NEARLY SEMI -2-ABSORBING SUBMODULES AND RELATED CONCEPTS

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Abstract. In this article, R is commutative ring with identity and Y is a left unitary R-module. A proper submodule L of Y is called nearly semiprime submodule if whenever $r^n y \in L$, where $r \in R$ and $y \in Y$, $n \in Z^+$, implies that $ry \in L + J(Y)$, where $J(Y)$ is the Jacobson radical of Y. This concept in courage us to introduce the concept nearly semi-2-absorbing submodule as a generalization of nearly semiprime submodule, where a proper submodule L of Y is called nearly semi-2-absorbing submodule of Y if whenever $a^2 y \in L$, where $a \in R$, $y \in Y$, implies that either $ay \in L + J(Y)$ or $a^2 \in [L : Y]$. Many basic properties, and characterization of this concept are introduce. On the other hand the relation of this concept with other classes of modules are studied.

Keywords: semiprime submodule, semi 2-absorbing submodule, good ring.

1. Introduction

The notion of prime submodule was introduce by [6] . where a proper submodule N of an R-module Y is called prime submodule, if whenever $ry \in N$, $r \in R$, $y \in Y$, implies that either $y \in N$ or $r \in [N : Y]$ [6], where $[N : Y] = \{r \in R : rY \subseteq N\}$. Semiprime submodule as a generalization of prime submodule was introduced by [1], where a proper submodule N of Y is called semiprime if whenever $r^n y \in N$, $r \in R$, $y \in Y$, $n \in Z^+$ implies that $ry \in N$ [1]. This concept generalized in [7] to nearly semi prime sub module, where a proper sub module N of Y is called nearly semiprime if whenever $r^n y \in N$, $r \in R$, $y \in Y$, $n \in Z^+$ implies that $ry \in N + J(Y)$. Also, semiprime submodule generalized to semi -2- absorbing submodule in [3] ,where a proper submodule N of Y is called semi -2-absorbing sub module of Y if whenever $a^2 y \in N$, where $a \in R$, $y \in Y$, implies that either $ay \in N$ *or* $a^2 \in [N : Y]$. This led us to introduce the

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concept of nearly semi -2 absorbing sub module as a generalization of nearly semiprime submodule, where a proper submodule N of Y is called nearly semi-2-absorbing if whenever $a^2y \in N$, where $a \in R,\ y \in Y$, implies that either $y \in N + J(Y)$ *or* $a^2 \in [N + J(Y) : Y]$. We give many properties, characterizations and relationship between nearly semi-2-absorbing and other concepts.

2. Nearly semi -2-absorbing sub modules

We investigate in this section, the concept of nearly semi -2-absorbing sub module as a generalization of nearly semi prime sub module.

Definition 2.1. A proper submodule L of an R-module Y is called nearly Semi -2- absorbing, if whenever $a^2y \in L$, where $a \in R,\ y \in Y$, implies that either $ay \in L + J(Y)$ or $a^2 \in [L + J(Y) : Y]$.

A proper ideal J of a ring R is called nearly semi-2-absorbing ideal if J is nearly semi-2-absorbing sub module of an R-module R.

Proposition 2.2. *If L is a nearly semiprime submodule of an R-module Y, then L is nearly semi -2-absorbing sub module of Y.*

Proof. Assume that $a^2y \in L,\ a \in R\ ,\ y \in Y\ ,\ with\ a^2 \notin [L + J(Y) : Y]$, since L is a nearly semi prime , it follows that $ay \in L + J(Y)$. Thus L is nearly semi-2-absorbing sub module of Y.

The converse of Proposition 2.2 is not true in general, so the following example explain that.

Example 2.3. Consider the submodule $L = 4Z$ of the Z-module Z , L is nearly semi-2-absorbing but not nearly semi prime because if $2^2 \times 1 \in 4Z$, where $2 \in Z$, $1 \in Z$, implies that $2 \times 1 \notin 4Z + J(Z)$, but $2^2 \in [4Z + J(Z) : Z] = 4Z$. On the other hand if $2^2 \times 1 \in 4Z$, implies that $2 \times 1 \notin 4Z + J(Z)$ hence $4Z$ is not nearly semiprime Z-sub module.

