Z_3-CONNECTED GRAPHS WITH NEIGHBORHOOD UNIONS AND MINIMUM DEGREE CONDITION

Liangchen Li*
Department of Mathematics
Luoyang Normal University
Luoyang 471022
China
liangchen_li@163.com

Fan Yang
School of Mathematics and Physics
Jiangsu University of Science and Technology
Zhenjiang 212003
China

Abstract. Let G be a 2-edge-connected simple graph on $n \geq 15$ vertices, and let A denote an abelian group with the identity element 0. If a graph G^* is obtained by repeatedly contracting nontrivial A-connected subgraphs of G until no such a subgraph left, we say G can be A-reduced to G^*. In this paper, we prove that if for every $uv \notin E(G)$, $|N(u) \cup N(v)| + \delta(G) \geq n$, then G is not Z_3-connected if and only if G can be Z_3-reduced to one of $\{C_3, K_4, K_4^-, L\}$, where L is obtained from K_4 by adding a new vertex which is joined to two vertices of K_4. Our results extend the early theorem by Li et al. (Graphs and Combin., 29 (2013): 1891-1898).

Keywords: neighborhood unions, minimum degree, Z_3-connectivity, 3-flow.

1. Introduction

Graphs in this paper are finite, loopless, and may have multiple edges. Terminology and notation not defined here are from [1].

For $S \subseteq V(G)$, let $N_S(v)$ denote the set of vertices in S that are adjacent to v in G and $d_S(v) = |N_S(v)|$. If $S = V(G)$, we write $N(v) = N_G(v)$, $N[v] = N(v) \cup \{v\}$ and $d(v) = d_G(v)$. For a vertex v, $N(v)$ is called the *neighborhood* of v. For two subsets $A, B \subseteq V(G)$, let $e_G(A, B)$ ($e(A, B)$ for short) denote the number of edges with one endpoint in A and the other endpoint in B. For simplicity, if H_1 and H_2 are two subgraphs of G, we write $e(H_1, H_2)$ instead of $e(V(H_1), V(H_2))$. A complete graph on n vertices is denoted by K_n, and K_n^- is the graph obtained from K_n by deleting one edge. A *k-cycle*, denoted by C_k, is a cycle of length k. For simplicity, we use δ to denote $\delta(G)$, the minimum degree of G.

∗. Corresponding auhtor

Let G be a graph, and let D be an orientation of G. If an edge $e \in E(G)$ is directed from a vertex u to a vertex v, then let *tail*$(e) = u$ and *head*$(e) = v$. For a vertex $v \in V(G)$, let $E^+(v)$ denote the set of edges with tail v and $E^-(v)$ the set of edges with head v. Let A denote an (additive) abelian group with the identity element 0 and let $A^* = A - \{0\}$. We define $F(G, A) = \{f | f : E(G) \to A\}$ and $F^*(G, A) = \{f | f : E(G) \to A^*\}$.

Given a function $f \in F(G, A)$, define $\partial f : V(G) \to A$ by

$$\partial f(v) = \sum_{e \in E^+(v)} f(e) - \sum_{e \in E^-(v)} f(e),$$

where "$\sum$" refers to the addition in A. The value $\partial f(v)$ is known as the *net flow out of* v *under* f.

For a graph G, a function $b : V(G) \to A$ is an A*-valued zero-sum function* on G if $\sum_{v \in V(G)} b(v) = 0$. The set of all A-valued zero-sum functions on G is denoted by $\mathbb{Z}(G, A)$. Given $b \in \mathbb{Z}(G, A)$, a function $f \in F^*(G, A)$ is an (A, b)*-nowhere-zero flow* if G has an orientation D such that $\partial f = b$. A graph G is A*-connected* if for every $b \in \mathbb{Z}(G, A)$, G admits an (A, b)-nowhere-zero flow. A *nowhere-zero* A*-flow* is an $(A, 0)$-nowhere-zero flow. More specifically, a *nowhere-zero* k*-flow* is a nowhere-zero Z_k-flow, where Z_k is the cyclic group of order k. Tutte [18] proved that G admits a nowhere-zero A-flow with $|A| = k$ if and only if G admits a nowhere-zero k-flow.

