WEAKLY θ_I-PREOPEN SETS AND DECOMPOSITION OF CONTINUITY

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Abstract. In this paper we introduce and study the notion of weakly θ_I-preopen sets and weakly θ_I-precontinuous functions to obtain a decomposition of continuity. We also investigate their fundamental properties.
Keywords: ideal,ideal topological spaces, θ_I-pre open sets, weakly θ_I-pre open sets, weakly θ_I-precontinuous functions.

1. Introduction

The concept of ideals in general topological spaces was introduced and studied by Hamlett and Jankovic [9] (see also [10], [11]) and Vaidyanathaswamy [33]and other papers. Newcomb [27], Rancin [29], Samuels [31] and Hamlet et al. ([9], [10], [11]) motivated the research by applying topological ideals to generalize the most basic properties in general topology. Jankovic and Hamlet [18] introduced the notion of I-open sets in ideal topological space. El-Monsef et al.[25] further investigated I-open sets and I-continuous functions in ideal topological space. Some new forms of I-open sets are introduced in [7] (see [15]) and other papers. Yuksel et al. [35] and Acikgoz et al.[1] have investigated some new classes of functions in ideal topological spaces. Hatir and Noiri [15] introduced the

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notions of weakly semi-I-open sets and weakly semi-I-continuous functions in ideal topological space. Q.L.Shi [30] initiated and elaborated the notion of weakly α-I-open sets, weakly α-I-continuous, weakly α-I-open, weakly α-I-closed functions and weakly α-I-paracompact spaces in ideal topological spaces. In 2013 Mustafa and Al-Ghour [26] defined the notion of weakly b-I-open sets, weakly b-I-continuous, weakly b-I-open and weakly b-I-closed functions in ideal topological spaces. Quite recently in [4]some new forms of θ_I-open sets have introduced and studied and a new decomposition of continuity is obtained by Al-Omari and Noiri. The concept of θ_I-open sets is based on θ-open sets due to Veličko [34]. A set A is said to be θ-open [34], if every point of A has an open neighborhood whose closure is contained in A.

This new concept of θ_I-preopen sets motivated me to generalize this notion as weakly θ_I-preopen sets. The main theme of the present paper is to devise and elaborate the concept of weakly θ_I-preopen sets and to obtain new decomposition of continuity in ideal topological spaces. This paper is organized as follows, in section 3 we define weakly θ_I-preopen sets and establish its interrelationships with some other generalized open sets and also study its characterizations. In section 4 we define and study strong θ_{pre}-t-I sets, strong θ_{pre}-B-I sets and θ^B-sets. In section 5 we introduce and investigate weakly θ_I-precontinuous and weakly θ_I-preirresolute functions in ideal topological spaces.

2. Preliminaries

Throughout this paper, (X, τ) and (Y, σ) will denote topological spaces with no separation properties assumed. $Cl(V)$ and $Int(V)$ will denote the closure and the interior of V in X, respectively, for a subset V of a topological space (X, τ). $C(X)$ denotes the collection of closed subsets of X. An ideal I on a nonempty set X is a nonempty collection of subsets of X which satisfies the following:

1. $V \in I$ and $U \subset V$ implies $U \in I$,
2. $V \in I$ and $U \in I$ implies $V \cup U \in I$.

The pair (X, τ, I) of a topological space (X, τ) and an ideal I on X is called an ideal topological space or simply an ideal space. It is important that a family of sets is a filter if and only if the family of the complements of these sets is an ideal. One connection between an ideal and the topology on a given ideal space arises through the concept of the local function on a subset. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the collection of all subsets of X, a set operator $(.)$:$P(X) \to P(X)$ called a local function of A with respect to τ and I, is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X\colon (U \cap A) \notin I$, for every $U \in \tau(x)$ $\}$, where $\tau(x) = \{U \in \tau : x \in U\}$ [33] (c.f. [18], [19]). A Kuratowski closure operator $Cl^*(A) = A \cup A^*(I, \tau)$ induces a topology $\tau^*(I, \tau)$ called the $*$-topology which is finer than τ. It is generated by the base $\beta(I, \tau)$ $= \{U \setminus I : U \in \tau$ and $I \in I$ $\}$. In general $\beta(I, \tau)$ is not always a topology as shown in [18]. We will write A^* for $A^*(I,\tau)$ and τ^* for $\tau^*(I,\tau)$. In general X^* is a proper subset of X. Hayashi [17] used the hypothesis $X = X^*$ and Samuels [31] used the hypothesis $\tau \cap I = \phi$.

Although these two conditions are equivalent due to [18] and therefore the ideal topological spaces satisfying this hypothesis are called as Hayashi-Samuels spaces ([19], [33], [17]).

Now we recall some definitions and results which are used in this paper.

Definition 1. A subset A of a topological space (X, τ) is said to be

1. preopen [22] if $A \subset Int(Cl(A))$,
2. semi-open [20] if $A \subset Cl(Int(A))$,
3. α-open [23] if $A \subset Int(Cl(Int(A)))$,
4. β-open [24] if $A \subset Cl(Int(Cl(A)))$,
5. b-open [5] if $A \subset Int(Cl(A)) \cup Cl(Int(A))$,
6. a t-set [32] if $Int(A) = Int(Cl(A))$,
7. a B-set [32] if $A = U \cap V$, where $U \in \tau$ and V is a t-set,
8. a t-I-set [12] if $Int(A) = Int(Cl^*(A))$,
9. a B_I-set [12] if $A = U \cap V$, where $U \in \tau$ and V is a t-I-set,
10. a strong t-I-set [14] if $Int(A) = sCl(Int(Cl^*(A)))$,
11. a strong B_I-set [14] if $A = U \cap V$, where U is an open set and V is a strong t-I-set.