Lemma 2.4 ([1]). *A proper submodule L of an R-module Y is semiprime if and only if whenever $a^2y \in L$, where $r \in R,\ y \in Y$, then $ry \in L$.*

Remark 2.5. Every semiprime submodule of an R-module Y is nearly semi -2-absorbing, but the converse is not true in general.

Proof. Since every semiprime submodule is nearly semi prime[7], hence the proof is follows by Proposition 2.3.

For the converse ,consider the following example:

Consider the sub module $(\overline{0})$ of the Z-module Z_4 . $(\overline{0})$ is nearly semi-2-absorbing but not semiprime submodule of Z_4 because if $2^2 \times 1 \in (\overline{0})$ then $2 \times 1 \notin (\overline{0})$ hence $(\overline{0})$ is not semiprime, but $2^2 \in \left[(\overline{0}) + J(Z_4) : Z_4\right] = \left[(\overline{0}) + \{\overline{0}, \overline{2}\} : Z_4\right] = 2Z$, hence $(\overline{0})$ is nearly semi-2-absorbing in Z_4.

Proposition 2.6. *Let K be a submodule of an R-module Y, with $K + J(Y)$ is nearly semi-2-absorbing , then K is nearly semi-2-absorbing.*

Proof. Assume that $r^2y \in K$, *where* $r \in R$, $y \in Y$, then $r^2y \in K + J(y)$. But $K + J(Y)$ is nearly semi-2-absorbing then either $ry \in K + ?(Y) + J(Y) = K + J(Y)$ or $r^2 \in [K + J(Y) + J(Y) : Y]$ =$[K + J(Y) : Y]$. Hence K is nearly semi-2-absorbing in Y.

Lemma 2.7 ([5]). *Let N and K be two sub modules of an R-module Y, then*

1. *If $N \subset K$, then $[N : M] \subseteq [K : M]$.*
2. *If $N \subset K$,then $[N : M] \subseteq [N : K]$.*

Proposition 2.8. *Let E and F be two submodules of an R-module Y with $E \subsetneq F$. If E is a nearly semi-2-absorbing submodule of Y and $J(Y) \subseteq J(F)$, then E is nearly semi-2-absorbing in F.*

Proof. Assume that $r^2y \in E$, with $r \in R$, $y \in F$, since E is nearly semi-2-absorbing sub module of Y, then $ry \in E + J(Y)$ or $r^2 \in [E + J(Y) : Y]$. But $J(Y) \subseteq J(F)$, it follows that $ry \in E + J(F)$ or $r^2 \in [E + J(F) : Y] \subseteq [E + J(F) : F]$ by Lemma 2.7. Hence E is nearly semi-2-absorbing in F.

Remark 2.9. Every prime submodule of an R-module Y is nearly -2-absorbing submodule of Y ,while the converse is not true.

Proof. Since prime submodule is semiprime [1], hence the proof follows by Remark 2.5. For the converse consider the following example, let $Y = Z, R = Z$. $L = 9Z$ a submodule of $Y, 9Z$ is nearly semi-2-absorbing of Z since $3^2 \times 1 \in 9Z$ it follows that $3^2 \in [9Z + J(Z) : Z] = 9Z$. But $9Z$ is not prime submodule of Z, since $3 \times 3 \in 9Z$ but $3 \notin 9Z$ and $3 \notin [9Z : Z] = 9Z$.

Recall that a proper submodule L of an R-module is 2-absorbing if whenever $aby \in L$, with $a, b \in R$, $y \in Y$ implies that either $ay \in L$ or $by \in L$ or $ab \in [L : Y]$ [2].

