A NEW INTUITIONISTIC FUZZY DIVERGENCE MEASURE AND ITS APPLICATIONS TO HANDLE FAULT DIAGNOSIS OF TURBINE

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Abstract. The literature of probability, fuzzy and intuitionistic fuzzy divergence measures provides the applications of a variety of divergence measures to different disciplines dealing with real life problems. Many such divergence measures have been generated through different approaches but still there is a scope that better ones can be developed which will provide applications to variety of disciplines. The present communication involving the development of a new intuitionistic measure of divergence for fuzzy distributions is a motivation in this direction. The newly proposed mathematical model is helpful for the study of fault diagnosis of turbine. In the present paper, we have provided an algorithm which can handle the main faults in the turbine along with useful information for future trends and verified the results numerically.
Keywords: fuzzy sets, intuitionistic fuzzy sets, intuitionistic fuzzy divergence measure/cross-entropy, fault diagnosis, turbine, vibration fault.

1. Introduction

This is to be emphasized that while dealing with the phenomenon of steam turbine failure, many faults are responsible for the cause of vibration of turbine and consequently delivers complex relation between vibration and fault types of turbine. The occurrence of such turbine faults compels us to provide immediate attention, cause of fault and timely diagnose for its repair so as to avoid more accidents and much financial losses. This is a real life problem dealing with diagnose of the vibration fault of steam turbine, the various diagnosis method are available in the literature. These methods are outstanding contributions of (Ye, 2009; Salahshoor, Khoshro, & Kordestani, 2011; Zhao, Liu, & He, 2012; Yang, Lee, Junker, & Ghezel-Ayagh, 2010; Kyriazis, & Mathioudakis; 2009).

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It has been observed that while dealing with turbine faults problems, there may be a variety of reasons for the same symptom of the fault and as a consequence of the phenomenon the system becomes more complex and difficult for its analysis and precision. The subjectivity of the experiment suggests that the fuzzy set theory introduced by (Zadeh, 1965) can play a vital role in handling such a type of uncertain information and finds widespread applications. This is to add that fuzzy set theory was extended to intuitionistic fuzzy set theory by (Atanassov, 1986), who characterize a membership function and non-membership function so as to provide mathematical background in handling imperfect situations where proper decision cannot be made. Some work related with IFSs theory has been provided by (He & He, 2016; He, He & Chen, 2015; Li & Ren, 2015). This theory is an alternative approach which finds tremendous applications in various fields including image processing, pattern recognition, fault diagnose and medical science. Motivated by the well known distance measures in probability spaces due to (Kullback, & Leibler, 1951; Bhandari, & Pal, 1993) investigated and introduced a new fuzzy divergence measure. Some other pioneer, who contributed towards the fuzzy (Parkash & Sharma, 2005; Parkash & Kumar, 2016; He, He, Wang & Chen, 2015; He, He, Lee, Kim, Zhang & Yang, 2017; Fei & Li, 2016; Li, 2016, Li & Liu, 2015; Dey, Pradhan, Pal & Pal, 2015; Dey & Pal, 2016; Dey, Pal & Pal, 2016 etc.).

The present communication deals with the investigations and enlargement of a new intuitionistic fuzzy divergence measure responsible for detecting the fault diagnosis of turbine. In section 2, some drawbacks in the existing IF-divergence measures have been pointed out. To overcome the drawbacks, a new IF-divergence measure has been proposed and studied its elegant properties in the form of theorems. These results have presented in section 3 of the paper. In section 4, application of a new IF-divergence Measure to handle fault diagnosis of turbine is discussed and also numerical example has been presented to illustrate the procedure of proposed method based on fault diagnosis of turbine.

2. Preliminaries and review of existing intuitionistic fuzzy divergence measures

In this section, fundamental knowledge concerning about IFS and intuitionistic fuzzy divergence measure is introduced.