The θ-interior [34] of A in X is the union of all θ-open subsets contained in A and is denoted by $Int_\theta(A)$. The complement of a θ-open set is said to be θ-closed. The θ-closure of A is defined as $Cl_\theta(A) = \{x \in X\colon (Cl(U) \cap A) \neq \phi$, for all $U \in \tau(x)\}$ and a set A is θ-closed if and only if $A = Cl_\theta(A)$. All θ-open sets form a topology on X which is coarser than τ and denoted by τ_θ. A topological space (X, τ_θ) is regular if and only if $\tau = \tau_\theta$. The θ-closure of a given set need not be a θ-closed set. A point $x \in X$ is called a θ-I-closure point of A if $(Cl^*(U) \cap A) \neq \phi$ for each open set U containing x. The set of all θ_I-closure points of A is called the θ_I-closure of A and denoted by $Cl_{\theta_I}(A) = \{x \in X : (Cl^*(U) \cap A) \neq \phi$ for all $U \in \tau(x)\}$. A subset A is said to be θ_I-closed if $Cl_{\theta_I}(A) = A$. The complement of a θ_I-closed set is called a θ_I-open set. In other words A is said to be θ_I-open if $Cl_{\theta_I}(X \setminus A) = X \setminus A$.

Definition 2 ([3])**.** Let (X, τ, I) be an ideal topological space. A point $x \in X$ is called a θ_I-interior point of A if there exists an open set containing x such that $U \subseteq Cl^*(U) \subseteq A$. The set of all θ_I-interior points of A is called the θ_I-interior of A and denoted by $Int_{\theta_I}(A)$. A is θ_I-open if and only if $A = Int_{\theta_I}(A)$.

The following results are useful in the sequel:

Lemma 1 ([18]). *Let (X, τ, I) be an ideal topological space and A, B be any two subsets of X. Then the following properties hold:*

1. *If $A \subseteq B$, then $A^* \subseteq B^*$;*
2. *If $A^* = Cl(A^*) \subseteq Cl(A)$;*
3. *$(A^*)^* \subseteq A^*$;*
4. *$(A \cup B)^* = A^* \cup B^*$.*
5. *If $U \in \tau$, then $U \cap A^* \subset (U \cap A)^*$.*

Lemma 2 ([4]). *Let (X, τ, I) be an ideal topological space and A be a subset of X. Then the following properties hold.*

1. *If A is open, then $Cl(A) = Cl_{\theta_I}(A) = Cl_\theta(A)$.*
2. *If A is closed, then $Int(A) = Int_{\theta_I}(A) = Int_\theta(A)$.*

Definition 3. A subset A of an ideal topological space (X, τ, I) is said to be

1. I-open [25] if $A \subset Int(A^*)$.
2. α-I-open [12] if $A \subseteq Int(Cl^*(Int(A)))$
3. pre-I-open [7] if $A \subseteq Int(Cl^*(A))$.
4. semi-I-open,[12] if $A \subseteq Cl^*(Int(A))$.
5. β-I-open [15] if $A \subseteq Cl(Int(Cl^*(A)))$.
6. θ_I-preopen [4] if $A \subseteq Int(Cl_{\theta_I}(A)))$.
7. θ_I-semi-open [4] if $A \subseteq Cl(Int_{\theta_I}(A))$,
8. θ_I-β-open [4] if $A \subseteq Cl(Int(Cl_{\theta_I}(A)))$.
9. θ_I-α-open [4] if $A \subseteq Int(Cl(Int_{\theta_I}(A)))$.
10. weakly semi-I-open [16] if $A \subset Cl^*(Int(Cl(A)))$.
11. weakly pre-I-open [14] if $A \subset sCl(Int(Cl^*(A))$.
12. weakly b-I-open [26] if $A \subseteq Cl^*(Int(Cl(A))) \cup Cl(Int(Cl^*(A)))$.

Lemma 3 ([13]). *For a subset A of a topological space (X, τ), the following properties hold:*

1. *$sCl(A) = A \cup Int(Cl(A))$,*
2. *If A is open then $sCl(A) = Int(Cl(A))$.*

3. Weakly θ_I-preopen sets

Definition 4. A subset A of an ideal topological space (X, τ, I) is said to be weakly θ_I-preopen if $A \subseteq sCl(Int(Cl_{\theta_I}(A)))$.

The family of all weakly θ_I-preopen sets of the space (X,τ,I) will be denoted by Wθ_IPO(X, τ).

Theorem 1. *For any subset A of an ideal topological space (X, τ, I), the following properties hold:*

1. *Every θ_I-preopen set is weakly θ_I-preopen.*
2. *Every weakly θ_I-preopen set is θ_I-β-open.*
3. *Every preopen set is θ_I-preopen and hence weakly θ_I-preopen .*

Proof. Let A be any subset of an ideal topological space (X, τ, I).

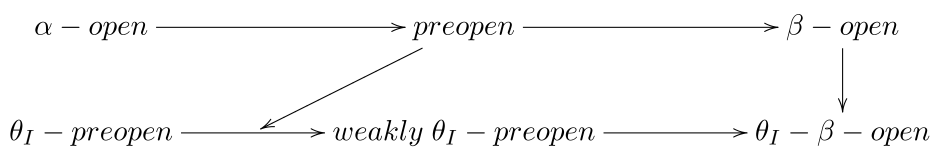
1. Suppose A is θ_I-preopen. By using the definition of a θ_I-preopen set, we have $A \subseteq Int(Cl_{\theta_I}(A)) \subseteq sCl(Int(Cl_{\theta_I}(A)))$.This shows that A is weakly θ_I-preopen. This shows that A is weakly θ_I-preopen.

2. Suppose A is weakly θ_I-preopen then we have $A \subseteq sCl(Int(Cl_{\theta_I}(A))) \subseteq Cl(Int(Cl_{\theta_I}(A))))$. This implies that A is θ_I-β-open.

3. Suppose A is preopen then we have $A \subseteq Int(Cl(A)) \subseteq Int(Cl_{\theta_I}(A)))$ and therefore A is θ_I-preopen and hence A is weakly θ_I-preopen. □

3.1 Interrelationship

The following diagram will describe the interrelations among a weakly θ_I-preopen set and some other existing open sets in an ideal topological space. None of these implications is reversible as shown by examples given below.



Example 1. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b\}, \{b, c, d\}\}$ and $I = P(X)$, then (X, τ, I) is an ideal topological space.

$C(X) = \{X, \phi, \{c, d\}, \{a, d\}, \{d\}, \{a, c, d\}, \{a\}\}$. Let $A = \{b, d\}$ be any subset of X, then $Cl_{\theta_I}(\{b, d\}) = \{b, d\}$ and $Int(\{b, d\}) = \{b\}$ and $A = \{b, d\} \subsetneq \{b\}$. This implies that A is not a θ_I-preopen set. But $sCl(\{b\}) = \{b\} \cup X = X$, consequently $A = \{b, d\} \subseteq X$. This shows that A is weakly θ_I-preopen.