Proposition 2.10. *Every 2-absorbing submodule of an R-module Y is nearly semi-2-absorbing submodule of Y.*

Proof. Let L be 2-absorbing submodule of Y, and $r^2y \in L$, where $r \in R$, $y \in Y$. Since L is 2-absorbing it follows that either $ry \in L$ or $r^2 \in [L : Y]$. That is either $ry \in L + J(Y)$ or $r^2 \in [L : Y] \leq [L + J(Y) : Y]$ by Lemma 2.7. Thus L is nearly semi-2-absorbing.

Proposition 2.11. *Let Y be an R-module, and K be a proper submodule of Y with $J(Y) \subseteq K$. Then K is semi-2-absorbing iff K is nearly semi-2-absorbing.*

Proof. $\Rightarrow$ Assume that $r^2y \in K$, where $r \in R$, $y \in Y$. Then either $ry \in K \subseteq K + J(Y)$ or $r^2 \in [K : Y] \subseteq [K + J(Y) : Y]$. Hence either $ry \in K + J(Y)$ or $r^2 \in [K + J(Y) : Y]$. Thus K is nearly semi-2-absorbing.

$\Leftarrow$ Assume that $a^2y \in K$ with $a \in R$, $y \in Y$. Then either $ay \in K + J(Y)$ or $a^2 \in [K + J(Y) : Y]$. But $J(Y) \subseteq K$, it follows that $K + J(Y) = K$. Hence either $ay \in K$ or $a^2 \in [K : Y]$. Thus K is semi-2-absorbing.

Remark 2.12. The intersection of two nearly semi-2-absorbing submodules of an R-module Y need not to be nearly semi-2-absorbing submodules. For example let $Y = Z$ and $R = Z, L = 2Z, K = 9Z$ are nearly semi-2-absorbing submodules of Y, but $2Z \cap 9Z = 18Z$ is not nearly semi-2-absorbing submodule of Y, since $3^2 \times 2 \in 18Z$, but $3 \times 2 \notin 18Z + J(Z)$ and $3^2 \notin [18Z + J(Z) : Z]$.

Proposition 2.13. *Let L be a nearly semi-2-absorbing submodule of an R-module Y, and K is a proper submodule of Y with $K \nsubseteq L$ and $J(K) = J(Y)$, then $L \cap K$ is nearly semi-2-absorbing submodule in K.*

Proof. Since $K \nsubseteq L$, then $L \cap K < K$. Let $r^2y \in K \cap L$, where $r \in R$, $y \in K$. Since L is nearly semi-2-absorbing and $r^2y \in L$, it follows that either $ry \in L + J(Y)$ or $r^2 \in [L + J(Y) : Y]$, but $J(K) = J(Y)$ then $ry \in L + J(K)$ or $r^2 \in [L + J(K) : Y]$. Since $y \in K$, then $ry \in K$, hence $ry \in (L + J(K)) \cap K$, it follows that $ry \in (L \cap K) + J(K)$, or $r^2 \in [L + J(K) : Y]$, implies that $r^2y \in L+J(Y)$ for all $y \in Y$. Since $ry \in K$, then $r^2y \in K$. Hence $r^2y \in (L + J(K)) \cap K$, implies that $r^2y \in (L \cap K) + J(K)$, hence $r^2 \in [(L \cap K) + J(K) : Y] \subseteq [(L \cap K) + J(K) : K]$. That is $r^2 \in [(L \cap K) + J(K) : K]$. Hence $L \cap K$ is nearly semi-2-absorbing in K.

Recall that a ring R is good ring if $J(R)Y = J(Y)$ where Y is an R-module [4].

Remark 2.14 ([4])**.** If R is good ring, then $J(Y) \cap N = J(N)$, where Y is an R-module, N submodule of Y.

