DETERMINATION OF THE SHORTEST PATH IN VAGUE NETWORKS

P.K. Kishore Kumar*
Department of Information Technology
Al Musanna College of Technology
Sultanate of Oman
kishorePK@act.edu.om

S. Lavanya
Department of Mathematics
Bharathi Women's College
Chennai
India
lavanyaprasad1@gmail.com

Hossein Rashmanlou
Department of Computer Science
University College of Adib
Sari
Iran
rashmanlou.1987@gmail.com

Mostafa Nouri Jouybari
Department of Mathematics
Payame Noor University(PNU)
P.O. Box 19395-3697
Tehran
Iran
m_njoybari@pnu.ac.ir

Abstract. We propose a new approach to determine the shortest path in a vague network(VN), a network in which vertices and edges remain crisp but each edge $(i, i+1)$ has an associated weight, which is a vague number of the form $[R_{it}, R_{if}]$ for each i. For each VN, we associate two vague networks called true and false limit fuzzy networks having the same set of vertices and edges but each edge $(i, i+1)$ is attached with a vague weight R_{it} and R_{if} respectively. We exhibit that the shortest path of weight $w = [w_t, w_f]$ an vague number in VN, the path for which the shortest path of weight w_t in the true fuzzy network coincides with the shortest path of weight w_t in the true limit vague network. The concept is illustrated with the help of a simple situation and the validation of mathematical verification is provided.

Keywords: fuzzy network, vague network (VN), shortest path, vague shortest path.

*. Corresponding author

1. Introduction

Graph theory has found its importance in many real time problems. Recent applications in graph theory is quite interesting analysing any complex situations and moreover in engineering applications. It has got numerous applications on operations research, system analysis, network routing, transportation and many more. In 1975, Rosenfeld [22] discussed the concept of fuzzy graphs whose ideas are implemented by Kauffman [16] in 1973. The fuzzy relation between fuzzy sets were also considered by Rosenfeld who developed the structure of fuzzy graphs, obtaining various analagous results of several graph theoretical concepts. Bhattacharya [4] gave some remarks of fuzzy graphs. The complement of fuzzy graphs was introduced by Mordeson [17]. Atanassov introduced the concept of intuitionistic fuzzy relation and intuitionistic fuzzy graphs [2, 3, 28, 29]. Talebi and Rashmanlou [41] studied the properties of isomorphism and complement of interval-valued fuzzy graphs. They defined isomorphism and some new operations on vague graphs [42, 43]. Borzooei and Rashmalou analysed new concepts of vague graphs [5], degree of vertices in vague graphs [6], more results on vague graphs [7], semi global domination sets in vague graphs with application [8] and degree and total degree of edges in bipolar fuzzy graphs with application [9]. Rashmanlou *et.al.* defined the complete interval-valued fuzzy graphs [23]. Rashmanlou and Pal studied intuitionistic fuzzy graphs with categorical properties [28], some properties of highly irregular interval-valued fuzzy graphs [27], more results on highly irregular bipolar fuzzy graphs [29], balanced interval-valued fuzzy graphs [25] and antipodal interval-valued fuzzy graphs [24]. Samanta and Pal investigated fuzzy k-competition and p-competition graphs, and concept of fuzzy planar graphs in [20, 21, 30]. Also they introduced fuzzy tolerance graph [39], bipolar fuzzy hypergraphs [40] and given several properties on it. Pal and Rashmanlou [19] defined many properties of irregular interval-valued fuzzy graphs. Ganesh *et al.* [12, 13] analysed the properties of Regular product vague graphs and product vague line graphs.

In graph theory the shortest path problem is the problem of finding a path between two vertices such that sum of the weight of its constituent edges is minimized. An example is finding the shortest way to get from one location to another on a road map. The vertices(or nodes) represents the locations and are weighted by the time needed to travel that segment and the edges(or links) represents the roads leading to various places connected through out the destination point. The shortest path problem has transportation, communication routing and scheduling. Now, in any network path the arc length may represent time or cost. Therefore in the real world, it can be considered to be a fuzzy set.To analyse any complete information we make intensive use of graphs and its properties. For working on partial information or incomplete information or to handle the systems containing the elements of uncertainty we understand that fuzzy logic and its involvement in graph theory is applied.

