

WEAKLY (I, J) -CONTINUOUS MULTIFUNCTIONS AND CONTRA (I, J) -CONTINUOUS MULTIFUNCTIONS

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Abstract. The purpose of the present paper is to introduce, study and characterize upper and lower weakly (I, J) -continuous multifunctions and contra (I, J) -continuous multifunctions. Also, we investigate its relation with another class of continuous multifunctions.

Keywords: weakly (I, J) -continuous multifunctions, I -open set, I -closed set, contra (I, J) -continuous multifunctions, (I, J) -continuous multifunctions.

1. Introduction

It is well known today, that the notion of multifunction playing a very important role in general topology, upper and lower continuity have been extensively studied on multifunctions $F : (X, \tau) \rightarrow (Y, \sigma)$. Currently using the notion of ideal topological space, different types of upper and lower continuity in a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma)$ have been studied and characterized [2], [8], [9], [15],

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[18]. The concept of ideal topological spaces has been introduced and studied by Kuratowski [12] and the local function of a subset A of a topological space (X, τ) was introduced by Vaidyanathaswamy [17] as follows: given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X , a set operator $(.)^* : P(X) \rightarrow P(X)$, called the local function of A with respect to τ and I , is defined as follows: for $A \subseteq X$, $A^*(\tau, I) = \{x \in X / U \cap A \notin I \text{ for every } U \in \tau_x\}$, where $\tau_x = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $cl^*(,)$ for a topology $\tau^*(\tau, I)$ called the $*$ -topology, finer than τ is defined by $cl^*(A) = A \cup A^*(\tau, I)$. We will denote $A^*(\tau, I)$ by A^* . In 1990, Jankovic and Hamlett[10], introduced the notion of I -open set in a topological space (X, τ) with an ideal I on X . In 1992, Abd El-Monsef et al.[1] further investigated I -open sets and I -continuous functions. In 2007, Akdag [2], introduced the concept of I -continuous multifunctions in a topological space with an ideal on it. In 2007, Al-Omari and Noorani [3], introduced the notions of contra- I -continuous and almost I -continuous functions. Given a multifunction $F : (X, \tau) \rightarrow (Y, \sigma)$, and two ideals I, J associate, now with the topological spaces (X, τ, I) and (Y, σ, J) , consider the multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$. We want to study some type of upper and lower continuity of F as doing Rosas et al. [14]. In this paper, we introduce and study two new classes of multifunctions called a weakly (I, J) -continuous multifunctions and contra (I, J) -continuous multifunctions in topological spaces. Investigate its relation with another classes of continuous multifunctions. Also its relation when the ideal $J = \{\emptyset\}$.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces in which no separation axioms are assumed, unless explicitly stated and if I is an ideal on X , (X, τ, I) mean an ideal topological space. For a subset A of (X, τ) , $Cl(A)$ and $\text{int}(A)$ denote the closure of A with respect to τ and the interior of A with respect to τ , respectively. A subset A is said to be regular open [16] (resp. semiopen [11], preopen[13], semi preopen [4]) if $A = \text{int}(Cl(A))$ (resp. $A \subseteq Cl(\text{int}(A))$, $A \subseteq \text{int}(Cl(A))$, $A \subseteq Cl(\text{int}(Cl(A)))$). The complement of a regular open (resp. semiopen, semi-preopen) set is called regular closed (resp. semiclosed, semi-preclosed) set. A subset S of (X, τ, I) is an I -open[10], if $S \subseteq \text{int}(S^*)$. The complement of an I -open set is called I -closed set. The I -closure and the I -interior, can be defined in the same way as $Cl(A)$ and $\text{int}(A)$, respectively, will be denoted by $I Cl(A)$ and $I \text{int}(A)$, respectively. The family of all I -open (resp. I -closed, regular open, regular closed, semiopen, semi closed, preopen, semi-preclosed) subsets of a (X, τ, I) , denoted by $IO(X)$ (resp. $IC(X)$, $RO(X)$, $RC(X)$, $SO(X)$, $SC(X)$, $PO(X)$, $SPO(X)$, $SPC(X)$). We set $IO(X, x) = \{A : A \in IO(X) \text{ and } x \in A\}$. It is well known that in an ideal topological space (X, τ, I) , $X^* \subseteq X$ but if the ideal is codense, that is $\tau \cap I = \emptyset$, then $X^* = X$.