Example 2. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{b\}, \{b, c, d\}, \{c, d\}\}$ and $I = P(X)$, then (X, τ, I) is an ideal topological space. $C(X) = \{X, \phi, \{a, c, d\}, \{a, b\}, \{a\}\}$. Let $A = \{a, b\}$ is β-open and hence θ_I-β-open. Because $cl(int(cl(A))) = int(cl(A)) = Cl(Int(Cl_{\theta_I}(\{a, b\}))) = \{a, b\} = A$. But $A = \{a, b\}$ is not weakly θ_I-preopen, since $sCl(Int(Cl_{\theta_I}(\{a, b\}))) = \{b\}$, which is not containing $\{a, b\}$.

Example 3. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b, c\}, \{a, c, d\}, \{a\}, \{a, b\}, \{a, c\}\}$ and $I = \{\phi, \{d\}\}$, then (X, τ, I) is an ideal topological space.

$C(X) = \{X, \phi, \{d\}, \{c, d\}, \{b, c, d\}, \{b\}, \{b, d\}\}$. Let $A = \{c\}$ be any subset of X, which is weakly θ_I-preopen, as $Cl_{\theta_I}(\{c\}) = X$ and $sCl(Int(Cl_{\theta_I}(\{c\})) = X$, which contains $\{c\}$. But it is not preopen, since $A \nsubseteq Int(Cl(\{c\})) = \phi$.

Theorem 2. *Let (X, τ, I) be an ideal topological space. Let V, A and A_α be the subsets of X. Then*

1. *If A_α is weakly θ_I-preopen for each $\alpha \in \wedge$, then $\cup_{\alpha\in\wedge} A_\alpha$ is weakly θ_I-preopen.*
2. *If A is weakly θ_I-preopen and V is α-open, then $A \cap V$ is weakly θ_I-preopen.*

Proof. 1. Since A_α is weakly θ_I-preopen for each $\alpha\in\wedge$, $A_\alpha \subseteq sCl(Int(Cl_{\theta_I}(A_\alpha)))$ for each $\alpha \in \wedge$. Therefore $A_\alpha \subseteq sCl(Int(Cl_{\theta_I}(\cup_{\alpha\in\wedge} A_\alpha)))$ for each $\alpha \in \wedge$ and $\cup_{\alpha\in\wedge} A_\alpha \subseteq sCl(Int(Cl_{\theta_I}(\cup_{\alpha\in\wedge} A_\alpha)))$.

Hence $\cup_{\alpha\in\wedge} A_\alpha$ is weakly θ_I-preopen.

2. $A \cap V \subseteq sCl(Int(Cl_{\theta_I}(A))) \cap Int(Cl(Int(V)))$
$= Int(Cl(Int(Cl_{\theta_I}(A))) \cap Int(Cl(Int(V)))$
$= Int[Cl(Int(Cl_{\theta_I}(A)) \cap Cl(Int(V))]$
$= Int[Cl[Int(Cl_{\theta_I}(A)) \cap (Int(V))]]$
$= sCl[Int(Cl_{\theta_I}(A) \cap Int(V))]$
$\subseteq sCl[Int(Cl_{\theta_I}(A \cap Int(V))] \subseteq sCl[Int(Cl_{\theta_I}(A \cap V))]$.

Therefore $(A \cap V) \subseteq sCl(Int(Cl_{\theta_I}(A \cap V))$. This shows that $A \cap V$ is weakly θ_I-preopen. □

Theorem 3. *For an ideal topological space (X, τ, I) and $A \subseteq X$, we have:*

1. *If $I = \phi$, then A is θ_I-open if and only if A is θ-open.*
2. *If $I = P(X)$, then A is θ_I-preopen if and only if A is preopen.*

Proof. 1. *Sufficiency*- It follows directly from [4].

Necessity-If $I = \phi$, then $A^* = Cl(A)$ and therefore $Cl_{\theta_I}(A) = Cl_\theta$.

2. *Sufficiency*- It follows from the Theorem 1.

Necessity-If $I = P(X)$, then $A^* = \phi$, therefore $Cl_{\theta_I}(A) = Cl(A)$, which implies the preopeness of A. □

Definition 5. A subset of an ideal topological space (X, τ, I) is said to be weakly θ_I-preclosed if its complement is weakly θ_I-preopen.

Theorem 4. *A subset A of an ideal topological space (X, τ, I) is said to be weakly θ_I-preclosed if and only if $sInt(Cl(Int_{\theta_I}(A))) \subset A$.*

Proof. Let A be a weakly θ_I-preclosed subset of the ideal topological space (X, τ, I), then $X \setminus A$ is weakly θ_I-preopen and hence $(X \setminus A) \subseteq sCl(Int(Cl_{\theta_I}(X \setminus A))) = X \setminus sInt(Cl(Int_{\theta_I}(A)))$. This implies $(X \setminus A) \subseteq (X \setminus (sInt(Cl(Int_{\theta_I}(A))))$.

Hence we have $sInt(Cl(Int_{\theta_I}(A))) \subseteq A$. Conversely, suppose $sInt(Cl(Int_{\theta_I}((A)))$ $\subseteq A$, then $X \setminus A \subseteq X \setminus sInt(Cl(Int_{\theta_I}(A))) = sCl(Int(Cl_{\theta_I}(X \setminus A)))$ and hence $(X \setminus A)$ is weakly θ_I-preopen. Therefore A is weakly θ_I-preclosed. □

Remark 1. The finite intersection of weakly θ_I-preopen sets need not be weakly θ_I-preopen.

Example 4. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b, c\}, \{a, b\}\}$ and

$I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ then (X, τ, I) is an ideal topological space. The family of closed subsets of X, $C(X) = \{X, \phi, \{d\}, \{c, d\}\}$. Then $A = \{a, d\}$ and $B = \{b, d\}$ are weakly θ_I-preopen, but their intersection $A \cap B = \{d\}$ is not weakly θ_I-preopen. Since $A \nsubseteq sCl(Int(Cl_{\theta_I}(\{d\}))) = \phi$.