We analyse the shortest path of any source to destination using vague networks(VN). We consider the directed network consisting of a finite set of vertices and finite set of edges. It is assumed that there is only one edge between any two vertices. The fuzzy shortest problem was first analysed by Dubois and Prade [11] . They used Floyd's algorithm and Ford's algorithm to treat the fuzzy shortest path problem. Although in their method of shortest length could be obtained but sometimes the corresponding path in the network does not exist. [15]Klein proposed a dynamical programming recursion based fuzzy algorithm [10] and later developed by many researchers. Recently, the concept of Interval valued fuzzy matrices(IVFM) as a generalization of fuzzy matrix was introduced by Shyamal and Pal [38], by extending the max-min operations on Fuzzy algebra $F = [0, 1]$ for elements $a, b \in F, a + b = \max\{a, b\}$ and $a.b = \min\{a, b\}$. Let F_{mn} be the set of all $m \times n$ fuzzy matrices over the fuzzy algebra with support $[0, 1]$, ie., the matrices whose entries are intervals and all the intervals are subintervals of the interval $[0, 1]$, then $\max\{a_i, b_i\} = [\max\{a_{iL}, b_{iL}\}, \max\{a_{iU}, b_{iU}\}]$. In earlier works, represented Interval Valued Fuzzy Matrices $A = (a_{ij}) = ([a_{ijL}, a_{ijU}])$ where each a_{ij} is a subinterval of the interval $[0, 1]$ as the interval matrix $A = [A_L, A_U]$ whose ij^{th} entry is the interval $[a_{ijL}, a_{ijU}]$, where the lower limit $A_L = (a_{ijL})$ and the upper limit $A_U = (a_{ijU})$ are fuzzy matrices such that the $A_L \leq A_U$ that is $a_{ijL} \leq a_{ijU}$ under the usual ordering of real numbers. In this paper, we adopt a similar technique to determine the shortest path for an vague network (VN), that is the path in which the sum of the weight of its constituent edges is minimized, by way of constructing two vague networks corresponding to the true and false limits for an VN as a generalisation of fuzzy shortest path technique presented in [15].Meenakshi *et al.* [18] determined the shortest path in interval-valued fuzzy networks. Sahoo *et al.* [31, 32, 33, 34, 35] analysed about different types of product on intuitionistic fuzzy graphs, intuitionistic fuzzy competition graph, intuitionistic fuzzy tolerance graph with application, product on intuitionistic fuzzy graphs and degree, covered and paired domination in intuitionistic fuzzy graphs. We propose a new approach to determine the shortest path in VN in which the edges representing the roads connecting the cities and each edge $(i, i+1)$ has an associated weight representing the traffic on the road connecting the cities i and $i+1$, which is an vague number of the form $R_i = [R_{iT}, R_{iF}]$ for each i and we apply the technique used in [15] to determine the shortest path in true and false limits of the fuzzy networks. We have defined the shortest path of VN as the path for which the shortest path in true limit vague network coincides with the shortest path in false limit vague networks and weight $[w_T, w_F]$ where w_T and w_F are the weights of the shortest path for true and false networks respectively.In this work we analyse the shortest path of vague networks using DP recursion algorithm. For further terminologies, the readers are referred to [1-6,14,15].

2. Preliminaries

A graph (V, E) be a set of points V and a set of pairs of these points E. The set V refers to the vertices of the graph and the set E refers to the edges of the graph. An edge is denoted by a pair of vertices $\{i, j\}$. If E is changed to a set of ordered pairs of distinct elements of V, then $G : (V, E)$ is a directed graph and E is the set of ordered pairs (i, j) . The ordered pairs (i, j) are referred to as arcs or edges and an arc goes from vertex i to vertex j. An arc (i, i) is referred to as a loop. A path from a vertex s to a vertex t is a sequence of arcs of the form $(p, i_1), (i_1, i_2)...(i_k, q)$.