By a multifunction $F : X \rightarrow Y$, we mean a point-to-set correspondence from X into Y , also we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \rightarrow Y$, the upper and lower inverse of any subset A of Y denoted by $F^+(A)$ and $F^-(A)$, respectively, that is $F^+(A) = \{x \in X : F(x) \subseteq A\}$ and $F^-(A) = \{x \in X : F(x) \cap A \neq \emptyset\}$. In particular, $F^+(y) = \{x \in X : y \in F(x)\}$ for each point $y \in Y$.

Definition 2.1 ([7]). A multifunction $F : (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

1. upper semi continuous at a point $x \in X$ if for each open set V of Y with $x \in F^+(V)$, there exists an open set U containing x such that $F(U) \subseteq V$.
2. lower semi continuous at a point $x \in X$ if for each open set V of Y with $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $F(a) \cap V \neq \emptyset$ for all $a \in U$.

Definition 2.2 ([15]). A multifunction $F : (X, \tau) \rightarrow (Y, \sigma)$ is said to be:

1. upper weakly continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an open set U containing x such that $U \subseteq F^+(Cl(V))$.
2. lower weakly continuous if for each $x \in X$ and each open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists an open set U containing x such that $F(u) \cap Cl(V) \neq \emptyset$ for every $u \in U$.
3. weakly continuous if it is both upper weakly continuous and lower weakly continuous.

Definition 2.3 ([2]). A multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be:

1. upper I -continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I -open set U containing x such that $U \subseteq F^+(V)$.
2. lower I -continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an I -open set U containing x such that $U \subseteq F^-(V)$.
3. I -continuous if it is both upper and lower I -continuous.

Definition 2.4 ([5]). A multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be:

1. upper weakly I -continuous if for each $x \in X$ and each open set V of Y such that $x \in F^+(V)$, there exists an I -open set U containing x such that $U \subseteq F^+(Cl(V))$.
2. lower weakly I -continuous if for each $x \in X$ and each open set V of Y such that $x \in F^-(V)$, there exists an I -open set U containing x such that $U \subseteq F^-(Cl(V))$.
3. weakly I -continuous if it is both upper weakly I -continuous and lower I -weakly continuous.

3. Weakly (I, J) -continuous multifunctions

Definition 3.1. A multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be:

1. upper weakly (I, J) -continuous at a point $x \in X$ if for each J -open set V such that $x \in F^+(V)$, there exists an I -open set U containing x such that $U \subseteq F^+(JCl(V))$
2. lower weakly (I, J) -continuous at a point $x \in X$ if for each J -open set V of Y such that $x \in F^-(V)$, there exists an I -open set U of X containing x such that $U \subseteq F^-(JCl(V))$.
3. upper (resp. lower) weakly (I, J) -continuous on X if it has this property at every point of X .

Example 3.2. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(a) = \{a\}$, $F(b) = \{c\}$ and $F(c) = \{b\}$. It is easy to see that:

The set of all I -open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

The set of all J -open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.

In consequence, F is upper (resp. lower) weakly (I, J) -continuous on X .

Example 3.3. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{b\}\}$ and two ideals $I = J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(a) = \{a\}$, $F(b) = \{c\}$ and $F(c) = \{b\}$. It is easy to see that:

The set of all I -open is $\{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.

The set of all J -open is $\{\emptyset, Y, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. In consequence, F is not upper (resp. lower) weakly (I, J) -continuous.

Recall that if (X, τ, I) is an ideal topological space and I is the empty ideal, then for each $A \subseteq X$, $A^* = cl(A)$, that is to said, every I -open set is a pre-open set, in consequence, if $F : (X, \tau, I) \rightarrow (Y, \sigma, \{\emptyset\})$ is upper weakly $(I, \{\emptyset\})$ -continuous, then F is upper weakly I -continuous.

Example 3.4. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a, c\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset\}$. Define a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(a) = \{b\}$, $F(b) = \{c\}$ and $F(c) = \{a\}$. It is easy to see that:

The set of all I -open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

The set of all J -open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.

