NEW CONCEPTS IN INTERVAL-VALUED INTUITIONISTIC FUZZY GRAPHS

S. Lavanya
Department of Mathematics
Bharathi Women's College
Chennai
India
lavanyaprasad1@gmail.com

P.K. Kishore Kumar*
Department of Information Technology
Al Musanna College of Technology
Sultanate of Oman
kishorePK@act.edu.om

Hossein Rashmanlou
Department of Computer Science
University College of Adib
Sari
Iran
rashmanlou.1987@gmail.com

Mostafa Nouri Jouybari
Department of Mathematics
Payame Noor University(PNU)
P.O. Box 19395-3697
Tehran
Iran
m_njoybari@pnu.ac.ir

Abstract. Intuitionistic fuzzy graphs is a highly growing research area as it is the generalization of the fuzzy graphs. In this paper, we introduce the concept of Interval-valued Intuitionistic fuzzy graphs(IVIFG), we also analyse some properties of IVIFG based on morphism such as weak isomorphism, co-weak isomorphism and some concepts on automorphism.

Keywords: IVIFG, weak isomorphism of IVIFG, co-weak isomorphism of IVIFG.

1. Introduction

Graph theory has found its importance in many real time problems. Recent applications in graph theory is quite interesting analysing any complex situations and moreover in engineering applications. It has got numerous applications

*. Corresponding author

on operations research, system analysis, network routing, transportation and many more. To analyse any complete information we make intensive use of graphs and its properties. For working on partial information or incomplete information or to handle the systems containing the elements of uncertainty we understand that fuzzy logic and its involvement in graph theory is applied. In 1975, Rosenfeld [21] discussed the concept of fuzzy graphs whose ideas are implemented by Kauffman [18] in 1973. The fuzzy relation between fuzzy sets were also considered by Rosenfeld who developed the structure of fuzzy graphs, obtaining various analagous results of several graph theoretical concepts. Bhattacharya [4] gave some remarks of fuzzy graphs. The complement of fuzzy graphs was introduced by Mordeson [19]. Atanassov introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs [2, 3, 32, 33]. Talebi and Rashmanlou [36] studied the properties of isomorphism and complement of interval-valued fuzzy graphs. They defined isomorphism and some new operations on vague graphs [37, 38]. Borzooei and Rashmalou analysed new concepts of vague graphs [5], degree of vertices in vague graphs [6], more results on vague graphs [7], semi global domination sets in vague graphs with application [8] and degree and total degree of edges in bipolar fuzzy graphs with application [9]. Rashmanlou et.al., defined the complete interval-valued fuzzy graphs [24]. Rashmanlou and Pal studied intuitionistic fuzzy graphs with categorical properties [29], some properties of highly irregular interval-valued fuzzy graphs [28], more results on highly irregular bipolar fuzzy graphs [30], balanced interval-valued fuzzy graphs [26] and antipodal interval-valued fuzzy graphs [25]. Samanta and Pal investigated fuzzy k-competition and p-competition graphs, and concept of fuzzy planar graphs in [21, 22, 31] . Also they introduced fuzzy tolerance graph [34], bipolar fuzzy hypergraphs [35] and given several properties on it. Pal and Rashmanlou [20] defined many properties of irregular interval-valued fuzzy graphs. Ganesh *et al.* [10, 11] analysed the properties of Regular product vague graphs and product vague line graphs. The article has been composed of four sections. Ganesh *et al.* [12, 13, 14, 15] has analysed some concepts on faces and dual of m-polar fuzzy graphs, regular bipolar fuzzy graphs, isomorphic properties of m-polar fuzzy graphs and novel concepts on strongly edge irregular m-polar fuzzy graphs.In section 1,we introduce the survey of Interval-valued intuitionistic fuzzy graphs. In section 2 we define the preliminaries of Intuitionistic fuzzy graphs and basic definitions, definition of IVIFG. In section 3 we define automorphic IVIFG and analyse the concepts of weak and co-weak isomorphic properties of IVIFG.For further terminologies, the readers are referred to [1-6,12,13].

2. Preliminaries

A fuzzy graph G=(V,σ,μ) where V is the vertex set, σ is a fuzzy subset of V and μ is a membership value on σ such that μ(u,v) $\leq$ σ(u) $\wedge$ σ(v) for every u,v $\in$ V.The underlying crisp graph of G is denoted by $G^* = (\sigma^*, \mu^*)$, where

$\sigma = \sup\rho(\sigma) = \{x \in V : \sigma(x) > 0\}and\mu = \sup\rho(\mu) = \{(x,y) \in V \times V : \mu(x,y) > 0\}$. H = $(\sigma^{\shortmid};\mu^{\shortmid})$ is a fuzzy subgraph of G if there exists X $\subseteq$ V such that, $\sigma^{\shortmid} : X \to [0,1]$ is a fuzzy subset and$\mu^{\shortmid} : X \times X \to [0,1]$ is a fuzzy relation on $\sigma^{\shortmid}$ such that μ(u,v) $\leq \sigma$(u) $\wedge$ σ(v), for all x,y $\in$ X.

Definition 2.1. *By an interval-valued fuzzy graph of a graph G we mean a pair $G^* = (A,B)$ where $A = [\mu_A^-, \mu_A^+]$ and $\mu_B : V \times V \to [0,1]$ are bijective such that membership value of nodes and edges are distinct and $\mu_B(x,y) \leq \mu_v(x) \wedge \mu_v(y) \forall x,y \in V$*

Definition 2.2. *An interval $[\mu - \epsilon, \mu + \epsilon]$ is said to be an ϵ-neighborhood of any membership value(ie., corresponding to any nodes or edges) μ for any ϵ satisfying the following conditions.*

(i) $\epsilon \not> min\{\mu_v(v_i), \mu_e(e_{ij})\}$;

(ii) $\epsilon \not> 1 - max\{\mu_v(v_i), \mu_e(e_{ij})\}$;

(iii) $\epsilon \neq d(\mu(x),\mu(y)) or \frac{1}{2} d(\mu(x),\mu(y))$ where $d(\mu(x),\mu(y)) = |\mu(x) - \mu(y)|$ and $\mu(x), \mu(y)$ are the membership or nodes or edges.