Lemma 2.15 ([4], Lemma 2.3.15)**.** *Let Y be an R-module , and L, F and E are submodule of Y with $F \subsetneq E$. Then $(L+F) \cap E = (L\cap E)+F = (L\cap E) + (F\cap E)$.*

Proposition 2.16. *Let Y be an R-module over good ring, and L is nearly semi-2-absorbing submodule of Y, and K be a proper submodule of Y with $K \nsubseteq L$ and $J(Y) \leq K$, then $(L \cap K)$ is nearly semi-2-absorbing in K.*

Proof. Since $K \nsubseteq L$, then $(L \cap K) \subset K$. Assume that $r^2y \in (L \cap K)$, where $r \in R$, $y \in K$. Since L is a nearly semi-2-absorbing, and $r^2y \in L$, then either $ry \in L + J(Y)$ or $r^2 \in [L + J(Y) : Y]$. Since $y \in K$ then $ry \in K$ and $r^2y \in K$. Hence $ry \in (L + J(Y)) \cap K$ or $r^2y \in (L + J(Y)) \cap K$ for all $y \in Y$. Thus by Lemma 2.15, $ry \in (L \cap K) + J(Y) \cap K$ or $r^2y \in (L \cap K) + (J(Y) \cap K)$. Since R is good ring, then $J(Y) \cap K = J(K)$. That is $ry \in (L \cap K) + J(K)$ or $r^2y \in (L \cap K) + J(K)$. Hence $ry \in (L \cap K) + J(K)$ or

$r^2 \in [(L \cap K) + J(K) : Y] \leq [(L \cap K) + J(K) : K]$ by Lemma 2.7, (2), implies that $r^2 \in [(L \cap K) + J(K) : K]$. Thus $L \cap K$ is nearly semi-2-absorbing in K.

Proposition 2.17. *Let L and K be nearly semi-2-absorbing proper submodules of an R-module Y, with $K \nsubseteq L$ and either $J(Y) \subseteq L$ or $J(Y) \subseteq K$, then $L \cap K$ is semi-2-absorbing of Y.*

Proof. Since $K \nsubseteq L$, then $L \cap K \subsetneq L \subsetneq Y$, it follows that $L \cap K \subsetneq Y$. Assume that $r^2 y \in L \cap K$, $r \in R$, $y \in Y$. Then $r^2 y \in K$ and $r^2 y \in L$, but both K and L are nearly semi-2-absorbing in Y, then either $ry \in K + J(Y)$ or $r^2 \in [K + J(Y) : Y]$ and either $ry \in L + J(Y)$ or $r^2 \in [L + J(Y) : Y]$. It follows that either $ry \in (K + J(Y)) \cap (L + J(Y))$ or $r^2 \in [K + J(Y) : Y] \cap [L + J(Y) : Y]$. If $J(Y) \subseteq L$, then $ry \in (K + J(Y)) \cap L$, it follows that by Lemma 2.15, $ry \in (L \cap K) + J(Y)$. If $J(Y) \subseteq K$, then it follows that by Lemma 2.15, $ry \in (L + J(Y)) \cap K$, then $ry \in (L \cap K) + J(Y)$. Also $r^2 \in [K + J(Y) : Y] \cap [L + J(Y) : Y] \subseteq [(L \cap K) + J(Y) : Y]$, implies that $r^2 \in [L \cap K + J(Y) : Y]$. That is either $ry \in (L \cap K) + J(Y)$ or $r^2 \in [L \cap K + J(Y) : Y]$. Hence $L \cap K$ is nearly semi-2-absorbing submodule of Y.

Proposition 2.18. *Let Y be an R-module over a good ring R, and L, K be submodules of Y, with $L \subsetneq K$ and $J(Y) \subseteq K$. If L is a nearly semi-2-absorbing submodule of Y, then L is a nearly semi-2-absorbing submodule of K.*

Proof. Assume that $r^2 y \in L$, $r \in R$, $y \in K$, since L is a nearly semi-2-absorbing submodule of Y, and $y \in K \subseteq Y$, then either $ry \in L + J(Y)$ or $r^2 \in [L + J(Y) : Y]$. But $y \in K$, then $ry \in K$. Hence either $ry \in (L + J(Y)) \cap K$ or $r^2 y \in (L + J(Y)) \cap K$ for each y in K. Hence either $ry \in (L \cap K) + J(Y) \cap K$ or $r^2 y \in (L \cap K) + (J(Y) \cap K)$. Since R is a good ring, and $L \subsetneq K$, it follows that either $ry \in L + J(K)$ or $r^2 y \in L + J(K)$. Hence either $ry \in L + J(K)$ or $r^2 \in [L + J(K) : K]$. Thus L is a nearly semi-2-absorbing in K.