If each arc (i, j) has an associated weight or length C_{ij} , then an (p, q) path has an associated weight or length equal to sum of the weights of the constituent arcs in the path. This in turn gives rise to the shortest path problem, which is to find the path with minimal weight between two vertices p and q. There are different ways to find one shortest path for a network. Some of the more general methods such as the labeling algorithm follow from dynamic progamming. It is assumed that graphs for the models to be presented or directed graphs, that is graph without cycles.

For an acyclic directed graphs $G : (V, E)$ with N vertices numbered from 1 to N such that '1' is the source and 'N' is the sink, a dynamic programming(DP) formulation for the shortest path problem is given as in

$$f_i(S_{i+1}) = \min_{x_i}(R_i(X_i, S_{i+1}) + f_{i-1}(S_1)), \tag{2.1}$$

where $f_{i-1}(S_i)$ denotes the optimal value of the objective function corresponding to the last $i-1$ stages and S_i is the input to the stage $i-1$, X_i denotes the vector of decision variable at stage i, $R_i(X_i, S_{i+1})$ is the return function of the stage i and $f_i(S_{i+1})$ denotes the optimal value of the objective function corresponding to the last i stages and S_{i+1} is the input to the stage i. Throughout the algorithm, vertex i is labeled with $f(i)$, and labels allow the determination of the path.

Through Belman's principle of optimality this recursion is very flexible and has many applications. One obvious flexibility is that the sum in can be replaced by almost any binary operator and the recursion will hold in . for the fuzzy optimization problems under that max-min composition, the sum in is the fuzzy addition and is reformulated as

$$f_i(S_{i+1}) = \min_{x_i}(R_i(X_i, S_{i+1}) + f_{i-1}(S_1)). \tag{2.2}$$

3. Shortest path of an VN

A vague network includes nodes and directed links. Each node represents a city. Each directed links $(i, i + 1)$ connects city i to $i + 1$. Let $X_i = \{X_1, X_2, X_3, ..., X_{i-1}\}$ denotes the vector of decision variable at stage i and $S_i = \{S_1, S_2, ..., S_{i+1}\}$ is the input to the stage $i - 1$. f_{i-1} denotes the fuzzy optimal value of the objective function corresponding to the last $i - 1$ stages.

If $X_i : R_i \to S_{i+1}$, then it indicates that the degree of relevance from stage i to stage $i+1$ is R_i , where R_i is a sub interval of $[0,1]$. Let $R_i = [R_{iT}, R_{iF}]$.

Since R_i is an interval of $[0,1]$, $R_{it,R_{if}}$, $R_i(X_i, S_{i+1})$ is the weight of the corresponding arc $(i, i+1)$. For this vague network(VN), let us construct two networks which we call as true limit vague network $(VN)_T$ and false limit vague network $(VN)_F$ with the same set of nodes and links, the weight of the corresponding arc $(i, i+1)$ in the lower limit vague network is R_{iT} and in the upper limit vague network in R_{iF}.

The vague shortest path networks can also be viewed in terms of the Dynamic programming (DP) recursion given in equation (2.1) . This recursion is very close to Ford's Algorithm and is easily extended to vague numbers as in equation (2.2). Then the DP recursion for lower vague network is

$$f_{iT}(S_{i+1}) = \min_{X_i}\{\max[R_{iT}(X_i, S_{i+1}), f_{(i-1)T}(S_i)]\}, \tag{3.1}$$

where $f_{(i-1)T}(S_i)$ denotes the optimal value of the objective function corresponding to the last $i-1$ stages S_i is the input to the stage $i-1$ of lower vague networks $(VN)_t$, X_i denotes the vector of decision variable at stage i, $R_{iT}(X_i, S_{i+1})$ is the return function of the stage i and $f_{iT}(S_{i+1})$ denotes the optimal value of the objective function corresponding to the last i stages and S_{i+1} is the input to the stage i of lower vague networks $(VN)_T$. DP recursion for upper vague network is

$$f_{iF}(S_{i+1} = \min_{X_i}\{\max[R_{iF}(X_i, S_{i+1}), f_{(i-1)F}(S_i)]\}. \tag{3.2}$$