Lemma 4. *For two subsets A and U of an ideal topological space (X, τ, I), the following is true:* $U \cap Cl_{\theta_I}(A) \subseteq Cl_{\theta_I}(U \cap A)$ *if U is θ_I-open.*

Proof. Let $x \in U \cap Cl_{\theta_I}(A)$. Then for every θ_I-open set V containing x, $V \cap U$ is a θ_I-open set containing x and hence $(V \cap U) \cap A \neq \phi$. This implies that $x \in Cl_{\theta_I}(U \cap A)$ and therefore we get the desired result. □

Lemma 5 ([18])**.** *Let (X, τ, I) be an ideal topological space and B be any subset of X such that* $B \subset A \subset X$*. Then* $B^*(\tau|A, I|A) = B^*(\tau, I) \cap A$*.*

If (X, τ, I) is an ideal topological space and A is subset of X; we denote by $\tau|A$ the relative topology on A and $I|A = \{A \cap I : I \in I\}$ is an ideal on A.

Lemma 6 ([16])**.** *Let (X, τ, I) be an ideal topological space,* $A \subset X$ *and* $U \in \tau$*. Then* $Cl^*(A) \cap U = Cl^*_U(A \cap U)$*.*

Theorem 5. *Let (X, τ, I) be an ideal topological space and* $A \subseteq U \in \tau$ *then A is weakly θ_I-preopen in (X, τ, I)if and only if A is weakly θ_I-preopen in (U, τ|U, I|U).*

Proof. *Necessity.* Let A be any weakly θ_I-preopen set in (X, τ, I), then we have $A \subseteq sCl(Int(Cl_{\theta_I}(A))) = Int(Cl(Int(Cl_{\theta_I}(A))))$ as $Int(Cl_{\theta_I}(A))$ is an open set.

Now $A = U \cap A \subseteq U \cap Int(Cl(Int(Cl_{\theta_I}(A))))$
$= Int(U \cap Int(Cl(Int(Cl_{\theta_I}(A))))$, being an open set.
$= Int_U(U \cap Int(Cl(Int(Cl_{\theta_I}(A))))$
$\subset Int_U(U \cap Cl(U \cap Int(Cl_{\theta_I}(A)))))$
$\subseteq Int_U(Cl_U(U \cap Int(Cl_{\theta_I}(A)))$
$= sCl_U(Int(U \cap (Cl_{\theta_I}(A))))$
$\subset sCl_U(Int_U(U \cap (Cl_{\theta_I}(A)))$
$= sCl_U(Int_U((Cl_{\theta_I})_U)(A))))$ by Lemma 6. This shows that A is weakly θ_I-preopen in $(U, \tau \mid U, I \mid U)$.

Sufficiency. Let A be weakly θ_I-preopen in $(U, \tau \mid U, I \mid U)$. Then we have
$A \subset sCl_U(Int_U((Cl_{\theta_I})_U)(A))))$
$= sCl_U(Int_U(Cl_{\theta_I}(A) \cap U)))$

$= sCl_U(U \cap Int(Cl_{\theta_I}(A) \cap U)))$, by Lemma 6
$= sCl_U(Int(Cl_{\theta_I}(A) \cap U))) = Int_U(Cl_U(Int(U \cap (Cl_{\theta_I}(A))))$
$= Int_U(U \cap Cl(Int(U \cap (Cl_{\theta_I}(A))))$
$= U \cap Int(Cl(Int(Cl_{\theta_I}(A))))$
$\subseteq Int(Cl(Int(Cl_{\theta_I}(A))))$
$= sCl(Int(Cl_{\theta_I}(A))))$.

This implies that A is weakly θ_I-preopen in (X, τ, I) □

Corollary 1. *Let(X, τ, I) be an ideal topological space. If $U \in \tau$ and A is weakly θ_I-preopen, then $U \cap A$ is weakly θ_I-preopen in $(U, \tau|U, I|U)$.*

Proof. Since $U \in \tau$ and A is weakly θ_I-preopen in (X, τ, I).

Since every open set is alpha-open, therefore by by Theorem 2, $U \cap A$ is weakly θ_I-preopen in (X, τ, I). Since $U \in \tau$ and by Theorem 5, $U \cap A$ is weakly θ_I-preopen in $(U, \tau|U, I|U)$. □

Definition 6. [8] A space (X, τ) is called submaximal if every dense subset of X is open.

Lemma 7. *[21] If (X, τ) is submaximal, then PO(X, τ) = τ.*

Corollary 2. *If (X, τ) is submaximal, then for any ideal I on X, $\theta_I PO(X) = \tau$.*

Proof. It follows directly from the fact that every preopen set is θ_I-preopen. □

Remark 2. If (X, τ) is submaximal, then for any ideal I on X, Wθ_IPO(X) $= \tau$.

Theorem 6 ([4])**.** *Let (X, τ, I) be an ideal topological space. The following are equivalent;*

1. *The θ_I-closure of every θ_I-open subset of X is θ_I-open;*
2. *$Cl(Int_{\theta_I}(A)) \subseteq Int(Cl_{\theta_I}(A))$ for every subset A of X;*
3. *$\theta_I PO(X) \subseteq \theta_I SO(X)$;*
4. *The θ_I-closure of every θ_I-β-open subset of X is θ_I-open;*
5. *$\theta_I \beta O(X) \subseteq \theta_I PO(X)$.*

Definition 7. A subset A of an ideal topological space (X, τ, I) is called θ_I-dense if $Cl_{\theta_I}(A) = X$.

Remark 3. Every θ_I-dense subset of an ideal topological space (X, τ, I) is θ_I-preopen.

Proof. It is obvious. □

Theorem 7. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

1. $\theta_I PO(X) \subseteq \theta_I SO(X)$,
2. *Every θ_I-dense subset of X is θ_I-semiopen,*
3. $Int_{\theta_I}(A)$ *is θ_I-dense for every θ_I-dense subset A,*
4. $Int_{\theta_I}[\theta_I\text{-}F_r(A)] = \phi$ *for every subset A,*
5. $\theta_I \beta O(X) \subseteq \theta_I SO(X)$,

Proof. (1) $\Rightarrow$ (2)It follows directly from the Remark 3.