$F : (X, \tau, I) \rightarrow (Y, \sigma)$ is upper weakly I -continuous but $F : (X, \tau, I) \rightarrow (Y, \sigma, \{\emptyset\})$ is not upper weakly $(I, \{\emptyset\})$ -continuous.

Now consider (X, τ, I) and (Y, σ, J) two ideals topological spaces. If $J \neq \{\emptyset\}$, then the concepts of upper weakly (I, J) -continuous and upper weakly I -continuous are independent, as we can see in the following examples.

Example 3.5. In the Example 3.4, the multifunction F is upper weakly (I, J) -continuous on X but is not upper weakly I -continuous on X .

Example 3.6. In the Example 3.3, the multifunction F is upper weakly I -continuous on X but is not upper weakly (I, J) -continuous on X .

Remark 3.7. It is easy to see that if $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is a multifunction and $JO(Y) \subset \sigma$ and F is upper (lower) weakly I -continuous, then F is upper (lower) weakly (I, J) -continuous. Even more, if $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is a multifunction and $JO(Y) \not\subseteq \sigma$, we can find upper (resp. lower) weakly (I, J) -continuous on X that are not upper (lower) weakly I -continuous.

The following theorem characterize the upper weakly (I, J) continuous multifunctions.

Theorem 3.8. For a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$, the following statements are equivalent:

1. F is upper weakly (I, J) -continuous.
2. $F^+(V) \subseteq \text{Int}(F^+(J \text{Cl}(V)))$ for any J -open set V of Y .
3. $I \text{Cl}(F^-(J \text{Int}(B))) \subset F^-(B)$ for any every J -closed subset B of Y .

Proof. (1) \Rightarrow (2): Let $x \in F^+(V)$ and V be any J - open set of Y . From (1), there exists an I -open set U_x containing x such that $U_x \subset F^+(J \text{Cl}(V))$. It follows that $x \in \text{Int}(F^+(J \text{Cl}(V)))$, in consequence, $F^+(V) \subseteq \text{Int}(F^+(J \text{Cl}(V)))$ for any J -open set V of Y .

(2) \Rightarrow (1): Let V any J -open subset of Y such that $x \in F^+(V)$. By (2), $x \in F^+(V) \subseteq \text{Int}(F^+(J \text{Cl}(V))) \subseteq F^+(J \text{Cl}(V))$. Choose $U = \text{Int}(F^+(J \text{Cl}(V)))$. U is an I -open subset of X , containing x . It follows that F is upper weakly (I, J) -continuous.

(2) \Rightarrow (3): Let B be any J - closed set of Y .

Then by (2), $F^+(Y \setminus B) = X \setminus F^-(B) \subseteq \text{Int}(F^+(J \text{Cl}(Y \setminus B)))$
 $= \text{Int}(F^+(J \text{Cl}(Y \setminus \text{Int}(B)))) = X \setminus I \text{Cl}(F^-(J \text{Int}(B)))$.

Thus, $I \text{Cl}(F^-(J \text{Int}(B))) \subset F^-(B)$.

(3) \Rightarrow (2): Let V be any J - open set of Y . Then by (3), $I \text{Cl}(F^-(J \text{Int}(Y \setminus V))) \subset F^-(Y \setminus V) = X \setminus F^+(V)$.

It follows that $I \text{Cl}(X \setminus F^+(I \text{Cl}(V))) = I \text{Cl}(F^-(Y \setminus I \text{Cl}(V)))$
 $= I \text{Cl}(F^-(J \text{Int}(Y \setminus V))) \subset X \setminus F^+(V)$, and then $X \setminus \text{Int}(F^+(I \text{Cl}(V)))$
 $\subseteq X \setminus F^+(V)$. Therefore the result follows. \square

Theorem 3.9. For a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$, the following statements are equivalent:

1. F is lower weakly (I, J) -continuous.
2. $F^-(V) \subseteq \text{Int}(F^-(J \text{Cl}(V)))$ for any J -open set V of Y .

3. $I\text{Cl}(F^+(J\text{int}(B))) \subset F^+(B)$ for any every J -closed subset B of Y .