Definition 2.3. *By an interval-valued intuitionistic fuzzy graph of a graph G we mean a pair $G^* = (A,B)$ where $A = [(\mu_A^-, \mu_A^+), (\nu_A^-, \nu_A^+)]$ and $\mu_e : V \times V \to [0,1]$ and $\nu_e : V \times V \to [0,1]$ are bijective such that true and false membership value of nodes and edges are distinct and $\mu_e(x,y) \leq \mu_v(x) \wedge \mu_v(y) \forall x,y \in V$, $\nu_e(x,y) \geq \nu_v(x) \vee \nu_v(y) \forall x,y \in V$*

Definition 2.4. *An interval-valued intuitionistic fuzzy graph(IVIFG) is said to be strong for the lower and upper bounds (μ^-, μ^+) and (ν^-, ν^+) of the edges and vertices satisfying the following conditions $\mu_e(x,y) = \mu_v(x) \wedge \mu_v(y) \forall x,y \in V$, $\nu_e(x,y) = \nu_v(x) \vee \nu_v(y) \forall x,y \in V$*

Definition 2.5. *Let $G = (V,E)$ be an IVIFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\nu(v))$ where $d_\mu(v) = \sum_{u \neq v} (\mu_e^-(v,u), \mu_e^+(v,u))$ and $d_\nu(v) = \sum_{u \neq v} (\nu_e^-(v,u), \nu_e^+(v,u))$*

Definition 2.6. *Let $G = (V,E)$ be an IVIFG. Then the total degree of a vertex v is defined by $td(v) = (td_\mu(v), td_\nu(v))$ where $td_\mu(v) = \sum_{u \neq v} (\mu_e^-(v,u) + \mu_v^-(v), \mu_e^+(v,u) + \mu_v^+(v))$ and $td_\nu(v) = \sum_{u \neq v} (\nu_e^-(v,u) + \nu_v^-(v), \nu_e^+(v,u) + \nu_v^+(v))$*

Definition 2.7. *Let $G = (V,E)$ be an IVIFG. If all the vertices of G have same degree then G is said to be regular IVIFG.*

Definition 2.8. *Let $G = (V,E)$ be an IVIFG. Then the order of G is defined as $O(G) = [\sum_{v \in V} \mu_v^-(v), \sum_{v \in V} \mu_v^+(v)]$, $[\sum_{v \in V} \nu_v^-(v), \sum_{v \in V} \nu_v^+(v)]$*

Definition 2.9. *Let $G = (V,E)$ be an IVIFG. Then the size of G is defined as $S(G) = [\sum_{u \neq v} \mu_e^-(v,u), \sum_{u \neq v} \mu_e^+(v,u)]$, $[\sum_{u \neq v} \nu_e^-(v,u), \sum_{u \neq v} \nu_e^+(v,u)]$*

Remark 2.1. In any IVIFG G, we have

$$\sum_{v \in V} d_G(v) = 2\{(\sum_{u \neq v} \mu_e^-(v,u), \sum_{u \neq v} \mu_e^+(v,u)]), (\sum_{u \neq v} \nu_e^-(v,u), \sum_{u \neq v} \nu_e^+(v,u))\} = 2S(G).$$

Definition 2.10. *Let $G = (V, E)$ be an IVIFG. Let $e_ij \in B$ be an edge of G where e_ij has its lower and upper bounds μ_e^-, ν_e^- and μ_e^+, ν_e^+. Then the degree of an edge e_ij defined as $d_\mu(e_ij) = d_\mu(v_i) + d_\mu(v_j) - 2\mu_e(e_ij)$ and $d_\nu(e_ij) = d_\nu(v_i) + d_\nu(v_j) - 2\nu_e(e_ij)$, for all its the vertices having the lower and upper bounds μ^-, ν^- and μ^+, ν^+ respectively.*

Definition 2.11. *Let $G = (V, E)$ be an IVIFS. Let $e_ij \in B$ be an edge of G where e_ij has its lower and upper bounds μ_e^-, ν_e^- and μ_e^+, ν_e^+. Then the total degree of an edge e_ij defined as $td_\mu(e_{ij}) = d_\mu(e_ij) + \mu(e_{ij})$ and $td_\nu(e_{ij}) = d_\nu(e_ij) + \nu(e_{ij})$, for all the lower and upper bounds μ^-, ν^- and μ^+, ν^+ respectively.*

3. Automorphic IVIFG

In this section we introduce the isomorphic properties of IVIFG.

Example 3.1. The below figure represents the IVIFG G of a crisp graph G^*

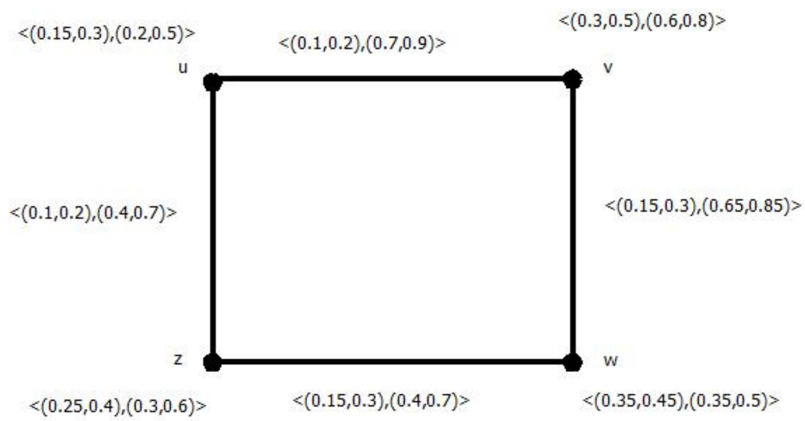

Fig 1 : Interval-valued Intuitionistic Fuzzy Graph

Throughout this work G^* is a crisp graph and G is a IVIFG.

Definition 3.1. *Let G_1 and G_2 be the IVIFGs. A homomorphism $f : G_1 \to G_2$ is a mapping $f : V_1 \to V_2$ which satisfies the following conditions:*

(i) $\mu_{A_1}^-(x_1) \leq \mu_{A_2}^-(f(x_1))$, $\mu_{A_1}^+(x_1) \leq \mu_{A_2}^+(f(x_1))$;

(ii) $\nu_{A_1}^-(x_1) \geq \nu_{A_2}^-(f(x_1))$, $\nu_{A_1}^+(x_1) \geq \nu_{A_2}^+(f(x_1))$;

(iii) $\mu_{B_1}^- x_1y_1 \leq \mu_{B_2}^-(f(x_1f(y_1)), \mu_{B_1}^+ x_1y_1 \leq \mu_{B_2}^+(f(x_1f(y_1))$;

(iv) $\nu_{B_1}^- x_1y_1 \geq \nu_{B_2}^-(f(x_1f(y_1)), \nu_{B_1}^+ x_1y_1 \geq \nu_{B_2}^+(f(x_1f(y_1))$, *for all* $x_1 \in V_1, x_1y_1 \in E_1$.