(2) $\Rightarrow$ (3) Let A be θ_I-dense, then A is θ_I-semiopen. Therefore $A \subseteq Cl(Int_{\theta_I}(A)) \subseteq Cl_{\theta_I}(Int_{\theta_I}(A)) \subseteq Cl_{\theta_I}(Cl_{\theta_I})(A) = Cl_{\theta_I}(A) = X$, as A is θ_I-dense, we have $Cl_{\theta_I}(Int_{\theta_I}(A)) = X$. Thus $Int_{\theta_I}(A)$ is θ_I-dense.

(3) $\Rightarrow$ (4) Suppose A be any subset of X, we have $X = Cl_{\theta_I}(A) \cup (X \setminus Cl_{\theta_I}(A)) = Cl_{\theta_I}(A) \cup Int_{\theta_I}(X \setminus A) \subseteq Cl_{\theta_I}(A) \cup Cl_{\theta_I}(Int_{\theta_I}(X \setminus A)) = Cl_{\theta_I}(A \cup Int_{\theta_I}(X \setminus A))$. This shows that $A \cup Int_{\theta_I}(X \setminus A)$ is θ_I-dense and therefore $Int_{\theta_I}(A \cup Int_{\theta_I}(X \setminus A))$ is θ_I-dense. $Int_{\theta_I}[(A \cup Int_{\theta_I}(X \setminus A)) \cap ((X \setminus A) \cup Int_{\theta_I}(A))] = X \setminus \theta_I\text{-}F_r(A)$. Since $X \setminus (\theta_I\text{-}F_r(A))$ is the intersection of the two θ_I-dense sets therefore $X \setminus (\theta_I\text{-}F_r(A))$ is θ_I-dense.

(4) $\Rightarrow$ (5) Let $A \in \theta_I \beta O(X)$. Then by (4) and Theorem 3.15 of [4] $A \in \theta_I SO(X)$.

(5) $\Rightarrow$ (1) It is obvious. □

Definition 8. A space (X, τ) is extremally disconnected [36] if the closure of every open set in X is open.

Theorem 8. *If a topological space (X, τ) is extremally disconnected and* $A \in \theta_I SO(X)$, *then* $A \in \theta_I \alpha O(X)$.

Proof. Let $A \in \theta_I SO(X)$, then we have $A \subseteq Cl(Int_{\theta_I}(A))$. Since X is extremally disconnected, we have $Cl(Int_{\theta_I}(A)) = Int(Cl(Int_{\theta_I}(A)))$. Hence $A \subseteq Cl(Int_{\theta_I}(A)) = Int(Cl(Int_{\theta_I}(A)))$. □

Theorem 9. *If a topological space (X, τ) is extremally disconnected and* $A \in \theta_I \beta O(X)$, *then* $A \in W\theta_I PO(X)$.

Proof. Let $A \in \theta_I \beta O(X)$, then we have $A \subseteq Cl(Int(Cl_{\theta_I}(A)))$. Since X is extremally disconnected, we have $Cl(Int(Cl_{\theta_I}(A))) = Int[Cl(Int(Cl_{\theta_I}(A)))]$. Therefore $A \subseteq Cl(Int(Cl_{\theta_I}(A))) = Int[Cl(Int(Cl_{\theta_I}(A)))] = sCl(Int(Cl_{\theta_I}(A)))$. This implies that $A \in W\theta_I PO(X)$. □

4. Strong θ_{pre}-t-I-sets

Definition 9. A subset of an ideal topological space is called a

1. Strong θ_{pre}-t-I-set if $sCl(sInt(Cl_{\theta_I}(A))) = Int(A)$.
2. θ_{pre}-t-I-set [4] if $Int(Cl_{\theta_I}(A))) = Int(A)$.

Theorem 10. *Let A and B be subsets of an ideal topological space (X, τ, I). If A and B are strong θ_{pre}-t-I-sets, then $A \cap B$ is a strong θ_{pre}-t-I-set.*

Proof. Since A and B are strong θ_{pre}-t-I-sets, then we have $sCl(sInt(Cl_{\theta_I}(A))) = Int(A)$ and $sCl(sInt(Cl_{\theta_I}(B))) = Int(B)$.

Now $Int(A \cap B) \subseteq Int(Cl_{\theta_I}(A \cap B)))$
$\subseteq sInt(Cl_{\theta_I}(A \cap B)))$
$\subseteq sCl(sInt(Cl_{\theta_I}(A \cap B)))$
$\subseteq sCl(sInt[(Cl_{\theta_I}(A)) \cap (Cl_{\theta_I}(B))]))$
$\subseteq sCl[(sInt(Cl_{\theta_I}(A)))) \cap (sInt(Cl_{\theta_I}(B)))]$
$\subseteq sCl(sInt(Cl_{\theta_I}(A)))) \cap sCl(sInt(Cl_{\theta_I}(B)))$
$= Int(A) \cap Int(B) = Int(A \cap B)$.

Therefore $sCl(sInt(Cl_{\theta_I}(A \cap B))) = Int(A \cap B)$ and hence $A \cap B$ is a strong θ_{pre}-t-I-set. □

Theorem 11. *Every strong θ_{pre}-t-I-set is a θ_{pre}-t-I-set.*

Proof. Let A be any strong θ_{pre}-t-I-set, then we have $sCl(sInt(Cl_{\theta_I}(A)))) = Int(A)$.

Therefore $Int(Cl_{\theta_I}(A))) \subseteq sCl(sInt(Cl_{\theta_I}(A)))) = Int(A) \subseteq Int(Cl_{\theta_I}(A)))$ and hence $Int(Cl_{\theta_I}(A))) = Int(A)$. □

Theorem 12. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

1. *A is regular open.*
2. *$sCl(Int(Cl_{\theta_I}(A))) = A$ and A is open.*
3. *A is a strong θ_{pre}-t-I-set and weakly θ_I-preopen.*

Proof. (1) ⇒ (2) Since A is regular open, we have $Int(Cl(A)) = A$ and A is open. Therefore by Lemma 2, we have $Cl_{\theta_I}(A) = Cl(A)$.

Hence $sCl(Int(Cl_{\theta_I}(A))) = sCl(lnt(Cl(A))) = sCl(A) = A \cup Int(Cl(A)) = A \cup A = A$.

(2) ⇒ (3) It is direct from the definition.