Proposition 2.19. *Let E be a submodule of an R-module Y. Then $E + J(Y)$ is a nearly semi-2-absorbing submodule of Y if and only if $\left[E + J(Y) : r^2 y\right] = [E + J(Y) : ry]$ for each $y \in Y$ or $r^2 \in [E + J(Y) : Y]$.*

Proof. $\Rightarrow$Assume that $r^2 \notin [E + J(Y) : Y]$. To prove that $\left[E + J(Y) : r^2 y\right] = [E + J(Y) : ry]$. It is clear that $[E + J(Y) : ry] \subseteq \left[E + J(Y) : r^2 y\right]$. Now, let $a \in \left[E + J(Y) : r^2 y\right]$, then $r^2 ay \in E + J(Y)$ since $E + J(Y)$ is nearly semi-2-absorbing in Y and $r^2 \notin [E + J(Y) : Y]$ so $a \in [E + J(Y) : ry]$. Thus $\left[E + J(Y) : r^2 y\right] = [E + J(Y) : ry]$.

$\Leftarrow$ Let $r^2 y \in E + J(Y)$, by hypothesis $[E + J(Y) : r^2 y] = [E + J(Y) : ry]$ or $r^2 \in [E + J(Y) : Y]$. If $[E + J(Y) : r^2 y] = [E + J(Y) : ry]$, then $[E + J(Y) : r^2 y] = R$ because $r^2 y \in E + J(Y)$. Implies that $[N + J(Y) : ry] = R$ and hance $ry \in E + J(Y)$. Thus either $ry \in E + J(Y)$ or $r^2 \in [E + J(Y) : Y]$. Hence $E + J(Y)$ is nearly semi 2-absorbing in Y.

Proposition 2.20. *Let Y be an R-module, and E be a submodule of Y. Then E is a nearly semi 2-absorbing in Y if and only if $r^2F \subseteq E$, implies that $rF \subseteq E + J(Y)$ or $a^2 \in [E + J(Y) : Y]$.*

Proof. $\Rightarrow$ Assume that $r^2F \subseteq E$, and suppose that there exist $y \in F$ such that $ry \notin E + J(Y)$. Since $r^2F \subseteq E$, so $r^2m \in E$ for each $m \in F$. But E is a nearly semi 2-absorbing in Y, and $ry \notin E + J(Y)$. Hence $r^2 \in [E + J(Y) : Y]$.

$\Leftarrow$ It is clear.

Proposition 2.21. *Let E be a submodule of an R-module Y such that $J(Y) \subseteq E$. If E is nearly semi 2-absorbing of Y, then$[E : Y]$ is a semi 2-absorbing ideal in R.*

Proof. Assume that $a^2b \in [E : Y]$, $a, b \in R$, implies that $a^2by \in E$ for each $y \in Y$. But E is a nearly semi 2-absorbing in Y, then either $aby \in E + J(Y)$ or $a^2 \in [E + J(Y) : Y]$, But $J(Y) \subseteq E$, it follows that $aby \in E$ or $a^2 \in [E : Y]$. That is $ab \in [E : Y]$ or $a^2 \in [E : Y]$.

$\Leftarrow$ Assume that $a^2y \in E$, $a \in R$, $y \in Y$, with the converse Proposition 2.21 hold under the class of cyclic modules.

Proposition 2.22. *Let E be a proper submodule of cyclic module Y. If $[E : Y]$ is semi 2-absorbing ideal of R, then E is a nearly semi 2-absorbing submodule of Y.*

Proof. Assume that $[E : Y]$ is semi 2-absorbing ideal of R then by [3], we have E is a semi 2-absorbing submodule of Y. Hence by Proposition 2.11, we get E is nearly semi 2-absorbing submodule of Y.