Definition 3.2. *Let G_1 and G_2 be the IVIFGs. An isomorphism $f : G_1 \to G_2$ is a bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions:*

(i) $\mu_{A_1}^-(x_1) = \mu_{A_2}^-(f(x_1))$, $\mu_{A_1}^+(x_1) = \mu_{A_2}^+(f(x_1))$;

(ii) $\nu^-_{A_1}(x_1) = \nu^-_{A_2}(f(x_1))$, $\nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1))$;

(iii) $\mu^-_{B_1}(x_1y_1) = \mu^-_{B_2}(f(x_1f(y_1)), \mu^+_{B_1}(x_1y_1) = \mu^+_{B_2}(f(x_1f(y_1))$;

(iv) $\nu^-_{B_1}(x_1y_1) = \nu^-_{B_2}(f(x_1f(y_1)), \nu^+_{B_1}(x_1y_1) = \nu^+_{B_2}(f(x_1f(y_1))$, *for all* $x_1 \in V_1$, $x_1y_1 \in E_1$.

Definition 3.3. *Let* G_1 *and* G_2 *be the IVIFGs. Then a weak isomorphism* $f : G_1 \to G_2$ *is a bijective mapping* $f : V_1 \to V_2$ *which satisfies the following conditions:*

(i) f *is a homomorphism;*

(ii) $\mu^-_{A_1}(x_1) = \mu^-_{A_2}(f(x_1))$, $\mu^+_{A_1}(x_1) = \mu^+_{A_2}(f(x_1))$;

(iii) $\nu^-_{A_1}(x_1) = \nu^-_{A_2}(f(x_1))$, $\nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1))$.

It is clear that a weak isomorphism maintains only the weights of the nodes.

Example 3.2. Consider the IVIFGs G_1 and G_2 of G_1^* and G_2^* respectively,

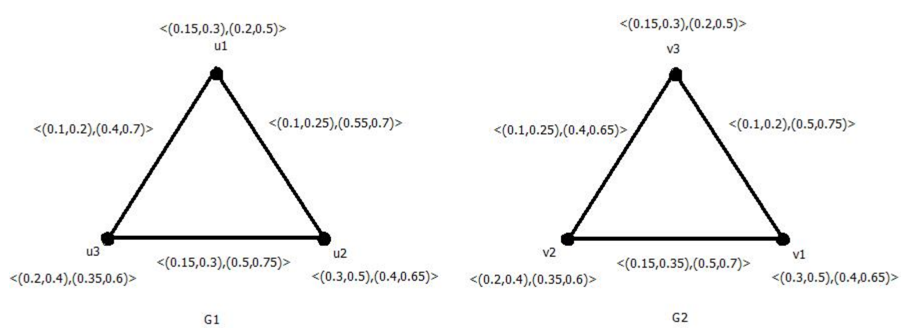

Fig 2 : Weak Isomorphism of IVIFG

A map $f : V_1 \to V_2$ defined by $f(u_1) = v_3, f(u_2) = v_1$ and $f(u_3) = v_2$. Then we have:

$$\mu^-_{A_1}(u_1) = \mu^-_{A_2}(v_3), \mu^+_{A_1}(u_1) = \mu^+_{A_2}(v_3),$$
$$\nu^-_{A_1}(u_1) = \nu^-_{A_2}(v_3), \nu^+_{A_1}(u_1) = \nu^+_{A_2}(v_3).$$
$$\mu^-_{A_1}(u_2) = \mu^-_{A_2}(v_1), \mu^+_{A_1}(u_2) = \mu^+_{A_2}(v_1),$$
$$\nu^-_{A_1}(u_2) = \nu^-_{A_2}(v_1), \nu^+_{A_1}(u_2) = \nu^+_{A_2}(v_1).$$
$$\mu^-_{A_1}(u_3) = \mu^-_{A_2}(v_2), \mu^+_{A_1}(u_3) = \mu^+_{A_2}(v_2),$$
$$\nu^-_{A_1}(u_3) = \nu^-_{A_2}(v_2), \nu^+_{A_1}(u_3) = \nu^+_{A_2}(v_2).$$

But we see that:

$$\mu^-_{B_1}(u_1u_2) = \mu^-_{B_2}(v_3v_1), \mu^+_{B_1}(u_1u_2) \neq \mu^+_{B_2}(v_3v_1),$$
$$\nu^-_{B_1}(u_1u_2) \neq \nu^-_{B_2}(v_3v_1), \nu^+_{B_1}(u_1u_2) \neq \nu^+_{B_2}(v_3v_1)$$
$$\mu^-_{B_1}(u_1u_3) = \mu^-_{B_2}(v_3v_2), \mu^+_{B_1}(u_1u_3) \neq \mu^+_{B_2}(v_3v_2),$$
$$\nu^-_{B_1}(u_1u_3) = \nu^-_{B_2}(v_3v_2), \nu^+_{B_1}(u_1u_3) \neq \nu^+_{B_2}(v_3v_2)$$
$$\mu^-_{B_1}(u_3u_2) = \mu^-_{B_2}(v_2v_1), \mu^+_{B_1}(u_3u_2) \neq \mu^+_{B_2}(v_2v_1),$$
$$\nu^-_{B_1}(u_3u_2) = \nu^-_{B_2}(v_2v_1), \nu^+_{B_1}(u_3u_2) \neq \nu^+_{B_2}(v_2v_1).$$

Hence the map is a weak isomorpism but not an isomorphism.

Definition 3.4. *Let G_1 and G_2 be the IVIFGs. Then a co-weak isomorphism $f : G_1 \to G_2$ is a bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions:*

(i) f is a homomorphism;

(ii) $\mu^-_{B_1}(x_1y_1) = \mu^-_{B_2}(f(x_1)f(y_1)), \mu^+_{B_1}(x_1y_1) = \mu^+_{B_2}(f(x_1)f(y_1))$;

(iii) $\nu^-_{B_1}(x_1y_1) = \nu^-_{B_2}(f(x_1)f(y_1)), \nu^+_{B_1}(x_1y_1) = \nu^+_{B_2}(f(x_1)f(y_1))$, for all $x_1 \in V_1, x_1y_1 \in E_1$.

It is clear that a co-weak isomorphism maintains only the weights of the arcs.