(3) ⇒ (1) Let A be strong θ_{pre}-t-I-set and weakly θ_I-preopen, then we have $A \subseteq sCl(Int(Cl_{\theta_I}(A)))$ and $sCl(Int(Cl_{\theta_I}(A))) = Int(A)$. We have $A \subseteq sCl(Int(Cl_{\theta_I}(A))) = Int(A) \subseteq A$, then A is open. Therefore by Lemma 3, $A = sCl(Int(Cl(A))) = Int(Cl(Int(Cl(A)))) = Int(Cl(A)))$. Hence A is regular open. □

5. Strong θ_{pre}-*B*-*I*-sets and θ^B sets

Definition 10. A subset A of an ideal topological space (X, τ, I) is called

1. a strong θ_{pre}-B-I set if $A = U \cap V$, where $U \in \tau$ and and V is a strong θ_{pre}-t-I-set.
2. a θ_{pre}-B-I set [4] if there exist $U \in \tau$ and a θ_{pre}-t-I-set V in X such that $A = U \cap V$.

Theorem 13. *For a subset A of an ideal topological space (X, τ, I), the following properties hold:*

1. *If A is a strong θ_{pre}-t-I-set, then it is a strong θ_{pre}-B-I set.*
2. *If A is a strong θ_{pre}-B-I set, then it is a θ_{pre}-B-I set.*

Proof. 1. Let A be a strong θ_{pre}-t-I-set, then we have $sCl(sInt(Cl_{\theta_I}(A))) = Int(A)$. $A = A \cap X$ and X is open. This implies that A is a strong θ_{pre}-B-I set.

2. Let A be a strong θ_{pre}-B-I set, then we have $A = U \cap V$, where U is an open set and V is strong θ_{pre}-t-I-set. By Theorem 11, V is θ_{pre}-t-I-set and hence A is a θ_{pre}-B-I set. □

Definition 11. [4] A subset A of an ideal topological space (X, τ, I) is called a θ^Aset if $A = U \cap V$, where $U \in \tau$ and V is strongly θ_I-semi-closed i.e. V is θ_I-semi-closed and $Int(Cl_{\theta_I}(A)) = Cl(Int_{\theta_I}(A))$.

Definition 12. [4] A subset A of an ideal topological space (X, τ, I) is said to be θ_I-β-closed if $Int(Cl(Int_{\theta_I}(A)))) \subseteq A$.

Definition 13. A subset A of an ideal topological space (X, τ, I) is called a θ^B-set if $A = U \cap V$, where $U \in \tau$ and V is θ_I-β-closed.

Theorem 14. *Every θ^A-set is θ^B-set.*

Proof. Let V be strongly θ_I-semi-closed, then $Int(Cl_{\theta_I}(V)) \subseteq V$ and $Int(Cl_{\theta_I}(V)) = Cl(Int_{\theta_I}(V))$. Now $Int(Cl(Int_{\theta_I}(V)))$
$= Int(Int(Cl_{\theta_I}(V)))$
$\subseteq Int(Cl_{\theta_I}(V)) \subseteq V$. We get $Int(Cl(Int_{\theta_I}(V)))) \subseteq V$. Therefore V is θ_I-α-closed. This implies that every θ^A-set is θ^B-set. □

But the converse of Theorem 14 need not be true as shown by the following example.

Example 5. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b, c\}, \{a, c\}\}$ and $I = \{\phi, \{a, c\}\}$ then (X, τ, I) is an ideal topological space. $C(X) = \{X, \phi, \{d\}, \{b, d\}\}$. If $A = \{a, b\}$ then we can write A as $A = X \cap A$, where X is an open set and A is θ_I-β-closed, since $Int(Cl(Int_{\theta_I}(A))) = \phi \subseteq A$. Hence A is θ^B-set. But A is not a θ^A-set, since A is not a θ_I-semi-closed, as $Int(Cl_{\theta_I}(A)) = X \nsubseteq A$.

Theorem 15. *For a subset A of an ideal topological space (X, τ, I), the following properties are equivalent:*

1. *A is open.*
2. *A is preopen and a θ^B-set.*
3. *A is θ_I-preopen and a θ^B-set.*
4. *A is weakly θ_I-preopen and a θ^B-set.*
5. *A is θ_I-β-open and a θ^B-set.*
6. *A is weakly θ_I-preopen and a strong θ_{pre}-B-I-set.*

Proof. Here (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Rightarrow$ (4) $\Rightarrow$ (5) follows from Theorem 1.

(5) $\Rightarrow$ (1) It follows directly from Theorems 1, 6 and 14.

(1) $\Rightarrow$ (6): It is obvious.

(6) $\Rightarrow$ (1): Let A be weakly θ_I-preopen and a strong θ_{pre}-B-I-set, then we have $A \subseteq sCl(Int(Cl_{\theta_I}(A)))$ and $A = U \cap V$, where U is open and V is a strong θ_{pre}-t-I-set so that $sCl(sInt(Cl_{\theta_I}(V))) = Int(V)$. Hence we get
$A = A \cap U \subseteq sCl(Int(Cl_{\theta_I}(A))) \cap U = \{(sCl(Int(Cl_{\theta_I}(U \cap V)))) \cap U\}$
$\subseteq \{sCl(Int(Cl_{\theta_I}(U)))\} \cap \{sCl(Int(Cl_{\theta_I}(V)))\} \cap U$
$= \{sCl(Int(Cl_{\theta_I}(V)))\} \cap U$
$\subseteq \{sCl(sInt(Cl_{\theta_I}(V)))\} \cap U$
$= (Int(V) \cap U) \subseteq (V \cap U) = A$.
Thus A is an open set. □

6. Weakly θ_I-precontinuous

Definition 14. A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be

1. precontinuous [22] if preimage of every open set in Y is preopen in X.
2. θ_I-α-continuous if the preimage of every open set in Y is θ_I-α-open in X.
3. θ_I-precontinuous [4] if the preimage of every open set in Y is θ_I-preopen in X.
4. θ_I-β-continuous [4] if the preimage of every open set in Y is a θ_I-β-open in X.
5. θ_{pre}-B-I-continuous [4] if the preimage of every open set in Y is a θ_{pre}-B-I-set.
6. θ^A-continuous [4] if the preimage of every open set in Y is a θ^A set.