Corollary 2.23. *Let E be a proper submodule of cyclic R-module Y with $J(Y) \subseteq E$. Then E is a nearly semi 2-absorbing submodule of Y if and only if $[E : Y]$ is a semi 2-absorbing ideal of R.*

Proposition 2.24. *Let $\varphi : Y \longrightarrow Y$ be an R-epimorphism with $Ker\ \varphi \subseteq E$ whene E is a proper submodule of Y. Then*

1. *If E is a nearly semi 2-absorbing in Y, then $\varphi(E)$ is a nearly semi 2-absorbing submodule in $Y'e$.*
2. *If E^1 is a nearly semi 2-absorbing submodule in $Y'e$ and $Ker\varphi$ is small submodule in Y, then$\varphi^{-1}(E'e)$ is a nearly semi 2-absorbing submodule in Y.*

Proof.

1. $\varphi(E)$ is a proper submodule of $Y'e$, if not, that is $\varphi(E) = Y'e$, then for each $y \in Y$,$\varphi(y) \in Y'e = \varphi(E)$, implies that $\varphi(y) = \varphi(\eta)$ for some $n \in E$, hence $\varphi(y - n) = 0$, then $y - n \in Ker\varphi \subseteq E$, implies that $y \in E$,

hence E=Y contradiction (since $E \subsetneq Y$) Now, assume that $r^2y'e \in \varphi(E)$, $r \in R, y'e \in Y'e$, since φ is onto, then there exist $y \in Y$ such that $\varphi(y) = y'e$ hence $r^2\varphi(y) \in \varphi(E)$, that is $\varphi\left(r^2y\right) \in \varphi(E)$ then there exist $e \in E$ such that $\varphi\left(r^2y\right) = \varphi(e)$, hence $\varphi\left(e - r^2y\right) = 0$, implies that $e - r^2y \in Ker\varphi \subseteq E$, hence $r^2y \in E$, but E is a nearly semi 2-absorbing in Y, then either $ry \in E + J(Y)$ or $r^2 \in [E + J(Y) : Y]$. That is $ry \in E+J(Y)$ or $r^2Y \in E+J(Y)$. It follows $r\varphi(y) \in \varphi(E)+\varphi(J(Y))$ or $r^2\varphi(y) \subseteq \varphi(E)+\varphi(J(Y))$. Hence $ry'e \in \varphi(E)+\varphi(J(Y'e))$ or $r^2y'e \subseteq \varphi(E) + (J(Y'e))$. Hence either $ry'e \in \varphi(E) + J(Y')$ or $r^2 \in [\varphi(E) + J(Y'e) : Y'e]$. Therefore $\varphi(E)$ is a nearly semi 2-absorbing submodule in$Y'e$.

2. It is clear that $\varphi(E'e) \subsetneq Y$.Now, assume that $r^2y \in \varphi^{-1}(E'e)$, $r \in R$, $y \in Y$, then $r^2\varphi(y) \in E'e$, implies that $r^2y'e \in E'e$, $y'e \in Y'e$, since $E'e$ is a nearly semi 2-absorbing in $Y'e$, then either $ry'e \in E'e + J(Y'e)$ or $r^2 \in [E'e + J(Y'e) : Y'e]$. That is either $\varphi(ry) \in E + J(Y'e)$ or $r^2y'e \subseteq E'e + (J(Y'e))$. Thus either $\varphi(ry) \in E + J(Y'e)$or $\varphi(r^2y) \subseteq E'e + J(Y'e)$. It follows that either $ry \in \varphi^{-1}(E'e) + \varphi^{-1}(J(Y'e))$ or $r^2y \subseteq \varphi^{-1}(E'e) + \varphi^{-1}(J(Y'e))$. Hence either $ry \in \varphi^{-1}(E'e) + J(Y)$ or $r^2 \in [\varphi^{-1}(E'e) + J(Y) : Y]$. Therefore $\varphi^{-1}(E'e)$ is a nearly semi 2-absorbing in Y.