An edge is *contracted* if it is deleted and its two ends are identified into a single vertex. Let H be a connected subgraph of G. Let G/H denote the graph obtained from G by contracting all edges of H and deleting all the loops. A graph G is A*-reduced* if it contains no nontrivial A-connected subgraph. We say that a graph G^* is an A*-reduction* of G if G^* is A-reduced and if G^* can be obtained from G by contracting all maximally A-connected subgraphs of G. It is known that the A-reduction of a graph is A-reduced and an A-reduction of a reduced graph is itself.

Integer flow problems were introduced by Tutte [17, 18]. Group connectivity was introduced by Jaeger *et al.* [7] as a generalization of nowhere-zero flows. The following conjecture is due to Jaeger *et al.*.

Conjecture 1.1. ([7]) *Every* 5*-edge-connected graph is* Z_3*-connected.*

Recently, Thomassen [16] confirmed the weak 3-flow conjecture, and Lovász *et al.* [13] proved that every 6-edge-connected graph is Z_3-connected. However, Conjecture 1.1 is still open.

On the other hand, degree conditions, local structure and forbidden subgraphs are used to investigate the existence of nowhere-zero 3-flows and Z_3-connectivity of graphs. One can find sufficient conditions for the existence of nowhere-zero 3-flows and Z_3-connectivity, and such conditions are related with ones for hamiltonian graphs. It is known that every graph which contains a hamiltonian cycle admits a nowhere-zero 4-flow and there are infinite graphs containing a hamiltonian cycle do not admit a nowhere-zero 3-flow [15]. For the literature, some results can be seen in [8, 14, 19, 20, 21].

In this paper, we still focus on the neighborhood unions condition, which was first introduced by Faudree *et al.* [6] as sufficient conditions for the existence of hamiltonian graphs. Faudree *et al.* [6] proved that if G is a 2-connected simple graph on $n \geq 3$ vertices such that $|N(u) \cup N(v)| \geq (2n-1)/3$ for each pair of nonadjacent vertices u and v, then G is hamiltonian. For this Faudree *et al.*'s result, the first author and X. Li proved that if $|N(u) \cup N(v)| \geq \lceil \frac{2n}{3} \rceil$ for any pair of nonadjacent vertices u and v, then G is Z_3-connected if and only if G cannot be Z_3-reduced one of four specified graphs $\{C_3, K_4, K_4^-, L\}$, where G is a 2-edge-connected graph. On the other hand, Faudree *et al.* [5] proved that if G is a graph on n vertices such that $|N(u) \cup N(v)| + \delta \geq n$ for each pair of nonadjacent vertices u and v, then G is hamiltonian, which improved the result of Faudree *et al.* [6]. Motivated by above observations, we present the following theorem in this paper.

Theorem 1.2. *Let G be a 2-edge-connected simple graph on $n \geq 15$ vertices. If $|N(u) \cup N(v)| + \delta \geq n$ for every $uv \notin E(G)$, then G is not Z_3-connected if and only if G can be Z_3-reduced to one of $\{C_3, K_4, K_4^-, L\}$, where L is obtained from K_4 by adding a new vertex which is joined to two vertices of K_4.*

2. Proof of the main result

For simplicity, define $\mathcal{F}$ to be the set of all 2-edge-connected simple graphs on $n \geq 15$ vertices such that $G \in \mathcal{F}$ if and only if $|N(u) \cup N(v)| + \delta \geq n$ for each $uv \notin E(G)$.

In order to prove Theorem 1.2, we need some lemmas. Some results [2, 3, 8, 9] on group connectivity are summarized as follows.