Let us define DP recursion for Interval valued fuzzy network as,

$$f_{i-1}(S_i) = [f_{(i-1)T}(S_i), f_{(i-1)F}(S_i)]. \tag{3.3}$$

Then by recursion

$$f_i(S_{i+1}) = [f_{(i-1)T}(S_{i+1}), f_{(i-1)F}(S_{i+1})]. \tag{3.4}$$

By previous equations we get the equation

$$\begin{aligned} f_i(S_{i+1}) &= [\min_{X_i}\{\max[R_{iT}(X_i, S_{i+1}), f_{(i-1)T}(S_i)]\}, \\ &\quad \min\{\max[R_{iF}(X_i, S_{i+1}), f_{(i-1)F}(S_i)]\} \\ &= [\min_{X_i}\{\max\{R_{iT}(X_i, S_{i+1}), R_{iF}(X_i, S_{i+1})], [f_{(i-1)T}(S_i), f_{(i-1)F}(S_i)\}\}] \\ &= [\min_{X_i}\{\max[R_iF(X_i, S_{i+1}), f_{i-1}(S_i)]\}], \end{aligned} \tag{3.5}$$

where $f_i(S_{i+1})$ denotes the optimal value of the objective function corresponding to the last i stages and S_{i+1} is the input to the stage i of vague networks (VN) , $f_{(i-1)}(S_i)$ denotes the optimal value of the objective function corresponding to the last $i-1$ stages and S_i is the input to the stage $i-1$ of vague networks (VN) , X_i denotes the vector of decision variable at stage i , $R_i(X_i, S_{i+1})$ is the return function of the stage i of vague networks (VN).

Definition 3.1. *Shortest path in VN = Shortest path in true limit vague network* $(VN)_T$ *= Shortest path in false limit vague network* $(VN)_F$*. Weight of the shortest path of* $VN = [W_T, W_F]$ *where* W_T *and* W_F *are weights of the fuzzy shortest path in* $(VN)_T$ *and* $(VN)_F$ *respectively.*

Algorithm.

Step 1: Identify the decision variables and specify objective function to be optimized for vague networks.

Step 2: Decompose the network into a number of smaller sub intervals. Identify the stage variable at each stage and write down the vague transformation function as a function of the state variable and decision variable at the next stage.

Step 3: Write down a general recursive relationship for completing the vague optimal policy of VN by using the interval valued fuzzy dynamic programming recursion in (3.4) and (3.7).

Step 4: Construct appropriate stage to show the required values of the return function at each Stage in VN.

Step 5: Determine the overall fuzzy optimal decision or policy and its value at each stage of an VN.

Step 6: We get the shortest path of IVFN.

Now, A_N^t be the vague networks, representing the weight of N during time interval t.

$$A_N^t = [A_{NT}^t, A_{NF}^T], \tag{3.6}$$

where A_{NT}^t is the true limit (R_{iT}) of the vague network and A_{NF}^T is false limit (R_{iF}) of the vague network. Then,

$$\text{shortest path in } A_N^t = \text{ shortest path in } A_{NT}^t = \text{shortest path in } A_{NF}^t. \tag{3.7}$$

Weight of the shortest path of VN =

$$\begin{aligned} &[\text{Weight of the shortest path in } A_{NT}^t,\\ &\ \text{Weight of the shortest path in } A_{NF}^t]. \end{aligned} \tag{3.8}$$

We shall illustrate the technique with a simple example and provide the mathematical verification.

Example 3.1. We consider a network $N = (V, E)$ consisting n nodes (cities) and m edges (roads) connecting the cities of a country. If we measure the crowdness that is traffic of the roads of the network for particular time duration. It is quite tough to measure the crowdness in a duration as it is not fixed, but varies from time to time. So, appropriate technique to grade the crowdness deals with an interval and not a fixed point. Hence we use the concept of vague measures true and false limits to analyse the crowdness range.

The network N is a vague network in which the weight of each arc $(i, i+1)$ depends upon the crowdness.