Example 3.3. Consider the IVIFGs G_1 and G_2 of G^*_1 and G^*_2 respectively,

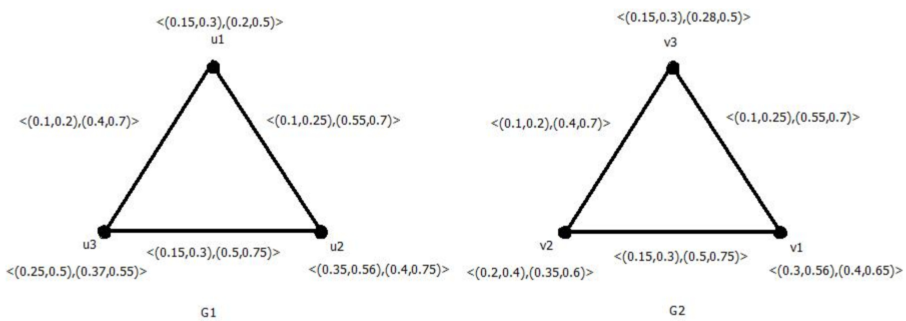

Fig 3 : co-weak Isomorphism of IVIFG

A map $f : V_1 \to V_2$ defined by $f(u_1) = v_3, f(u_2) = v_1$ and $f(u_3) = v_2$. Then we have:

$$\mu^-_{A_1}(u_1) = \mu^-_{A_2}(v_3), \mu^+_{A_1}(u_1) = \mu^+_{A_2}(v_3),$$
$$\nu^-_{A_1}(u_1) \neq \nu^-_{A_2}(v_3), \nu^+_{A_1}(u_1) = \nu^+_{A_2}(v_3).$$
$$\mu^-_{A_1}(u_2) \neq \mu^-_{A_2}(v_1), \mu^+_{A_1}(u_2) = \mu^+_{A_2}(v_1),$$
$$\nu^-_{A_1}(u_2) = \nu^-_{A_2}(v_1), \nu^+_{A_1}(u_2) \neq \nu^+_{A_2}(v_1).$$
$$\mu^-_{A_1}(u_3) \neq \mu^-_{A_2}(v_2), \mu^+_{A_1}(u_3) \neq \mu^+_{A_2}(v_2),$$
$$\nu^-_{A_1}(u_3) \neq \nu^-_{A_2}(v_2), \nu^+_{A_1}(u_3) \neq \nu^+_{A_2}(v_2).$$

But we see that

$$\begin{aligned}
&\mu^-_{B_1}(u_1u_2) = \mu^-_{B_2}(v_3v_1), \mu^+_{B_1}(u_1u_2) = \mu^+_{B_2}(v_3v_1),\\
&\nu^-_{B_1}(u_1u_2) = \nu^-_{B_2}(v_3v_1), \nu^+_{B_1}(u_1u_2) = \nu^+_{B_2}(v_3v_1)\\
&\mu^-_{B_1}(u_1u_3) = \mu^-_{B_2}(v_3v_2), \mu^+_{B_1}(u_1u_3) = \mu^+_{B_2}(v_3v_2),\\
&\nu^-_{B_1}(u_1u_3) = \nu^-_{B_2}(v_3v_2), \nu^+_{B_1}(u_1u_3) = \nu^+_{B_2}(v_3v_2)\\
&\mu^-_{B_1}(u_3u_2) = \mu^-_{B_2}(v_2v_1), \mu^+_{B_1}(u_3u_2) = \mu^+_{B_2}(v_2v_1),\\
&\nu^-_{B_1}(u_3u_2) = \nu^-_{B_2}(v_2v_1), \nu^+_{B_1}(u_3u_2) = \nu^+_{B_2}(v_2v_1).
\end{aligned}$$

Hence the map is a co-weak isomorpism but not an isomorphism.

Remark 3.1.

1. If $G_1 = G_2 = G$, then the homomorphism f over itself is called an endomorphism. An Isomorphism f over G is called an automorphism.
2. Let $A = (\mu^-_A, \mu^+_A, \nu^-_A, \nu^+_A)$ be an IVIFG with an underlying set V. Let $Aut(G)$ be the set of all bipolar intuitionistic automorphism of G. Let $e : G \to G$ be a map defined by $e(x) = x$, for all $x \in V$ clearly $e \in Aut(G)$.
3. If $G_1 = G_2$, then the weak and co-weak isomorphishms actually become isomorphic.
4. If $f : V_1 \to V_2$ is a bijective map then $f^{-1} : V_1 \to V_2$ is also a bijective map.

Definition 3.5. *An Interval-valued intuitionistic fuzzy set $A = (\mu^-_A, \mu^+_A, \nu^-_A, \nu^+_A)$ in a semigroup S is called a interval-valued intuitionistic subsemigroup of S if it satisfies the following conditions:*

$\mu^-_B(xy) \le (\mu^-_A(x) \wedge \mu^-_A(y))$, $\mu^+_B(xy) \le (\mu^+_A(x) \wedge \mu^+_A(y))$
$\nu^-_B(xy) \ge (\nu^-_A(x) \vee \mu^-_A(y))$, $\nu^+_B(xy) \ge (\nu^+_A(x) \vee \nu^+_A(y))$, *for all* $x, y \in S$.

Definition 3.6. *An Interval-valued intuitionistic fuzzy set $A = (\mu^-_A, \mu^+_A, \nu^-_A, \nu^+_A)$ in a group G is called a interval-valued intuitionistic fuzzy subgroup of a group G if it is a interval-valued intuitionistic fuzzy sub-semigroup of G and satisfies* $\mu^-_A(x^{-1}) = \mu^-_A(x)$, $\mu^+_A(x^{-1}) = \mu^+_A(x)$, $\nu^-_A(x^{-1}) = \nu^-_A(x)$, $\nu^+_A(x^{-1}) = \nu^+_A(x)$

We now show how to associate an interval-valued intuitionistic fuzzy group with a interval-valued intuitionistic fuzzy graph in a natural way.

Proposition 3.1. *Let $G = (A, B)$ be an IVIFG and let $Aut(G)$ be the set of all automorphisms of G. Then $(Aut(G), \circ)$ forms a group.*

Proof. We have the following conditions:

$$\begin{aligned}
\mu^-_A((\phi \circ \psi)(x)) &= \mu^-_A(\phi(\psi(x))) \le \mu^-_A(\phi(x)) \ge \mu^-_A(x),\\
\mu^+_A((\phi \circ \psi)(x)) &= \mu^+_A(\phi(\psi(x))) \le \mu^+_A(\phi(x)) \ge \mu^+_A(x),\\
\nu^-_A((\phi \circ \psi)(x)) &= \nu^-_A(\phi(\psi(x))) \ge \nu^-_A(\phi(x)) \ge \nu^-_A(x),\\
\nu^+_A((\phi \circ \psi)(x)) &= \nu^+_A(\phi(\psi(x))) \ge \nu^+_A(\phi(x)) \ge \nu^+_A(x),
\end{aligned}$$

$$\mu_B^-((\phi\circ\psi)(x))(\phi\circ\psi)(y))) = \mu_B^-(\phi(\psi(x)))\phi(\psi(y)) \leq \mu_B^-(\phi(x)\phi(y)) \leq \mu_B^-(xy),$$
$$\mu_B^+((\phi\circ\psi)(x))(\phi\circ\psi)(y))) = \mu_B^+(\phi(\psi(x)))\phi(\psi(y)) \leq \mu_B^+(\phi(x)\phi(y)) \leq \mu_B^+(xy),$$
$$\nu_B^-((\phi\circ\psi)(x))(\phi\circ\psi)(y))) = \nu_B^-(\phi(\psi(x)))\phi(\psi(y)) \geq \nu_B^-(\phi(x)\phi(y)) \geq \nu_B^-(xy),$$
$$\nu_B^+((\phi\circ\psi)(x))(\phi\circ\psi)(y))) = \nu_B^+(\phi(\psi(x)))\phi(\psi(y)) \geq \nu_B^+(\phi(x)\phi(y)) \geq \nu_B^+(xy).$$

Thus $\phi\circ\psi \in Aut(G)$. Clearly, $Aut(G)$ satisfies associativity under the operation $\circ$, $\phi\circ e = e\circ\phi$.