Lemma 2.1 ([2, 3, 8, 9])**.** *Let A be an abelian group. Then the following results are known:*

(1) *K_1 is A-connected.*

(2) *If $e \in E(G)$ and if G is A-connected, then G/e is A-connected.*

(3) *If H is a subgraph of G and if both H and G/H are A-connected, then G is A-connected.*

(4) *Each even wheel is Z_3-connected and each odd wheel is not.*

(5) *Let G be a simple graph and H a nontrivial subgraphs of G. If H is Z_3-connected, then $|V(H)| \geq 5$.*

(6) *Let H be a Z_3-connected subgraph of G. If $e(v, V(H)) \geq 2$ for $v \in V(G-H)$, then the subgraph induced by $V(H) \cup \{v\}$ is Z_3-connected.*

Let G be a graph and let u, v, w be three vertices of G with $uv, uw \in E(G)$. $G_{[uv,uw]}$ is defined to be the graph obtained from G by deleting two edges uv and uw and adding one edge vw. It is clear that $d_{G_{[uv,uw]}}(u) = d(u) - 2$.

Lemma 2.2 ([2, 9])**.** *Let A be an abelian group. Let G be a graph and let u, v, w be three vertices of G with $d(u) \geq 4$ and $uv, uw \in E(G)$. If $G_{[uv,uw]}$ is A-connected, then so is G.*

Next we give two Theorems of Z_3-connectivity about degree conditions, which are important to prove our main Theorem.

Theorem 2.3 (Theorem 1.8 of [14])**.** *If G is a simple graph satisfying the Ore-condition with at least three vertices, then G is not Z_3-connected if and only if G is one of the 12 specified graphs shown in Fig. 1.*

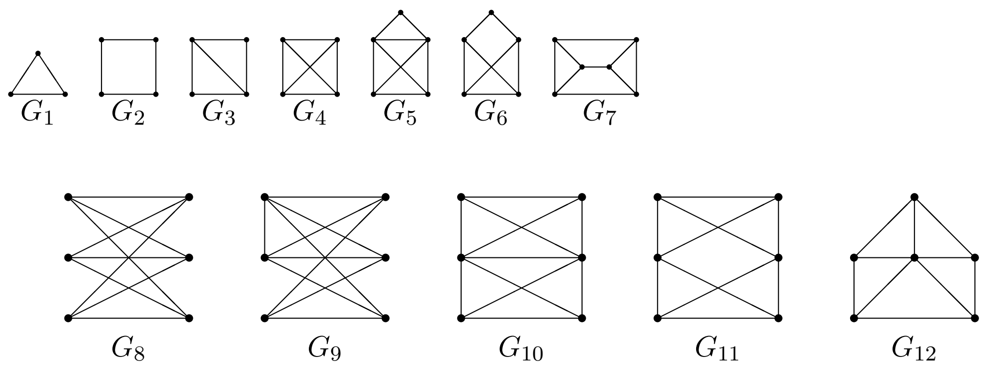

Fig. 1: 12 specified graphs for Theorem 2.3

Theorem 2.4 (Theorem 1.5 of [10])**.** *Let G be a 2-edge-connected simple graphs on $n \geq 14$ vertices. If $|N(u) \cup N(v)| \geq \lceil \frac{2n}{3} \rceil$ for every $uv \notin E(G)$, then G is not Z_3-connected if and only if G can be Z_3-reduced to one of $\{C_3, K_4, K_4^-, L\}$.*

Lemma 2.5 ([13])**.** *Every 6-edge-connected graph is Z_3-connected.*

Before proving Theorem 1.2, we summarize some characterizes of graphs in $\mathcal{F}$ with $\delta \geq \lfloor \frac{n}{3} \rfloor + 1$.