Proof. The proof is similar to that of Theorem 3.8. \square

Definition 3.10 ([14]). A multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be:

1. upper (I, J) -continuous at a point $x \in X$ if for each J -open set V containing $F(x)$, there exists an I -open set U containing x such that $F(U) \subset V$.
2. lower (I, J) -continuous at a point $x \in X$ if for each J -open set V of Y meeting $F(x)$, there exists an I -open set U of X containing x such that $F(u) \cap V \neq \emptyset$ for each $u \in U$.
3. upper (resp. lower) (I, J) -continuous on X if it has this property at every point of X .

Example 3.11. The multifunction defined in Example 3.2 is upper weakly (I, J) -continuous on X but is not upper (I, J) -continuous on X .

Remark 3.12. Every upper (resp. lower) (I, J) -continuous multifunction on X is upper (resp. lower) weakly (I, J) -continuous multifunction on X , but the converse is not necessarily true, as we can see in the following example.

Example 3.13. Let $X = \mathbb{R}$ the set of real numbers with the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$, $Y = \mathbb{R}$ with the topology $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $I = \{\emptyset\} = J$. Define $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(x) = \mathbb{Q}$ if $x \in \mathbb{Q}$ and $F(x) = \mathbb{R} \setminus \mathbb{Q}$ if $x \in \mathbb{R} \setminus \mathbb{Q}$. Recall that in this case the I -open sets are the preopen sets. f is upper (resp. lower) weakly (I, J) -continuous on X , but is not upper (resp. lower) (I, J) -continuous on X .

Theorem 3.14 ([14]). For a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$, the following statements are equivalent:

1. F is upper (I, J) -continuous.
2. $F^+(V)$ is I -open for each J -open set V of Y .
3. $F^-(K)$ is I -closed for every J -closed subset K of Y .
4. $I\text{Cl}(F^-(B)) \subset F^-(J\text{Cl}(B))$ for every subset B of Y .
5. For each point $x \in X$ and each J -open set V containing $F(x)$, $F^+(V)$ is an I -open containing x .

There exist any additional condition in order to say that every upper (resp. lower) (I, J) -continuous if upper (resp. lower) weakly (I, J) -continuous.

Theorem 3.15. Let $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a multifunction such that $F(x)$ is a J -open subset of Y for each $x \in X$. Then F is lower (I, J) -continuous if and only if lower weakly (I, J) -continuous.

Proof. Let $x \in X$ and V any J -open subset of Y such that $x \in F^-(V)$. Then there exists an I -open subset U of X containing x such that $U \subset F^-(J Cl(V))$. It follows that $F(u) \cap J Cl(V) \neq \emptyset$ for each $u \in U$. Since $F(u)$ is a J -open subset of Y for each $u \in U$, it follows that $F(u) \cap V \neq \emptyset$ and then F is lower (I, J) -continuous. The converse is clear because every (I, J) -continuous multifunction is weakly (I, J) -continuous. \square

Theorem 3.16. *Let $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ be a multifunction such that $F(x)$ is a J -open subset of Y for each $x \in X$. Then F is upper (I, J) -continuous if and only if upper weakly (I, J) -continuous.*

Proof. The proof is similar to the above Theorem. \square

4. Contra (I, J) -continuous multifunctions

Definition 4.1. *A multifunction $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is said to be:*

1. *upper contra (I, J) -continuous if for each $x \in X$ and each J -closed set V such that $x \in F^+(V)$, there exists an I -open set U containing x such that $F(U) \subset V$.*
2. *lower contra (I, J) -continuous if for each $x \in X$ and each J -closed set V of Y such that $x \in F^-(V)$, there exists an I -open set U of X containing x such that $U \subseteq F^-(V)$.*
3. *Contra (I, J) -continuous if it is upper contra (I, J) -continuous and lower contra (I, J) -continuous.*

Example 4.2. Let $X = \mathbb{R}$ the set of real numbers with the topology $\tau = \{\emptyset, \mathbb{R}, \mathbb{R} \setminus \mathbb{Q}\}$, $Y = \mathbb{R}$ with the topology $\sigma = \{\emptyset, \mathbb{R}, \mathbb{Q}\}$ and $I = \{\emptyset\} = J$. Define $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(x) = \mathbb{Q}$ if $x \in \mathbb{Q}$ and $F(x) = \mathbb{R} \setminus \mathbb{Q}$ if $x \in \mathbb{R} \setminus \mathbb{Q}$. Recall that in this case the I -open sets are the preopen sets. It is easy to see that F is upper (resp. lower) contra (I, J) -continuous.