$\mu_A^-(\phi^{-1}) = \mu_A^-(\phi)$,$\mu_A^+(\phi^{-1}) = \mu_A^+(\phi)$,$\nu_A^-(\phi^{-1}) = \nu_A^-(\phi)$,$\nu_A^+(\phi^{-1}) = \nu_A^+(\phi)$, for all $\phi \in Aut(G)$.

Hence $(Aut(G), \circ)$ forms a group. □

Now we state some propositions without their proofs as follows.

Proposition 3.2. *Let $G = (A, B)$ be an IVIFG and let $Aut(G)$ be the set of all automorphisms of G. Let $g = (\mu_g^-, \mu_g^+, \nu_g^-, \nu_g^+)$ be an interval-valued intuitionistic fuzzy set in $Aut(G)$ defined by*

$$\mu_g^-(\phi) = \inf\{\mu_B^-(\phi(x), \phi(y)) : (x, y) \in V \times V\},$$
$$\mu_g^+(\phi) = \inf\{\mu_B^+(\phi(x), \phi(y)) : (x, y) \in V \times V\},$$
$$\nu_g^-(\phi) = \sup\{\nu_B^-(\phi(x), \phi(y)) : (x, y) \in V \times V\},$$
$$\nu_g^+(\phi) = \sup\{\nu_B^+(\phi(x), \phi(y)) : (x, y) \in V \times V\},$$

for all $\phi \in Aut(G)$. Then $g = (\mu_g^-, \mu_g^+, \nu_g^-, \nu_g^+)$ is an interval-valued intuitionistic fuzzy group on $Aut(G)$.

Proposition 3.3. *Every interval-valued intuitionistic fuzzy group has an embedding into the interval-valued intuitionistic fuzzy group of the group of automorphisms of some IVIFG.*

We now prove that the isomorphism (weak isomorphism) between IVIFG is an equivalence solution (partial order relation).

Proposition 3.4. *Let G_1, G_2, G_3 be IVIFGs. Then the isomorphism between these IVIFGs is an equivalence relation.*

Proof. Reflexivity property is obvious. To prove the symmetry, let $f : V_1 \to V_2$ be an isomorphism of G_1 onto G_2. Then f is bijective map defined by

$$f(x_1) = x_2, \quad \forall x_1 \in V_1 \tag{3.1}$$

satistying the following conditions:

(i) $\mu_{A_1}^-(x_1) = \mu_{A_2}^-(f(x_1))$, $\mu_{A_1}^+(x_1) = \mu_{A_2}^+(f(x_1))$;

(ii) $\nu_{A_1}^-(x_1) = \nu_{A_2}^-(f(x_1))$, $\nu_{A_1}^+(x_1) = \nu_{A_2}^+(f(x_1))$;

(iii) $\mu_{B_1}^-(x_1y_1) = \mu_{B_2}^-(f(x_1)f(y_1))$, $\mu_{B_1}^+(x_1y_1) = \mu_{B_2}^+(f(x_1)f(x_2))$;

(iv) $\nu_{B_1}^-(x_1y_1) = \nu_{B_2}^-(f(x_1)f(y_1))$, $\nu_{B_1}^+(x_1y_1) = \nu_{B_2}^+(f(x_1)f(x_2))$, for all $x_1 \in V_1, x_1y_1 \in E_1$.

Since f is bijective from 3.1 it follows that: $f^{-1}(x_2) = x_1$, for all $x_2 \in V_2$. Thus:

(i) $\mu^-_{A_1}(f^{-1}(x_2)) = \mu^-_{A_2}(x_2)$, $\mu^+_{A_1}(f^{-1}(x_2)) = \mu^+_{A_2}(x_2)$;

(ii) $\nu^-_{A_1}(f^{-1}(x_2)) = \nu^-_{A_2}(x_2)$, $\nu^+_{A_1}(f^{-1}(x_2)) = \nu^+_{A_2}(x_2)$, for all $x_2 \in V_2$;

(iii) $\mu^-_{B_1}(f^{-1}(x_2y_2)) = \mu^-_{A_2}(x_2y_2)$, $\mu^+_{A_1}(f^{-1}(x_2y_2)) = \mu^+_{A_2}(x_2y_2)$;

(iv) $\nu^-_{B_1}(f^{-1}(x_2y_2)) = \nu^-_{A_2}(x_2y_2)$, $\nu^+_{A_1}(f^{-1}(x_2y_2)) = \nu^+_{A_2}(x_2y_2)$, for all $x_2y_2 \in E_2$.

Hence a bijective map $f^{-1} : V_2 \to V_1$ is an isomorphism from G_2 onto G_2.

To prove the transitivity, let $f : V_1 \to V_2$ and $g : V_2 \to V_3$ be the isomorphisms of G_1 onto G_2 and G_2 onto G_3, respectively. Then $g \circ f : V_1 \to V_3$ is a bijective map from V_1 and V_3, where $(g \circ f)(x_1) = g(f(x_1))$, for all $x_1 \in V_1$. Since a map $f : V_1 \to V_2$ defined by $f(x_1) = x_2$, for all $x_1 \in V_1$ is an isomorphism, so we have

$$\begin{aligned}
&\mu^-_{A_1}(x_1) = \mu^-_{A_2}(f(x_1)) = \mu^-_{A_2}(x_2),\\
&\mu^+_{A_1}(x_1) = \mu^+_{A_2}(f(x_1)) = \mu^+_{A_2}(x_2),\\
&\nu^-_{A_1}(x_1) = \nu^-_{A_2}(f(x_1)) = \nu^-_{A_2}(x_2),\\
&\nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1)) = \nu^+_{A_2}(x_2), \forall x_1 \in V_1.
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
&\mu^-_{B_1}(x_1y_1) = \mu^-_{B_2}(f(x_1)f(y_1)) = \mu^-_{B_2}(x_2y_2),\\
&\mu^+_{B_1}(x_1y_1) = \mu^+_{B_2}(f(x_1)f(y_1)) = \mu^+_{B_2}(x_2y_2),\\
&\nu^-_{B_1}(x_1y_1) = \nu^-_{B_2}(f(x_1)f(y_1)) = \nu^-_{B_2}(x_2y_2),\\
&\nu^+_{B_1}(x_1y_1) = \nu^+_{B_2}(f(x_1)f(y_1)) = \nu^+_{B_2}(x_2y_2), \forall x_1y_1 \in E_2.
\end{aligned} \tag{3.3}$$