Lemma 2.6. *Suppose that $G \in \mathcal{F}$ with $\delta \geq \lfloor \frac{n}{3} \rfloor + 1$. If G contains a nontrivial Z_3-connected subgraph, then G is Z_3-connected.*

Proof. Assume that H is the maximum nontrivial Z_3-connected subgraph of G. If $H = G$, then we are done. Otherwise H is a proper subgraph of G. Let $G' = G/H$ and let v' denote the new vertex which H is contracted to. By the choice of H, each vertex of $V(G - H)$ has at most one neighbor in $V(H)$. It follows that G' is a simple graph. Since G is 2-edge-connected, G' is 2-edge-connected, and so $d_{G'}(v') \geq 2$.

We claim that $|V(H)| > \lfloor \frac{n}{3} \rfloor + 1$. Firstly, we prove it for $n \geq 21$. Suppose otherwise that $|V(H)| \leq \lfloor \frac{n}{3} \rfloor + 1$. By Lemma 2.1(5), $5 \leq |V(H)| \leq \lfloor \frac{n}{3} \rfloor + 1$. Assume $|V(H)| = t$. Thus H contains at most $t(t-1)/2$ edges. Since $\delta \geq \lfloor \frac{n}{3} \rfloor + 1$, $d_{G'}(v') \geq t(\lfloor \frac{n}{3} \rfloor + 1) - t(t-1) = t(\lfloor \frac{n}{3} \rfloor + 2) - t^2$. Define a real value function

$f(t) = t(\lfloor \frac{n}{3} \rfloor + 2) - t^2 - (n-t) = t(\lfloor \frac{n}{3} \rfloor + 3) - t^2 - n$, where $t \in [5, \lfloor \frac{n}{3} \rfloor + 1]$. When $t \in [5, \lfloor \frac{n}{3} \rfloor - 1]$, it is easy to verify that $f(t) > 0$. In this case, we get $d_{G'}(v') > n - t = |V(G' - v')|$. This contradicts that G' is a simple graph. This implies that $t = \lfloor \frac{n}{3} \rfloor$ or $\lfloor \frac{n}{3} \rfloor + 1$. We firstly assume that $t = \lfloor \frac{n}{3} \rfloor$. In this case, note that $f(\lfloor \frac{n}{3} \rfloor) = 3\lfloor \frac{n}{3} \rfloor - n$ and $d_{G'}(v') \geq 2\lfloor \frac{n}{3} \rfloor \geq 14$. Let u and v be two adjacent vertices of $N(v')$. By the choice of H and Lemma 2.1 (4), $|(N_{G'}(u) \cap N_{G'}(v)) \cap N(v')| \leq 1$. When $(N_{G'}(u) \cap N_{G'}(v)) \cap N(v') = \{w\}$, then $N_{G'}(u) \cup N_{G'}(v)$ has $2\lfloor \frac{n}{3} \rfloor - 4$ vertices in $N(v')$ other than w since $d_{G'}(u) + d_{G'}(v) \geq 2\lfloor \frac{n}{3} \rfloor + 2$. It is easy to see that $G'_{[uw,vw]}$ contains a 2-cycle. Iteratively contracting 2-cycles generated in the processing leads eventually to a K_1, which is Z_3-connected. By Lemma 2.2 and 2.1(3), G is Z_3-connected. When $(N_{G'}(u) \cap N_{G'}(v)) \cap N(v') = \emptyset$, we know that $|N_{G'}(u) \cup N_{G'}(v)| \geq 2\lfloor \frac{n}{3} \rfloor$. Let z be a neighbor of u in $N(v')$. It is easy to see that $G'_{[zu,zv']}$ contains a 2-cycle. Iteratively contracting 2-cycles generated in the processing leads eventually to a K_1, which is Z_3-connected. Therefore G is Z_3-connected by Lemmas 2.2 and 2.1(3). Now we assume that $|V(H)| = \lfloor \frac{n}{3} \rfloor + 1$. Clearly $G' - v' = G - H$. In this case, $d_{G'}(v') \geq (\lfloor \frac{n}{3} \rfloor + 1)(\lfloor \frac{n}{3} \rfloor + 2) - (\lfloor \frac{n}{3} \rfloor + 1)^2 = \lfloor \frac{n}{3} \rfloor + 1$. Hence $d_{G'}(x) + d_{G'}(y) \geq 2(\lfloor \frac{n}{3} \rfloor + 1) \geq n - (\lfloor \frac{n}{3} \rfloor + 1) + 1 \geq |G'|$ for each two nonadjacent vertices x and y in G' and $|V(G')| \geq d_{G'}(v) + 1 \geq \lfloor \frac{n}{3} \rfloor + 2 \geq 7$. By Lemmas 2.3 and 2.1 (3), G is Z_3-connected.

