $\mathcal{U}$ be partially ordered by the set inclusion and let $\mathcal{V}$ be a totally ordered subfamily of $\mathcal{U}$. Take a soft set $F_0 = \bigsqcap\{F : F \in \mathcal{V}\}$. From Lemma 3.10 it follows that F_0 is a non-empty soft set on X. Then, we have $F_0 \in \mathcal{U}$ since the other conditions are easily verified. Also, it is a lower bound of $\mathcal{V}$ and so, by Zorn's Lemma, $\mathcal{U}$ has a minimal element H. Therefore, using the minimality of H, we obtain $\overline{\varphi_\psi(H)} = H$. Now, suppose that $\varphi_\psi(x^e) \neq x^e$ for some $x^e \widetilde{\in} H$. By hypothesis about φ_ψ, we obtain $x^e \widetilde{\notin} \overline{O_s\big(\varphi_\psi^2(x^e)\big)}$. From the fact that $\overline{O_s\big(\varphi_\psi^2(x^e)\big)} \sqsubseteq H$ and H is a minimal soft set of $\mathcal{U}$ it follows that $\overline{O_s\big(\varphi_\psi^2(x^e)\big)} = H$. Hence, we get $x^e \widetilde{\notin} H$, which leads to a contradiction. Thus, since $H \sqsubseteq \overline{O_s(x_0^{e_0})}$, there exists a soft point $y^t \widetilde{\in} \overline{O_s(x_0^{e_0})}$ such that $\varphi_\psi(y^t) = y^t$.

5. Conclusion

Soft set theory is a general method for solving problems of uncertainty. Recently, many authors have already studied the notion of topology on soft sets, because topology is an important area of mathematics with many applications in the domains of computer sciences and information sciences. On the other hand, metric spaces and their various generalizations (such as b-metric spaces) and also fixed point theory have attracted the attention of many mathematicians, physicists and computer scientists. Hence, there exists a considerable literature of fixed point theory dealing with results on fixed or common fixed points in b-metric spaces. Thus, softification of these concepts is highly desirable.

In this paper, we introduce a new soft topology with the help of a self soft mapping and study some of its basic properties. Also, we prove some fixed soft point theorems on this soft topology. Then, we define the notion of a soft b-metric space and investigate its relation with a soft metric space. Later, we present the concept of a soft orbit and establish a fixed soft point theorem using this concept.

In the fields of soft topology and soft metric, the findings in this paper will open up new areas of research. Since there exist close relations between soft sets and information systems, one can use the results inferred from this research to improve these kinds of relations. Also, a soft b-metric space can be extended in fuzzy soft and intuitionistic fuzzy soft b-metric spaces in order to have more affirmative solution in decision making problems in real life situations. Moreover, one can obtain fixed point results for self soft mapping by using suitable conditions in soft b-metric spaces.

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