Since a map $g : V_2 \to V_3$ defined by $g(x_2) = x_3$ for $x_2 \in V_2$ is an isomorphism, We have

$$\begin{aligned}
&\mu^-_{A_2}(x_2) = \mu^-_{A_3}(g(x_2)) = \mu^-_{A_3}(x_3),\\
&\mu^+_{A_2}(x_2) = \mu^+_{A_3}(g(x_2)) = \mu^+_{A_3}(x_3),\\
&\nu^-_{A_2}(x_2) = \nu^-_{A_3}(g(x_2)) = \nu^-_{A_3}(x_3),\\
&\nu^+_{A_2}(x_2) = \nu^+_{A_3}(g(x_2)) = \nu^+_{A_3}(x_3), \forall x_2 \in V_2.
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
&\mu^-_{B_2}(x_2y_2) = \mu^-_{B_3}(g(x_2)g(y_2)) = \mu^-_{B_3}(x_3y_3),\\
&\mu^+_{B_2}(x_2y_2) = \mu^+_{B_3}(g(x_2)g(y_2)) = \mu^+_{B_3}(x_3y_3),\\
&\nu^-_{B_2}(x_2y_2) = \nu^-_{B_3}(g(x_2)g(y_2)) = \nu^-_{B_3}(x_3y_3),\\
&\nu^+_{B_2}(x_2y_2) = \nu^+_{B_3}(g(x_2)g(y_2)) = \nu^+_{B_3}(x_3y_3).
\end{aligned} \tag{3.5}$$

From 3.2 and 3.4 and $f(x_1) = x_2$,$x_1 \in V_1$, we have

$$\begin{aligned}
&\mu^-_{A_1}(x_1) = \mu^-_{A_2}(f(x_1)) = \mu^-_{A_3}(g(x_2)) = \mu_{A_3}(g(f(x_1)),\\
&\mu^+_{A_1}(x_1) = \mu^+_{A_2}(f(x_1)) = \mu^+_{A_3}(g(x_2)) = \mu_{A_3}(g(f(x_1)),\\
&\nu^-_{A_1}(x_1) = \nu^-_{A_2}(f(x_1)) = \nu^-_{A_3}(g(x_2)) = \nu_{A_3}(g(f(x_1)),\\
&\nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1)) = \nu^+_{A_3}(g(x_2)) = \nu_{A_3}(g(f(x_1)), \forall x_1 \in V_1.
\end{aligned} \tag{3.6}$$

From 3.3 and 3.5, we have

$$\begin{aligned}
\mu^-_{B_1}(x_1y_1) &= \mu^-_{B_2}(f(x_1)f(y_1)) = \mu^-_{B_2}(x_2y_2) = \mu^-_{B_3}(g(x_2)g(y_2))\\
&= \mu^-_{B_3}(g(f(x_1))g(f(y_1)),\\
\mu^+_{B_1}(x_1y_1) &= \mu^+_{B_2}(f(x_1)f(y_1)) = \mu^+_{B_2}(x_2y_2) = \mu^+_{B_3}(g(x_2)g(y_2))\\
&= \mu^+_{B_3}(g(f(x_1))g(f(y_1)),\\
\nu^-_{B_1}(x_1y_1) &= \nu^-_{B_2}(f(x_1)f(y_1)) = \nu^-_{B_2}(x_2y_2) = \nu^-_{B_3}(g(x_2)g(y_2))\\
&= \nu^-_{B_3}(g(f(x_1))g(f(y_1)),\\
\nu^+_{B_1}(x_1y_1) &= \nu^+_{B_2}(f(x_1)f(y_1)) = \nu^+_{B_2}(x_2y_2) = \nu^+_{B_3}(g(x_2)g(y_2))\\
&= \nu^+_{B_3}(g(f(x_1))g(f(y_1)), \forall x_1y_1 \in E_1.
\end{aligned} \tag{3.7}$$

Thus, we prove that $g \circ f$ is an isomorphism between G_1 and G_3.

Hence the proof. □

Proposition 3.5. *Let G_1, G_2, G_3 be IVIFGs. Then the weak isomorphism between these IVIFGs is a partial order relation.*

Proof. Reflexive property is obvious.

To prove the antisymmetry, let $f : V_1 \to V_2$ be a weak isomorphism of G_1 onto G_2. Then f is a bijective map defined by $f(x_1) = x_2$, for all $x_1 \in V_1$ satisfying the following

$$\begin{aligned}
&(i) \quad \mu^-_{A_1}(x_1) = \mu^-_{A_2}(f(x_1)), \mu^+_{A_1}(x_1) = \mu^+_{A_2}(f(x_2)),\\
&(ii) \quad \nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1)), \nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_2)),\\
&(iii) \quad \mu^-_{B_1}(x_2y_2) \le \mu^-_{B_2}(f(x_1)f(y_1)), \mu^+_{B_1}(x_1y_1) \le \mu^+_{B_1}(f(x_1)f(y_1),\\
&(iv) \quad \nu^-_{B_1}(x_2y_2) \ge \nu^-_{B_2}(f(x_1)f(y_1)), \nu^+_{B_1}(x_1y_1)\\
&\qquad \ge \nu^+_{B_1}(f(x_1)f(y_1).x_1 \in V_1, \forall x_1y_1 \in E_1.
\end{aligned} \tag{3.8}$$

Let $g : V_2 \to V_1$ be a weak isomorphism of G_2 onto G_1. Then g is a bijective map defined by $g(x_2) = x_1$, for all satisfying

$$\begin{aligned}
&\mu^-_{A_2}(x_2) = \mu^-_{A_1}(g(x_2)), \mu^+_{A_2}(x_2) = \mu^+_{A_1}(g(x_2)),\\
&\nu^-_{A_2}(x_2) = \nu^-_{A_1}(g(x_2)), \nu^+_{A_2}(x_2) = \nu^+_{A_1}(g(x_2)), \forall x_1 \in V_2\\
&\mu^-_{B_2}(x_2y_2) \le \mu^-_{B_1}(g(x_2)g(y_2)), \mu^+_{B_1}(x_2y_2) \le \mu^+_{B_1}(g(x_2)g(y_2),\\
&\nu^-_{B_2}(x_2y_2) \ge \nu^-_{B_1}(g(x_2)g(y_2)), \nu^+_{B_1}(x_2y_2) \ge \nu^+_{B_1}(g(x_2)g(y_2), \forall x_2y_2 \in E_2.
\end{aligned} \tag{3.9}$$

The inequalities 3.8 and 3.9 holds on the finite sets V_1 and V_2 only when G_1 and G_2 have the same number of edges and the corresponding edges have weight. Hence G_1 and G_2 are identical.