Now we claim that $|V(H)| \geq \lfloor \frac{n}{3} \rfloor + 2$ for $15 \leq n \leq 20$. Similarly, we get $5 \leq |V(H)| \leq \lfloor \frac{n}{3} \rfloor + 1$. For $15 \leq n \leq 17$, note that $\lfloor \frac{n}{3} \rfloor = 5$. In this case, the proof is similarly to the case $|V(H)| = \lfloor \frac{n}{3} \rfloor, \lfloor \frac{n}{3} \rfloor + 1$ for $n \geq 21$. Therefore $|V(H)| \geq \lfloor \frac{n}{3} \rfloor + 2$ for $15 \leq n \leq 17$. For $18 \leq n \leq 20$, we firstly verify that $|V(H)| \neq 5$. Suppose otherwise that $|V(H)| = 5$. Since $\delta \geq 7$, $d(v') \geq 15$. If $n = 18$ or 19, then $d(v') \geq |G - H|$. It contradicts that G' is simple. If $n = 20$, then we get $N(v') = V(G) - V(H)$. Let $x, y \in N(v')$ be two adjacent vertices in G'. Consider the graph $G'_{[xv',xy]}$. It is easy to see that $G'_{[xv',xy]}$ contains at least five 2-cycles with one common vertex v'. Iteratively contracting 2-cycles generated in the processing leads eventually to the graph G''. Denote the new vertex by v''. If $G'' = K_1$, then G' is Z_3-connected by Lemmas 2.2 and 2.1 (3). We may assume that $G'' \neq K_1$. It is easy to verify that $d_{G''-v''}(v) \geq 6$ for $v \in V(G'') - \{x, v''\}$. This implies that $G'' - v''$ satisfies Ore-condition. Therefore, $G'' - v''$ is Z_3-connected by Theorem 2.3. By Lemmas 2.1 and 2.2, G' is Z_3-connected. Thus, G is Z_3-connected by Lemma 2.1. Then we get $|V(H)| \geq \lfloor \frac{n}{3} \rfloor = 6$ for $18 \leq n \leq 20$. When $|V(H)| = \lfloor \frac{n}{3} \rfloor$or $\lfloor \frac{n}{3} \rfloor + 1$ for $18 \leq n \leq 20$, the proof is similarly to the case $n \geq 21$. Therefore, $|V(H)| \geq \lfloor \frac{n}{3} \rfloor + 2$.

Thus, we may assume that $|V(H)| \geq \lfloor \frac{n}{3} \rfloor + 2$. Note that $|V(G' - v')| = n - |V(H)| \leq n - \lfloor \frac{n}{3} \rfloor - 2 = \lceil \frac{2n}{3} \rceil - 2$. Since $e(v, H) \leq 1$ for each $v \in V(G - H)$ and $n \geq 15$, $\delta(G' - v') \geq \lfloor \frac{n}{3} \rfloor \geq 5$. Hence $d_{G'-v'}(x) + d_{G'-v'}(y) \geq 2\delta(G' - v') \geq 2\lfloor \frac{n}{3} \rfloor \geq |V(G' - v')|$ for every two nonadjacent vertices x and y of $G' - v'$. Hence $G' - v'$ satisfies the Ore-condition. Since $\delta(G' - v') \geq 5$, $G' - v'$ is Z_3-connected by Lemma 2.3. For $e(v', V(G-H)) \geq 2$, G' is Z_3-connected by Lemma 2.1 (6). Therefore, G is Z_3-connected by Lemma 2.1 (3). □