Suppose that we want to select the shortest highway route(path) between two cities. The following route network provices the possible routes between the starting city at node 1 and the destination city at node 7. The routes pass through intermediate cities designated by nodes 2 to 6. By using our

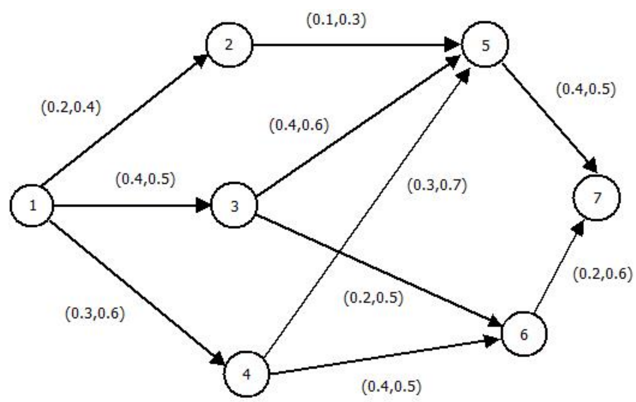

Fig 1: Vague graph with true and false limits

representation , $A_N^t = [A_{NT}^t, A_{NF}^t]$ Now we apply the algorithm to find a path

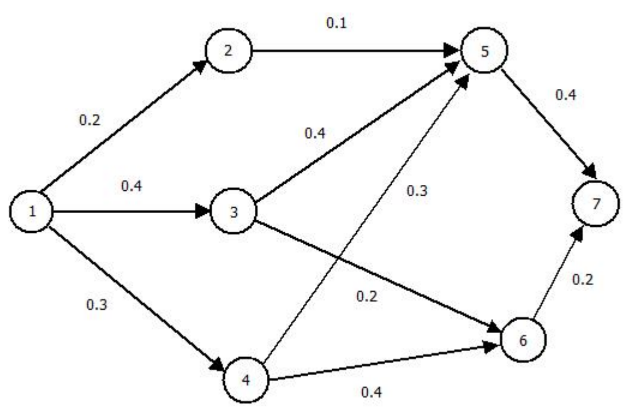

Fig 2 : Vague graph with true limits

between city 1 to city 7 which is minimum among all the paths between city 1 to city 7.

(i) Shortest path for the true limit vague network.

First we decompose the true limit vague network into sub networks or stages as Now S_1 is the state in which the node 1 lies also, S_1 has only state value $S_1 = 1$. State S_2 has only three possible values say 2,3 and 4 corresponding to stage 1 and so on. Possible alternative paths from one stage