2.1 Intuitionistic fuzzy sets

Definition 2.1. *An intuitionistic fuzzy set A defined on the universe of discourse $X = (x_1, x_2, ..., x_n)$ given by the expression (Atanassov, 1986)*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}, \tag{2.1}$$

where the function $\mu_A(x) : X \to [0, 1]$ and $\nu_A(x) : X \to [0, 1]$ denote the membership degree and the non-membership degree respectively with the condition *that*

$$0 \le \mu_A(x) + \nu_A(x) \le 1, \ for\ every\ x \in X. \tag{2.2}$$

Now, let IFS(X) denote the family of all intuitionistic fuzzy sets in the finite universe $X = (x_1, x_2, ..., x_n)$ and $A, B \in IFS(X)$ given by $A = \{\langle x, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i)\rangle | x_i \in X\}$.Then some set operations can be defined as follows:

Complement of A, $A^C = \{\langle x, \nu_A(x_i), \mu_A(x_i)\rangle | x_i \in X\}$. Intersection of A and B, $A \cap B = \{\langle x, \min\{\mu_A(x_i), \mu_B(x_i)\}, \max\{\nu_A(x_i), \nu_B(x_i)\}\rangle | x_i \in X\}$. Union of A and B, $A \cup B = \{\langle x, \max\{\mu_A(x_i), \mu_B(x_i)\}, \min\{\nu_A(x_i), \nu_B(x_i)\}\rangle | x_i \in X\}$. Inclusion Relation, $A \subseteq B$ if and only if $\mu_A(x_i) \le \mu_B(x_i)$ and $\nu_A(x_i) \ge \nu_B(x_i)$, $\forall x_i \in X$.

Definition 2.2. *Let $A = \{\langle x, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x, \mu_B(x_i), \nu_B(x_i)\rangle | x_i \in X\}$ be two intuitionistic fuzzy sets in X. A mapping $D : IFS(X) \times IFS(X) \to \mathbb{R}$ is a divergence measure for intuitionistic fuzzy sets if it satisfies the following axioms (Montes, Pal, Jains, & Montes, 2015):*

(a) $D(A, B) = D(B, A)$.

(b) $D(A, B) = 0$ *if and only if* $A = B$.

(c) $D(A \cap C, B \cap C) \le D(A, B)$ *for every* $C \in IFS(X)$.

(d) $D(A \cup B, B \cup C) \le D(A, B)$ *for every* $C \in IFS(X)$.

Again, the non negativity of the divergence measure is not required in the above axioms. However, it is trivially deduced from (b) and (c) (or (b) and (d)).

A variety of divergence measures for intuitionistic fuzzy sets have been introduced by various researchers with their own merits, demerits and limitations. We, now review the following existing divergence measures for IFSs to make a further meaningful study.

$$\begin{aligned} D_{VS}(A, B) = \sum_{i=1}^{n} \Bigg(\mu_A(x_i) \ln \left(\frac{2\mu_A(x_i)}{\mu_A(x_i) + \mu_B(x_i)} \right) \\ + \left(\nu_A(x_i) \ln \left(\frac{2\nu_A(x_i)}{\nu_A(x_i) + \nu_B(x_i)} \right) \right) \Bigg) \end{aligned} \tag{2.3}$$

which is (Vlachos & Sergiadis, 2007) measures of intuitionistic fuzzy sets. (Vlachos & Sergiadis, 2007) also defined the symmetric version of measure (2.3) given by

$$\begin{aligned} & D_{VS}^{sym}(A, B) = D_{VS}(A, B) + D_{VS}(B, A) \\ & = \sum_{i=1}^{n} \Bigg(\mu_A(x_i) \ln \left(\frac{2\mu_A(x_i)}{\mu_A(x_i) + \mu_B(x_i)} \right) + \left(\nu_A(x_i) \ln \left(\frac{2\nu_A(x_i)}{\nu_A(x_i) + \nu_B(x_i)} \right) \right) \\ & + \mu_B(x_i) \ln \left(\frac{2\mu_B(x_i)}{\mu_A(x_i) + \mu_B(x_i)} \right) + \left(\nu_B(x_i) \ln \left(\frac{2\nu_B(x_i)}{\nu_A(x_i) + \nu_B(x_i)} \right) \right) \Bigg) \end{aligned} \tag{2.4}$$

$$\begin{aligned}
D_{ZY}(A,B) = &\sum_{i=1}^{n}\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\\
&\ln\left(\frac{2(\mu_A(x_i)+1-\nu_A(x_i))}{(\mu_A(x_i+1-\nu_A(x_i))+(\mu_B(x_i+1-\nu_B(x_i))}\right)\\
&+\sum_{i=1}^{n}\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\\
&\ln\left(\frac{2(\nu_A(x_i)+1-\mu_A(x_i))}{(\nu_A(x_i)+1-\mu_A(x_i))+(\nu_B(x_i)+1-\mu_B(x_i))}\right)
\end{aligned} \tag{2.5}$$

which is (Zhang and Jiang, 2008) measures of intuitionistic fuzzy sets.