Lemma 2.7. *Let $G \in \mathcal{F}$ and $\delta \geq \lfloor \frac{n}{3} \rfloor + 1$. If G contains no Z_3-connected subgraph, then G is 6-edge-connected.*

Proof. Suppose that $E_0 = (X, Y)$ is minimum edge cut of the graph G such that $|X|$ is smallest. If $e(X, Y) \geq 6$, then we have done. Otherwise we assume that $2 \leq e(X, Y) \leq 5$. Now we claim that $G[X]$ contains a Z_3-connected subgraph. Note that $\lfloor \frac{n}{3} \rfloor + 1 \leq |X| \leq \frac{n}{2}$. Without loss of generality, we assume that $x_1, x_2, \ldots, x_l \in X$ are incident to the edge of E_0, where $1 \leq l \leq 5$. When $l = 1$, we consider the graph $H = G[X - \{x_1\}]$. Since $X - x_1$ is not adjacent to any vertex of Y, $\delta(H) \geq \lfloor \frac{n}{3} \rfloor \geq 5$. Thus, $d_H(x) + d_H(y) \geq 2\lfloor \frac{n}{3} \rfloor \geq |H|$ for nonadjacent two vertices x, y in H. By Theorem 2.3, H is Z_3-connected. When $l = 2$, we consider the graph $H = G[X - \{x_1, x_2\}]$. In this case $\delta(H) \geq \lfloor \frac{n}{3} \rfloor - 1 \geq 4$. Therefore, for nonadjacent two vertices x, y in H, $d_H(x) + d_H(y) \geq 2(\lfloor \frac{n}{3} \rfloor - 1) = 2\lfloor \frac{n}{3} \rfloor - 2 \geq |H|$. Thus H is Z_3-connected by Theorem 2.3. When $l = 3, 4, 5$, it is easy to verify that $G[X]$ satisfies the Ore-condition. Therefore, by Theroem 2.3, $G[X]$ is Z_3-connected. It contradicts that G contains no Z_3-connected subgraph. This complete the proof of the lemma. □

Proof of Theorem 1.2 If $|N(u) \cup N(v)| \geq \lceil \frac{2n}{3} \rceil$ for every pair of nonadjacent vertices u and v of G, then G is Z_3-connected or can be Z_3-reduced to one of $\{C_3, K_4, K_4^-, L\}$ by Theorem 2.4. Therefore, in the following, we may assume that there are at least a pair of nonadjacent vertices u and v such that $|N(u) \cup N(v)| \leq \lceil \frac{2n}{3} \rceil - 1$.

Since G is 2-edge-connected, $\delta \geq 2$. When $2 \leq \delta \leq \lfloor \frac{n}{3} \rfloor$, $|N(u) \cup N(v)| \geq \lceil \frac{2n}{3} \rceil$ for each $uv \notin E(G)$. In this case we are done. Therefore, without loss of generality, we may assume that $\delta(G) \geq \lfloor \frac{n}{3} \rfloor + 1$. If G contains a nontrivial Z_3-connected graph, then G is Z_3-connected by Lemma 2.6. If G contains no nontrivial Z_3-connected graph, then, by Lemma 2.7, G is 6-edge-connected. Thus, by Lemma 2.5, G is Z_3-connected. This complete the proof of the theorem.