Definition 15. A function $f : (X, \tau, I) \rightarrow (Y, \sigma)$ is said to be

1. weakly θ_I-precontinuous if the preimage of every open set in Y is weakly θ_I-preopen in X.
2. strong θ_{pre}-t-I-continuous if the preimage of every open set in Y is a strongly θ_{pre}-t-I-set.
3. strong θ_{pre}-B-I-continuous if the preimage of every open set in Y is a strongly θ_{pre}-B-I-set.
4. θ^B-continuous if the preimage of every open set in Y is a θ^B-set.

Theorem 16. *1. Every θ_I-precontinuous function is weakly θ_I-precontinuous.*

2. Every precontinuous function is weakly θ_I-precontinuous.

3. Every weakly θ_I-precontinuous function is θ_I-β-continuous.

4. Every θ^A-continuous function is θ^B-continuous.

Proof. It follows directly from the Theorems 1 and 14. □

The converse of (1)-(3) in Theorem 16 need not be true as shown in the following three examples.

Example 6. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, c, d\}\}$ and $I = P(X)$ then (X, τ, I) is an ideal topological space. $C(X) = \{\phi, X, \{b, c, d\}, \{c, d\}, \{b\}\}$. Let $Y = \{1, 2, 3, 4\}$ and $\sigma = \{Y, \phi, \{1, 2\}, \{1, 2, 3\}\}$ then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be the function defined as $f(a) = 2$,$f(b) = 3$,$f(c) = 4$, $f(d) = 1$. Then f is weakly θ_I-precontinuous function but it is not θ_I-precontinuous. Since the preimage of every open set in Y is weakly θ_I-preopen but it is not θ_I-preopen in X. For, let $A = \{1, 2\}$, then the preimage $f^{-1}(\{1, 2\}) = \{a, d\}$ is weakly θ_I-preopen but it is not θ_I-open.

Example 7. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, c\}, \{c\}\}$ and $I = \{\phi, \{a\}\}$ then (X, τ, I) is an ideal topological space. $C(X)$=$\{\phi, X, \{b, c, d\}, \{a, b, d\}, \{b, d\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\sigma = \{Y, \phi, \{2\}, \{2, 3\}, \{2, 4\}, \{2, 3, 4\}\}$ then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function defined as $f(a) = 2$, $f(b) = 3$, $f(c) = 1$, $f(d) = 4$. Then f is weakly θ_I-precontinuous function but it is not precontinuous, since the preimage of every open set in Y is weakly θ_I-preopen but it is not preopen in X. For, let $A = \{2, 3\}$, then its preimage $f^{-1}(\{2, 3\}) = \{a, b\}$ is not a preopen in X.

Example 8. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{b\}, \{a, b, d\}\}$ and $I = \{\phi, \{a\}, \{b\}, \{a, b, \}\}$ then (X, τ, I) is an ideal topological space. $C(X) = \{\phi, X, \{b, c, d\}, \{a, c, d\}, \{c, d\}, \{c\}\}$. Let $Y = \{1, 2, 3, 4\}$ and $\sigma = \{Y, \phi, \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\}$ then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be the function defined as $f(a) = 4$, $f(b) = 1$, $f(c) = 2$, $f(d) = 3$. Then f is θ_I-β-continuous but it is not weakly θ_I-precontinuous. Since the preimage of every open set in Y is θ_I-β-continuous but it is not weakly θ_I-preopen in X. For, let $A = \{1, 2\}$, then its preimage $f^{-1}(\{1, 2\}) = \{b, c\}$ is not a weakly θ_I-preopen set but it is θ_I-β-open in X.

Example 9. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b, c\}, \{a, c\}\}$ and $I = \{\phi, \{a, c, \}\}$ then (X, τ, I) is an ideal topological space. $C(X) = \{\phi, X, \{d\}, \{b, d\}\}$. Let $Y = \{1, 2, 3, 4\}$ and $\sigma = \{Y, \phi, \{2\}, \{2, 3\}\}$ then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be the function defined as $f(a) = 3$, $f(b) = 2$, $f(c) = 4$, $f(d) = 1$. Then f is θ^B-continuous but it is not θ^A-continuous, since the preimage of every open set in Y is a θ^B-set but it is not a θ^A-set. For, let $A = \{2, 3\}$, its preimage $f^{-1}(\{2, 3\} = \{a, b\}$ is θ_I-β-closed but it is not θ_I-semiclosed in X as $X = X \cap A$ where X is open and A is θ_I-β-closed(for θ^B-set) or strong θ_I-semiclosed in X(for θ^A-set).

Definition 16. Let A be a subset of the space (X, τ, I) and let $x \in X$. Then A is called a weakly θ_I-preneighborhood of x if there exists a weakly θ_I-preopen set V containing x such that $V \subseteq A$.

Theorem 17. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

1. *f is weakly θ_I-precontinuous.*
2. *For each $x \in X$ and for each $U \in \sigma$ containing $f(x)$, $f^{-1}(U)$ is weakly θ_I-preneighborhood of x.*

Proof. (1) $\Rightarrow$ (2) Suppose $x \in X$ and U is any open set in Y such that $f(x) \in U$. By Theorem 18 there exists a weakly θ_I-preopen set M containing x in X such that $f(M) \subseteq U$; hence $x \in M \subseteq f^{-1}(U)$. Therefore $f^{-1}(U)$ is weakly θ_I-preneighborhood of x.

(2) $\Rightarrow$ (1) Let U be any open set in Y and $x \in f^{-1}(U)$. Since $f^{-1}(U)$ is weakly θ_I-preneighborhood of x, therefore there exists a weakly θ_I-preopen set M_x such that $x \in M_x \subset f^{-1}(U)$. Thus we have $f^{-1}(U) = \bigcup\{M_x : x \in f^{-1}(U)\}$ and hence $f^{-1}(U)$ is weakly θ_I-preopen in X. □

Theorem 18. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

1. *f is is weakly θ_I-precontinuous.*
2. *For each $x \in X$ and each $V \in \sigma$ containing $f(x)$, there exists a weakly θ_I-preopen set U containing x such that $f(U) \subset V$.*
3. *For each $x \in X$ and each $V \in \sigma$ containing f(x), $Cl_{\theta_I}(f^{-1}(V))$ is weakly θ_I-preneighborhood of x.*
4. *The inverse image of each closed set in Y is weakly θ_I-preclosed.*

Proof. (1) $\Rightarrow$ (2) Let $x \in X$ and let V be any open set in Y such that $f(x) \in V$. Set $P = f^{-1}(V)$. By (1) P is weakly θ_I-preopen and therefore $x \in P$ implies that $f(P) \subset V$.