To prove the transitivity, let $f : V_1 \to V_2$ and $g : V_2 \to V_3$ be the isomorphisms of G_1 onto G_2 and G_2 onto G_3, respectively. Then $g \circ f : V_1 \to V_3$ is a bijective map from V_1 and V_3, where $(g \circ f)(x_1) = g(f(x_1))$, for all $x_1 \in V_1$. Since a map $f : V_1 \to V_2$ defined by $f(x_1) = x_2$, for all $x_1 \in V_1$ is a weak isomorphism, so we have

$$\begin{aligned} &\mu^-_{A_1}(x_1) = \mu^-_{A_2}(f(x_1)) = \mu^-_{A_2}(x_2), \\ &\mu^+_{A_1}(x_1) = \mu^+_{A_2}(f(x_1)) = \mu^+_{A_2}(x_2), \\ &\nu^-_{A_1}(x_1) = \nu^-_{A_2}(f(x_1)) = \nu^-_{A_2}(x_2), \\ &\nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1)) = \nu^+_{A_2}(x_2), \forall x_1 \in V_1. \end{aligned} \tag{3.10}$$

$$\begin{aligned} &\mu^-_{B_1}(x_1y_1) \leq \mu^-_{B_2}(f(x_1)f(y_1)) = \mu^-_{B_2}(x_2y_2) \\ &\mu^+_{B_1}(x_1y_1) \leq \mu^+_{B_2}(f(x_1)f(y_1)) = \mu^+_{B_2}(x_2y_2) \\ &\nu^-_{B_1}(x_1y_1) \geq \nu^-_{B_2}(f(x_1)f(y_1)) = \nu^-_{B_2}(x_2y_2) \\ &\nu^+_{B_1}(x_1y_1) \geq \nu^+_{B_2}(f(x_1)f(y_1)) = \nu^+_{B_2}(x_2y_2), \forall x_1y_1 \in E_1. \end{aligned} \tag{3.11}$$

Since a map $g : V_2 \to V_3$ defined by $g(x_2) = x_3$ for $x_2 \in V_2$ is a weak isomorphism, We have

$$\begin{aligned} &\mu^-_{A_2}(x_2) = \mu^-_{A_3}(g(x_2)) = \mu^-_{A_3}(x_3), \\ &\mu^+_{A_2}(x_2) = \mu^+_{A_3}(g(x_2)) = \mu^+_{A_3}(x_3), \\ &\nu^-_{A_2}(x_2) = \nu^-_{A_3}(g(x_2)) = \nu^-_{A_3}(x_3), \\ &\nu^+_{A_2}(x_2) = \nu^+_{A_3}(g(x_2)) = \nu^+_{A_3}(x_3), \forall x_2 \in V_2. \end{aligned} \tag{3.12}$$

$$\begin{aligned} &\mu^-_{B_2}(x_2y_2) \leq \mu^-_{B_3}(g(x_2)g(y_2)) = \mu^-_{B_3}(x_3y_3), \\ &\mu^+_{B_2}(x_2y_2) \leq \mu^+_{B_3}(g(x_2)g(y_2)) = \mu^+_{B_3}(x_3y_3), \\ &\nu^-_{B_2}(x_2y_2) \geq \nu^-_{B_3}(g(x_2)g(y_2)) = \nu^-_{B_3}(x_3y_3), \\ &\nu^+_{B_2}(x_2y_2) \geq \nu^+_{B_3}(g(x_2)g(y_2)) = \nu^+_{B_3}(x_3y_3), \forall x_1y_1 \in E_1. \end{aligned} \tag{3.13}$$

From 3.10 and 3.12 and $f(x_1) = x_2$,$x_1 \in V_1$, we have

$$\begin{aligned} &\mu^-_{A_1}(x_1) = \mu^-_{A_2}(f(x_1)) = \mu^-_{A_3}(g(x_2)) = \mu_{A_3}(g(f(x_1)), \\ &\mu^+_{A_1}(x_1) = \mu^+_{A_2}(f(x_1)) = \mu^+_{A_3}(g(x_2)) = \mu_{A_3}(g(f(x_1)), \\ &\nu^-_{A_1}(x_1) = \nu^-_{A_2}(f(x_1)) = \nu^-_{A_3}(g(x_2)) = \nu_{A_3}(g(f(x_1)), \\ &\nu^+_{A_1}(x_1) = \nu^+_{A_2}(f(x_1)) = \nu^+_{A_3}(g(x_2)) = \nu_{A_3}(g(f(x_1)), \forall x_1 \in V_1. \end{aligned} \tag{3.14}$$

From 3.11 and 3.13, we have

$$
\begin{aligned}
\mu^-_{B_1}(x_1y_1) &\le \mu^-_{B_2}(f(x_1)f(y_1)) = \mu^-_{B_2}(x_2y_2) = \mu^-_{B_3}(g(x_2)g(y_2))\\
&= \mu^-_{B_3}(g(f(x_1))g(f(y_1)),\\
\mu^+_{B_1}(x_1y_1) &\le \mu^+_{B_2}(f(x_1)f(y_1)) = \mu^+_{B_2}(x_2y_2) = \mu^+_{B_3}(g(x_2)g(y_2))\\
&= \mu^+_{B_3}(g(f(x_1))g(f(y_1)),\\
\nu^-_{B_1}(x_1y_1) &\ge \nu^-_{B_2}(f(x_1)f(y_1)) = \nu^-_{B_2}(x_2y_2)\\
&= \nu^-_{B_3}(g(x_2)g(y_2))\\
&= \nu^-_{B_3}(g(f(x_1))g(f(y_1)),\\
\nu^+_{B_1}(x_1y_1) &\ge \nu^+_{B_2}(f(x_1)f(y_1)) = \nu^+_{B_2}(x_2y_2) = \nu^+_{B_3}(g(x_2)g(y_2))\\
&= \nu^+_{B_3}(g(f(x_1))g(f(y_1)), \forall x_1y_1 \in E_1.
\end{aligned} \tag{3.15}
$$

Thus, we prove that $g \circ f$ is a weak isomorphism between G_1 and G_3. Hence the proof. □

4. Conclusion

Interval-valued intuitionistic fuzzy graph have numerous application in the real life systems and real life applications where the level of information inherited in the system varies with respect to time and have different level of precision. Most of the actions in real life situations are time dependent and also ambigous in partial information, symbolic models in expert system are more effective than traditional methods to identify the upper and lower bounds of the true and false membership values in an interval. In this paper, we introduced the concept of automorphism on IVIFG. Also we investigate the properties of morphism on IVIFG.

References

[1] M. Akram, M.G. Karunambigai, *Metric in bipolar fuzzy graphs*, World Applied Sciences Journal, 14 (2011), 1920-1927.