Example 4.3. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset, \{b\}\}$. Define a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(a) = \{b\}$, $F(b) = \{a\}$ and $F(c) = \{c\}$. It is easy to see that:

The set of all I -open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

The set of all J -open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.

The set of all J -closed is $\{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$.

In consequence, f is upper (resp. lower) (I, J) -continuous on X but is not upper (resp. lower) contra (I, J) -continuous.

Example 4.4. The multifunction F defined in Example 4.2 is upper (resp. lower) contra (I, J) -continuous but is not upper (resp. lower) (I, J) -continuous on X and the multifunction F defined in Example 4.3 is upper (resp. lower)

(I, J) -continuous but is not upper (resp. lower) contra (I, J) -continuous. In consequence both concepts are independent of each other.

Theorem 4.5. *For a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$, the following statements are equivalent:*

1. F is upper contra (I, J) -continuous.
2. $F^+(V)$ is I -open for each J -closed set V of Y .
3. $F^-(K)$ is I -closed for every J -open subset K of Y .

Proof. (1) \Leftrightarrow (2): Let $x \in F^+(V)$ and V be any J -closed set of Y . From (1), there exists an I -open set U_x containing x such that $U_x \subset F^+(V)$. It follows that $F^+(V) = \bigcup_{x \in F^+(V)} U_x$. Since any union of I -open sets is I -open, $F^+(V)$ is

I -open in (X, τ) . The converse is similar.

(2) \Leftrightarrow (3): Let K be any J -open set of Y . Then $Y \setminus K$ is a J -closed set of Y by (2), $F^+(Y \setminus K) = X \setminus F^-(K)$ is an I -open set. Then it is obtained that $F^-(K)$ is an I -closed set. The converse is similar. \square

Theorem 4.6. *For a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$, the following statements are equivalent:*

1. F is lower contra (I, J) -continuous.
2. $F^-(V)$ is I -open for each J -closed set V of Y .
3. $F^+(K)$ is I -closed for every J -open subset K of Y .
4. For each $x \in X$ and each J -closed set K of Y such that $F(x) \cap K \neq \emptyset$, there exists an I -open set U containing x such that $F(y) \cap K \neq \emptyset$ for each $y \in U$.

Proof. The proof is similar to the proof of Theorem 4.5. \square

Remark 4.7. It is easy to see that if $J = \{\emptyset\}$ and $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is upper (resp. lower) contra (I, J) -continuous then F is upper (resp. lower) contra I -continuous.

The following example shows the existence of upper (resp. lower) contra I -continuous that is not upper (resp. lower) contra $(I, \{\emptyset\})$ -continuous.

Example 4.8. Let $X = Y = \{a, b, c\}$ with two topologies $\tau = \{\emptyset, X, \{b\}\}$ $\sigma = \{\emptyset, Y, \{a, c\}\}$ and two ideals $I = \{\emptyset, \{a\}\}$, $J = \{\emptyset\}$. Define a multifunction $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ as follows: $F(a) = \{c\}$, $F(b) = \{b\}$ and $F(c) = \{a\}$. It is easy to see that:

The set of all I -open is $\{\emptyset, X, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

The set of all J -open is $\{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, Y\}$.

The set of all J -closed is $\{\emptyset, \{b\}, \{c\}, \{a, b\}, \{b, c\}, Y\}$.

Observe that $F : (X, \tau, I) \rightarrow (Y, \sigma)$ is upper contra I -continuous but $F : (X, \tau, I) \rightarrow (Y, \sigma, \{\emptyset\})$ is not upper contra $(I, \{\emptyset\})$ -continuous.

Remark 4.9. It is easy to see that if $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is a multifunction and $JO(Y) \subset \sigma$. If F is upper (lower) contra I -continuous, then F is upper (lower) (I, J) -continuous. Even more, if $F : (X, \tau, I) \rightarrow (Y, \sigma, J)$ is a multifunction and $JO(Y) \not\subset \sigma$, we can find upper (resp. lower) contra (I, J) -continuous on X that are not upper (lower) contra I -continuous.

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