Acknowledgements

The research of the first author is supported by NSFC (Grant No. 11301254), the Youth Backbone Teacher Foundation of Henan's University (Grant No. 2015GGJS-115) and Innovation Scientists Technicians Troop Construction Projects of Henan Province (Grant No. C20150027). The second author is supported by NSFC (Grant No. 11501256).

References

[1] J. A. Bondy, U. S. R. Murty, *Graphs theroy with applications*, Macmillan Press, New York, 1976.

[2] J. Chen, E. Eschen, H.-J. Lai, *Group connectivity of certain graphs*, Ars Combin., 89 (2008), 217–227.

[3] M. Devos, R. Xu, G. Yu, *Nowhere-zero Z_3-flows through Z_3-connectivity*, Discrete Math., 306 (2006), 26–30.

[4] G, Fan, C. Zhou, *Ore condition and nowhere-zero 3-flows,* Discrete Math. SIAM J., 22 (2008) ,288–294.

[5] R.J. Faudree, R.J. Gould, M.S. Jacobson, L. Lesnian, *Neighborhood unions and highly Hamilton graphs,* Ars Combin., 31 (1991), 139-148.

[6] R.J. Faudree, R.J. Gould, M.S. Jacobson, R.H. Schelp, *Neighborhood unions and Hamiltonian properties in graphs*, J. Combin. Theory, Ser. B, 47 (1989), 1-9.

[7] F. Jaeger, N. Linial, C. Payan, N. Tarsi, *Group connectivity of graphs-a nonhomogeneous analogue of nowhere zero flow properties*, J. Combin. Theory, Ser. B, 56 (1992), 165-182.

[8] H.-J. Lai, X. Li, Y. Shao, M. Zhan, *Group connectivity and group colorings of graphs-a survey*, Acta Math. Sinica, doi:10.1007/s10114-110-9746-3.

[9] H.-J. Lai, *Group connectivity of 3-edge-connected chordal graphs*, Graphs and Combin., 16 (2000), 165–176.

[10] L. Li, X. Li, *Neighborhood unions and Z_3-connectivity in graphs*, Graphs and Combinatorics, 29 (2013) 1891–1898.

[11] X. Li, Y. Liu, *Nowhere-zero 3-flows and Z_3-connectivity of graphs without two forbidden subgraphs*, Discrete Math., accepted.

[12] L. Li, X. Li, *Nowhere-zero 3-flows and Z_3-connectivity in bipartite graphs*, Discrete Math., 312(2012), 2238–2251.

[13] L. Lovász, C. Thomassen, Y. Wu, C. Zhang, *Nowhere-zero 3-flows and modulo k-orientations*, J. Combin. Theory, Ser. B, 103 (2013), 587-598.

[14] R. Luo, R. Xu, J. Yin, G. Yu, *Ore condition and Z_3-connectivity*, European J. Combin., 29 (2008) 1587-1595.

[15] J. Ma, X. Li, *Nowhere-Zero* 3*-flows of claw-free graphs*, Discrete Math., 336 (2014), 57–68.

[16] C. Thomassen, *The weak 3-flow conjecture and the weak circular flow conjecture*, J. Combin. Theory, Ser. B, 102 (2012), 521–529.

[17] W. T. Tutte, *A contribution on the theory of chromatic polynomial*, Canad. J. Math., 6 (1954), 80–91.

[18] W. T. Tutte, *On the algebraic theory of graph colorings*, J. Combin. Theory, 1 (1966), 15–50.

[19] X. Yao, X. Li, H. -J Lai, *Degree conditions for group connectivity*, Discrete Math., 310 (2010), 1050–1058.

[20] F. Yang, X. Li, L. Li, *Z_3-connectivity with independent number 2,* Graphs and Combin., 32 (2016), 419-429.

[21] X. Zhang, M. Zhan, Y. Shao, R. Xu, X. Li and H. J. Lai, *Degree sum condition for Z_3-connectivity in graphs*, Discrete Math., 310 (2010), 3390–3397.

Accepted: 1.06.2018