[2] K.T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20 (1986), 87-96.

[3] K.T. Atanassov, G. Pasi, R. Yager, V. Atanassova, *Intuitionistic fuzzy graph, interpretations of multi-person multi-criteria decision making*, Proceedings of EUSFLAT Conf, (2003), 177-182.

[4] P. Bhattacharya, *Some remarks on fuzzy graphs*, Pattern Recog. Lett., 7 (198), 297-302.

[5] R.A. Borzooei, H. Rashmanlou, *New concepts of vague graphs*, Int. J. Mach. Learn. Cybern.

[6] R.A. Borzooei, H. Rashmanlou, *Degree of vertices in vague graphs*, J. Appl. Math. Inf., 33 (2015), 545-557.

[7] R.A. Borzooei, H. Rashmanlou, *More results on vague graphs*, UPB Sci. Bul. Ser. A, 78, (2016), 109-122.

[8] R.A. Borzooei, H. Rashmanlou, *Semi global domination sets in vague graphs with application,* J. Intell. Fuzzy Syst., 30 (2016), 3645-3652.

[9] R.A. Borzooei, H. Rashmanlou, *Degree and total degree of edges in bipolar fuzzy graphs with application,* J. Intell. Fuzzy Syst., 30 (2016), 3271-3280.

[10] Ganesh Ghorai and Madhumangal Pal, *Regular product vague graphs and product vague line graphs*, Cogent Mathematics 3(1)(2016), 1-13 .

[11] Ganesh Ghorai and Madhumangal Pal, *Planarity in vague graphs with applications*, Acta Mathematica Academiae Paedagogicae Nyiregyhaziensis, 33 (2017), 1-21.

[12] Ganesh Ghorai and Madhumangal Pal, *Faces and Dual of m-polar fuzzy planar graphs,* Journal of Intelligent Fuzzy system, 31 (2016), 2043-2049.

[13] Ganesh Ghorai and Madhumangal Pal, *A Note on "Regular bipolar fuzzy graphs"*, Neural Computing and Applications, 21 2012, 197-205.

[14] Ganesh Ghorai and Madhumangal Pal, *Some isomorphic properties of m-polar fuzzy graphs with applications,* SpringerPlus, 5 (2016), 1-21.

[15] Ganesh Ghorai and Madhumangal Pal, *Novel concepts of strongly edge irregular m-polar fuzzy graphs*, International Journal of Applied and Computational Mathematics, DOI: 10.1007/s40819-016-0296-y, (2016).

[16] M.G. Karunambigai,R. Parvathi, O.K. Kalaivani, *A study on Atanassov's intuitionistic fuzzy graphs*, Proceedings of the International Conference on Fuzzy Systems, FUZZ-IEEE, Taipei, Taiwan, 2011, 157-167.

[17] M.G. Karunambigai, O.K. Kalaivani, *Self centered intuitionistic fuzzy graph*, World Applied Sciences Journal, 14 (2011), 1928-1936.

[18] A. Kauffman, *Introduction a la theorie des sous-emsembles flous*, Masson etCie, 1973.

[19] J.N. Mordeson, P.S. Nair, *Fuzzy graphs and fuzzy hypergraphs*, physicaVerlag, Heidelberg, 1998, Second Edition, 2001.

[20] M. Pal, H. Rashmanlou, *Irregular interval-valued fuzzy graphs*, Ann. Pure Appl. Math, 3 (2013), 56-66.

[21] A. Pal, S. Samanta, M. Pal, *Concept of fuzzy planar graphs*, Proceedings of Science and Information Conference, (2013), October 7-9, 2013, London, UK, 557-563.

[22] M. Pal, S. Samanta, A. Pal, *Fuzzy k-competition graphs*, Proceedings of Science and Information Conference, 2013, October 7-9, London, UK, 572-576.

[23] A. Rosenfeld. Fuzzy graphs, *Fuzzy sets and their applications*, (L. A. Zadeh, K. S. Fu, M.Shimura, Eds.), Academic Press New York, 1975, 77-95.

[24] H. Rashmanlou, Y.B. Jun, *Complete interval-valued fuzzy graphs*, Ann. Fuzzy Math. Inform., 3 (2013), 677-687.

[25] H. Rashmanlou, M. Pal, *Antipodal interval-valued fuzzy graphs,* International Journal of Applications of Fuzzy Sets and Artificial Intelligence, 3 (2013), 107-130.

[26] H. Rashmanlou, M. Pal, *Balanced interval-valued fuzzy graph,* Journal of Physical Sciences, 17 (2013), 43-57.

[27] H. Rashmanlou, M. Pal, *Isometry on interval-valued fuzzy graphs,* Int. J. Fuzzy Math. Arch., 3 (2013), 28 -35.

[28] H. Rashmanlou, M. Pal, *Some properties of highly irregular interval-valued fuzzy graphs,* World Applied Sciences Journal, 27 (2013), 1756-1773.

[29] H. Rashmanlou, M. Pal, *Intuitionistic fuzzy graphs with categorical properties,* Fuzzy Information and Engineering, 7 (2015), 317-384.

[30] H. Rashmanlou, Young Bae Jun and R.A. Borzoei, *More results on highly irregular bipolar fuzzy graphs*, Annals of Fuzzy Mathematics and Informatics, 2014, 2287-6235.

[31] S. Samanta, M. Pal, *Fuzzy k-competition graphs and p-competition fuzzy graphs*, Fuzzy Inf. Eng., 5 (2013), 191-204.

[32] A. Shannon, K.T. Atanassov, *Intuitionistic fuzzy graphs from α, β, and (a, b)levels*, Notes on Intuitionistic Fuzzy Sets, 1 (1995), 32-35.

[33] A. Shannon, K.T. Atanassov, *A first step to a theory of the intuitionistic fuzzy graphs*, Proceeding of FUBEST (Lakov, D., Ed.), Sofia, 1994, 59-61.

[34] S. Samanta, M. Pal, *Fuzzy tolerance graphs,* Int. J. Latest Trend. Math., 1 (2011), 57-67.

[35] S. Samanta, M. Pal, *Bipolar fuzzy hypergraphs*, Int. J. Fuzzy Logic Syst., 2 (2012), 17-28.

[36] A.A. Talebi, H. Rashmanlou, *Isomorphism on interval-valued fuzzy graphs*, Ann. Fuzzy Math. Inform., 6 (2013), 47-58.

[37] A.A. Talebi, H. Rashmalou, N. Mehdipoor, *Isomorphism on vague graphs*, Ann. Fuzzy Math. Inform., 6 (2013), 575-588.

[38] A.A. Talebi, H. Rashmanlou and Reza Ameri, *New concepts on product interval-valued fuzzy graphs*, Journal of Applied Mathematics and Informatics, 34 (2016), 179-192.

Accepted: 5.10.2017