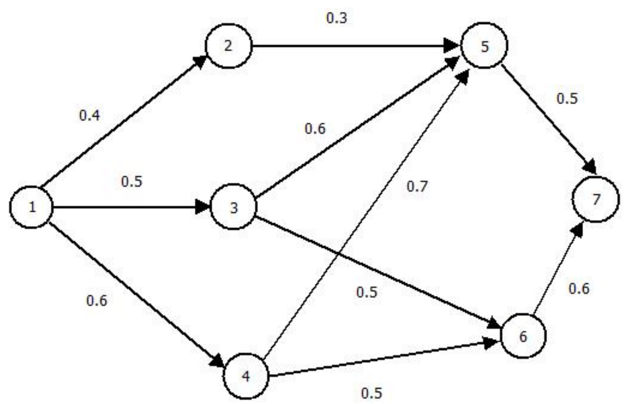

Fig 4: Vague graph with false limits

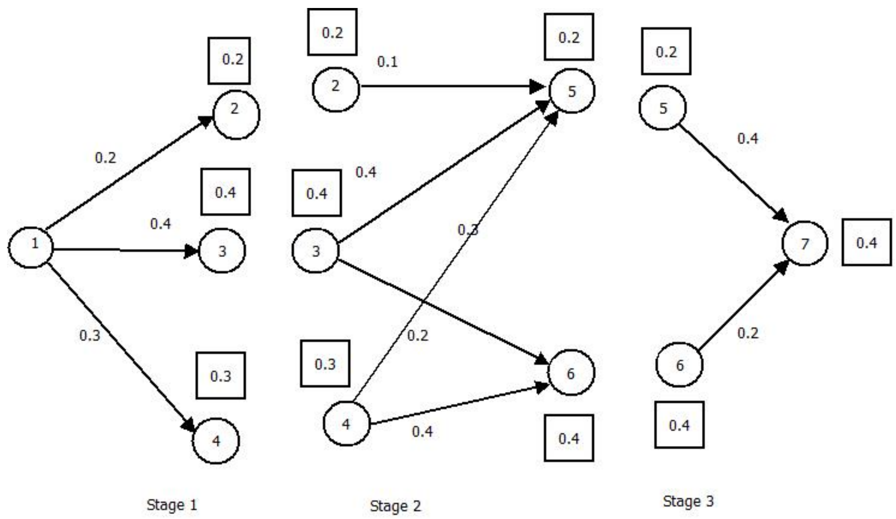

Fig 3 : Vague true network with stages

to the other will be called decision variables by X_i the decision which takes from S_{i-1} to S_i. The return or the gain which obviously being the function of decision will be denoted by $R_{iT}(X_i, S_{i+1})$. Here $R_{iT}(X_i, S_{i+1})$ can be identified with the true limit of the corresponding arc. By equation we have $f_{iT}(S_{i+1}) = \min_{X_i}\{\max[R_{iT}(X_i, S_{i+1}), f_{(i-1)T}(S_i)]\}$. Now initially for $i = 0$, $f_i(S_{i+1}) = f_0(S_1) = f_0(1) = 0$.

For Stage 1, (i=1), $f_1(S_2) = \min_{X_1}\{\max[R_{1T}(X_1, S_2), f_0(S_1)]\}$
$= \min_{X_1}[R_{1T}(X_1, S_2)]$.

Now tabulating the date for $f_1(S_2)$

S_1	S_2	X_i	$R_{iT}(X_1, S_2)$	$f_1(S_2)$	fuzzy optimal policy
1	2	1-2	0.2	0.2	1-2
	3	1-3	0.4	0.4	1-3
	4	1-4	0.3	0.3	1-4

For Stage 2 ($i = 2$), $f_2(S_3) = \min_{X_2}\{\max[R_{2T}(X_2, S_3), f_1(S_2)]\}$.

S_2	S_3	X_2	$R_{2T}(X_2, S_3)$	$\max(R_2, f_1)$	$f_2(S_3)$	fuzzy optimal policy
2		2-5	0.1	0.2	0.2	2-5
3	5	3-5	0.4	0.4	0.4	3-5
		3-6	0.3	0.4	0.4	3-6
4	6	4-5	0.3	0.3	0.3	4-5
		4-6	0.4	0.4	0.4	4-6

For last stage 3 ($i = 3$), $f_3(S_4) = \min_{X_3}\{\max[R_{3L}(X_3, S_4), f_2(S_x)]\}$.

S_2	S_3	X_2	$R_{2T}(X_2, S_3)$	$\max(R_2, f_1)$	$f_2(S_3)$	fuzzy optimal policy
5		5-7	0.2	0.4	0.4	5-7
6	7	6-7	0.4	0.4	0.4	6-7

Therefore, for the true limit vague network of the shortest path from city 1 to city 7 is $1 \to 4 \to 6 \to 7$

Weight of the shortest path $W_T = (0.2, 0.1, 0.4)$.

(ii) Shortest path for the false limit fuzzy matrices. Decompose the false limit fuzzy network into sub network or stage as follows