Proposition 2.25. *Let E be a proper submodule of an R-module Y such that E is a nearly semi-2-absorbing submodule of Y, then $S^{-1}E$ is a nearly semi-2-absorbing submodule of $S^{-1}R$-module $S^{-1}Y$.*

Proof. Assume that$(\overline{a})^2\overline{y} \in S^{-1}E$ where $\overline{a} = \frac{a}{s_1} \in S^{-1}R$ and $\overline{y} = \frac{y}{s_2} \in S^{-1}Y$, $a \in R$, $y \in Y$, s_1, $s_2 \in S$. Hence $\frac{a^2y}{{s_1}^2s_2} \in S^{-1}E$, implies that $\frac{a^2y}{t} \in S^{-1}E$, t $={s_1}^2s_2$, then there exists $t_1 \in S$ such that $a^2t_1y \in E$. Since E is a nearly semi-2-absorbing in Y, then either $at_1y \in$E+$J(Y)$ or $a^2t_1 \in$ [E+$J(Y) : Y$], it follows that either $\frac{a^2t_1y}{{s_1}^2t_1s_2} \in S^{-1}$ [E+$J(Y)$] or $\frac{a^2t_1}{{s_1}^2t_1} \in S^{-1}$ [E+$J(Y) : Y$]. Hence either $(\overline{a})^2\overline{y} \in S^{-1}E+S^{-1}(J(Y))$ or $(\overline{a})^2 \in \left[S^{-1}(E) + S^{-1}(J(Y)) : S^{-1}Y\right]$, implies that either $(\overline{a})^2\overline{y} \in S^{-1}E + J\left(S^{-1}Y\right)$ or $(\overline{a})^2 \in \left[S^{-1}(E) + J\left(S^{-1}Y\right) : S^{-1}Y\right]$. Therefore $S^{-1}E$ is a nearly semi-2-absorbing submodule of $S^{-1}Y$.

Proposition 2.26. *Let $Y = Y_1 \oplus Y_2$ be an R-module, where Y_1, Y_2 are R-modules, and let E and F be a proper submodules of Y_1 and Y_2 respectively, then*

1. *E is a nearly semi-2-absorbing submodule in Y_1 if and only if $E \oplus Y_2$ is a nearly semi-2-absorbing in Y.*

2. *F is a nearly semi-2-absorbing in Y_2 if and only if $Y_1 \oplus F$ is a nearly semi-2-absorbing in Y.*

Proof. It is easy, we omitted.

Proposition 2.27. *If E and F are nearly semi-2-absorbing submodules in Y_1 and Y_2, respectively such that*$[E + J(Y_1) : Y_1] = [F + J(Y_2) : Y_2]$*. Then* $K = E \oplus F$ *is a nearly semi-2-absorbing submodule in R-module* $Y = Y_1 \oplus Y_2$*, where* Y_1*,* Y_2 *are R-modules.*

Proof. Assume that $r^2(y_1, y_2) \in E \oplus F$, $a \in R$, $(y_1, y_2) \in Y$, $y_1 \in Y_1$, $y_2 \in Y_2$, implies that $r^2 y_1 \in E$ and $r^2 y_2 \in F$. Since E and F are nearly semi-2-absorbing, then either $ry_1 \in E + J(Y_1)$ or $r^2 \in [E + J(Y_1) : Y_1]$ and either $ry_2 \in F + J(Y_2)$ or $r^2 \in [F + J(Y_2) : Y_2] = [E + J(Y_1) : Y_1]$ so $ry_1 \in E + J(Y_1)$ and $ry_2 \in F + J(Y_2)$ or $r^2 \in [E + J(Y_1) : Y_1]$. Thus $r(y_1, y_2) \in (E + J(Y_1)) \int (F + J(Y_2))$ or $r^2 \in [E \oplus F + J(Y_1 + Y_2) : Y_1 + Y_2]$. It follows that either $r(y_1, y_2) \in E \oplus F + J(Y_1 + Y_2)$ or $r^2 \in [E \oplus F + J(Y_1 + Y_2) : Y_1 + Y_2]$.

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