(2) $\Rightarrow$ (3) Since V is open in Y and $f(x) \in V$, then by (2) there exists a weakly θ_I-preopen set P containing x such that $f(P) \subset V$. Therefore $x \in P \subseteq sCl(Int(Cl_{\theta_I}(P))) \subseteq sCl(Int(Cl_{\theta_I}(f^{-1}(V)))) \subseteq Cl_{\theta_I}(Cl_{\theta_I}(f^{-1}(V))) = Cl_{\theta_I}(f^{-1}(V))$. This shows that $Cl_{\theta_I}(f^{-1}(V))$ is a weakly θ_I-preneighborhood of x.

(3) $\Rightarrow$ (1) Let V be any open set in Y and $x \in f^{-1}(V)$. By (3), $Cl_{\theta_I}(f^{-1}(V)$ is weakly θ_I-preneighborhood of x, there exists a weakly θ_I-preopen set U_x in X such that $x \in U_x \subseteq Cl_{\theta_I}(f^{-1}(V)$. Hence $Cl_{\theta_I} f^{-1}(V) \subseteq \bigcup_{x \in f^{-1}(V)} U_x$. This implies that $f^{-1}(V)$ is weakly θ_I-preopen in X and therefore f is weakly θ_I-precontinuous.

(1) $\Leftrightarrow$ (4) It is obvious. □

Theorem 19. *If $f : (X, \tau, I) \to (Y, \sigma)$ is any weakly θ_I-precontinuous function and $U \in \tau$, then the restriction $f \mid U : (U, \tau \mid U, I \mid U) \to (Y, \sigma)$ is weakly θ_I-precontinuous.*

Proof. Let V be any open set in Y. Since f is weakly θ_I-precontinuous, $f^{-1}(V)$ is weakly θ_I-preopen. Since U is open, by Corollary 1, $U \cap f^{-1}(V)$ is weakly θ_I-preopen in $(U, \tau \mid U, I \mid U)$. Since $(f \mid U)^{-1}(V) = U \cap f^{-1}(V)$ and $(f \mid U)^{-1}(V)$ is weakly θ_I-preopen in $(U, \tau \mid U, I \mid U)$. This implies that $(f \mid U)$ is weakly θ_I-precontinuous function. □

Theorem 20. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function and $\{U_\alpha : \alpha \in \tau\}$ be an open cover of X. Then f is weakly θ_I-precontinuous if and only if the restriction $(f \mid U_\alpha)$:$(U_\alpha,\ \tau \mid U_\alpha,\ I \mid U_\alpha)$ is weakly θ_I-precontinuous for each $\alpha \in \triangle$.*

Proof. *Necessity.* It follows directly from the Theorem 19.

Sufficiency. Let V be any open set in Y. Since $(f \mid U_\alpha)$ is a weakly θ_I-precontinuous for each $\alpha \in \triangle$, $(f \mid U_\alpha)^{-1}(V)$ is a weakly θ_I-preopen set in $(U_\alpha,\ \tau \mid U_\alpha,\ I \mid U_\alpha)$. Hence by Theorem 5 $(f \mid U_\alpha)^{-1}(V)$ is weakly θ_I-preopen in $(X,\ \tau,\ I)$. Moreover we consider $f^{-1}(V) = X \cap f^{-1}(V) = \bigcup_{\alpha \in \triangle}(\ U_\alpha \cap f^{-1}(V))= \bigcup_{\alpha \in \triangle}(f \mid U_\alpha)^{-1}(V)$. By using Theorem 2, $\bigcup_{\alpha \in \triangle}(f \mid U_\alpha)^{-1}(V)$ is weakly θ_I-preopen in $(X,\ \tau,\ I)$.

Therefore $f^{-1}(V)$ is weakly θ_I-preopen in $(X,\ \tau,\ I)$. Hence f is weakly θ_I-precontinuous. □

Theorem 21. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is weakly θ_I-precontinuous if and only if the function $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, is weakly θ_I-precontinuous.*

Proof. *Necessity.* Let f be weakly θ_I-precontinuous. Let $x \in X$ and R be any open neighborhood of $g(x)$ in $X \times Y$. Then there exists an open set $P \times Q$ in $X \times Y$ such that $g(x) = (x, f(x)) \in (P \times Q) \subseteq R$. By assumption f is weakly

θ_I-precontinuous and there exists a weakly θ_I-preopen set P_0 in X containing x such that $f(P_0) \subset Q$. By Theorem 2 $P \cap P_0$ is weakly θ_I-preopen and $g(P \cap P_0) \subset (P \times Q) \subset R$. This implies that g is weakly θ_I-precontinuous.

*Sufficiency.*Suppose that the function g is weakly θ_I-precontinuous Let $x \in X$ and Q be any open set in Y containing $f(x)$. Then $X \times Q$ is open in $X \times Y$. Since g is weakly θ_I-precontinuous, by hypothesis there exits a weakly θ_I-preopen set P containing x such that $g(P) \subset X \times Q$ and hence we get $f(P) \subset Q$. This shows that f is weakly θ_I-precontinuous. □

Definition 17. A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be weakly θ_I-preirresolute if $f^{-1}(V)$ is weakly θ_I-preopen in (X, τ, I) for every weakly θ_J-preopen set V in (Y, σ, J).

Theorem 22. *Let $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \rho)$ be two functions, then the following properties hold:*

1. *if f is weakly θ_I-precontinuous and g is continuous, then $g \circ f$ is weakly θ_I-precontinuous.*
2. *if f is weakly θ_I-preirresolute and g is weakly θ_I-precontinuous then $g \circ f$ is weakly θ_I-precontinuous.*

Proof. It is obvious from the definitions. □

7. Decompositions of continuity

Theorem 23. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, the following properties are equivalent:*

1. *f is continuous;*
2. *f is precontinuous and θ^B-continuous;*
3. *f is θ_I-precontinuous and θ^B-continuous;*
4. *f is weakly θ_I-precontinuous and θ^B-continuous;*
5. *f is θ_I-β-continuous and θ^B-continuous;*
6. *f is weakly θ_I-precontinuous and strongly θ_{pre}-B-I-continuous.*

Proof. It follows directly from Theorem 15. □

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