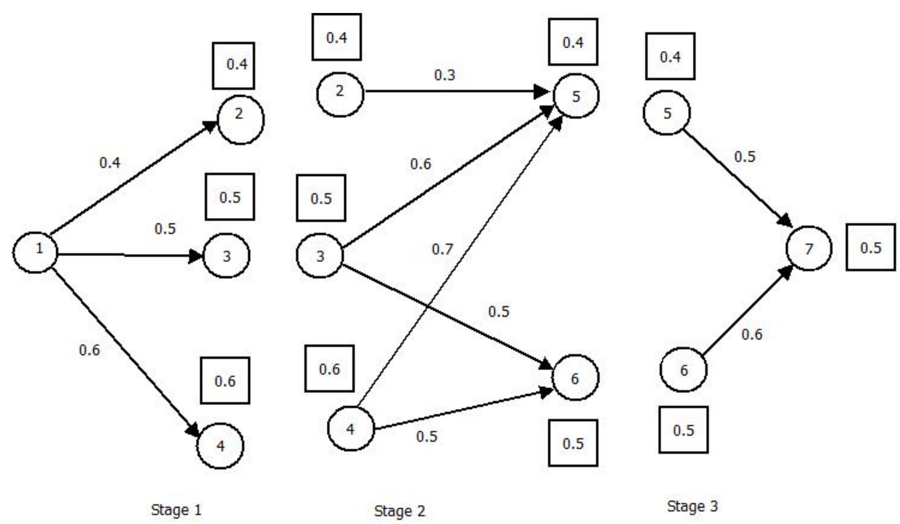

Fig 5 : Vague false network with stages

Similarly we have to find the false limit of the shortest path. Here $R_{iF}(X_i, S_{i+1})$ can be defined with the false limit of the corresponding arc.

By equation we have, $f_{iF}(S_{i+1}) = \min_{X_i}\{\max[R_{iF}(X_i, S_{i+1}), f_{(i-1)F}(S_i)]\}$.
Now, initially for $i = 0$, $f_i(S_{i+1}) = f_0(S_1) = f_0(1) = 0$.
For Stage $1(i = 1)$, $f_1(S_2) = \min_{X_1}\{\max[R_{1F}(X_1, S_2), f_0(S_1)]\}$
$= \min_{X_1}[R_{1F}(X_1, S_2)]$.

Now tabulating the data for $f_1(S_2)$

S_1	S_2	X_i	$R_{iT}(X_1, S_2)$	$f_1(S_2)$	fuzzy optimal policy
1	2	1-2	0.4	0.4	1-2
	3	1-3	0.5	0.5	1-3
	4	1-4	0.6	0.6	1-4

For stage 2 $(i = 2)$, $f_2(S_3) = \min_{X_2}\{\max[R_2(X_2, S_3), f_1(S_2)]\}$.

S_2	S_3	X_2	$R_{2T}(X_2, S_3)$	$\max(R_2, f_1)$	$f_2(S_3)$	fuzzy optimal policy
2		2-5	0.3	0.4	0.4	2-5
3	5	3-5	0.6	0.5	0.6	3-5
		3-6	0.5	0.5	0.5	3-6
4	6	4-5	0.7	0.6	0.7	4-5
		4-6	0.4	0.6	0.6	4-6

For last stage 3 $(i = 3)$, $f_2(S_4) = \min_{X_3}\{\max[R_3(X_3, S_4), f_2(S_3)]\}$.

S_2	S_3	X_2	$R_{2T}(X_2, S_3)$	$\max(R_2, f_1)$	$f_2(S_3)$	fuzzy optimal policy
5		5-7	0.5	0.4	0.5	5-7
6	7	6-7	0.6	0.5	0.6	6-7

Therefore the shortest path from city 1 to city 7 for the false limit vague network is $1 \to 2 \to 5 \to 7$. Weight of the shortest path $W_F = (0.4, 0.3, 0.6)$.

Now we conclude by equation. Shortest path in A^t_N = Shortest path in A^t_{NT} = Shortest path in $A^t_{NF} = 1 \to 2 \to 5 \to 7$, ie.., $W = [W_T, W_F] = [(0.2, 0.1, 0.4), (o.4, 0.3, 0.6)] = ([0.2, 0.4], [0.1, 0.3], [0.4, 0.6])$.

Therefore the shortest path of VN is $1 \to 2 \to 5 \to 7$

4. Conclusion

In this work we construct two vague networks namely $(FN)_T$ and $(FN)_F$ with the associated weight R_{iT} and R_{iF} respectively. Since the vertex sets and edge sets are same for VN, $(FN)_T$ and $(FN)_F$ and weight of the each node $(i, i+1)$ in VN is an interval of the form $W_i = [W_{iT}, W_{iF}]$. We conclude that the shortest path for an VN is the path for which the shortest path in true limit vague network coincides with the shortest path in false limit vague network and weight is $[W_T, W_F]$ where W_T andW_F are the weights of the shortest path for true and false limit vague networks.

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