$$\begin{aligned}
D_{WY}(A,B) = &\sum_{i=1}^{n}\left(\mu_A(x_i)\ln\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)\right.\\
&+\left(\nu_A(x_i)\ln\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right)+\pi_A(x_i)\ln\left(\left.\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right)\right)
\end{aligned} \tag{2.6}$$

which is (Wei & Yei, 2010; Hung, 2012) measures of intuitionistic fuzzy sets. The symmetric discrimination of measures (2.6) is given by

$$\begin{aligned}
D_{WY}^{sym}(A,B) &= D_{WY}(A,B)+D_{WY}(B,A)\\
&= \sum_{i=1}^{n}\left(\mu_A(x_i)\ln\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\left(\nu_A(x_i)\ln\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right)\right.\\
&+\mu_B(x_i)\ln\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\left(\nu_B(x_i)\ln\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right)\\
&+\pi_A(x_i)\ln\left(\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right)+\left.\left(\pi_B(x_i)\ln\left(\frac{2\pi_B(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right)\right)\right)
\end{aligned} \tag{2.7}$$

$$\begin{aligned}
D_J(A,B) = &\sum_{i=1}^{n}\left(\pi_A(x_i)\ln\left(\frac{2\pi_A(x_i)}{\pi_A(x_i)+\pi_B(x_i)}\right)\right.\\
&+\left.\left(\Delta_A(x_i)\ln\left(\frac{2\Delta_A(x_i)}{\Delta_A(x_i)+\Delta_B(x_i)}\right)\right)\right)
\end{aligned} \tag{2.8}$$

which is (Junjun, Dengbao, & Cuicui, 2013) measures of intuitionistic fuzzy sets. where $\Delta_A(x_i) = |\mu_A(x_i - \nu_A(x_i))|$, denotes that how close the membership and non membership degrees are. The symmetric divergence measure of (2.8) are defined as follows

$$D_J^{sym}(A,B) = D_J(A,B) + D_J(B,A). \tag{2.9}$$

The above listed measures have some drawbacks introduced by (Vlachos & Sergiadis, 2007; Zhang & Jiang, 2008; Junjun, Dengbao, & Cuicui, 2013) and thus needs a modification as defined in (Verma and Sharma, 2012). The present communication is a step in this direction of removing these drawbacks. Such a new proposal has been investigated and introduced in the section 3.

3. A new intuitionistic fuzzy divergence measure

We now propose the intuitionistic fuzzy divergence measure of A against B as given by the following mathematical expression

$$D(A,B) = -\log_2\left[\frac{1}{2}\left\{1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)} + \sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right\}\right], \tag{3.1}$$

where the functions $\mu_A(x), \mu_B(x) \& \nu_A(x), \nu_B(x)$ denote the membership degree and the non-membership degree respectively. The measure defined above overcomes all the drawbacks, i.e. it satisfies all the axioms defined in definition 2.2. To prove the validity of proposed measure, we now study the following properties.

3.1 Properties of the proposed intuitionistic fuzzy divergence measure

Theorem 3.1. *Let $A, B, C \in IFS(X)$, then the proposed measure $D(A,B)$ given by equation (3.1) satisfies the following properties are given as follows:*

(a) $D(A,B) = D(B,A)$ and $0 \leq D(A,B) \leq 1$.

(b) $D(A,B) = 0$ *if and only if* $A = B$.

(c) $D(A \cap B, B \cap C) \leq D(A,B)$ *for every* $C \in IFS(X)$.

(d) $D(A \cup B, B \cup C) \leq D(A,B)$ *for every* $C \in IFS(X)$.

(e) $D(A,B) = D(A^C, B^C)$.

(f) $D(A, B^C) = D(A^C, B)$.

(g) $D(A, A \cup B) = D(A \cap B, B) \leq D(A,B)$ for $A \subseteq B$ and $B \subseteq A$.

(h) $D(A \cap B, A \cup B) = D(A,B)$.

Proof. From equation (3.1), it is understood that $D(A,B) = D(B,A)$. Further for all $x_i \in X$, we have

$$\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)} \leq \frac{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}+\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}}{2}$$

$$\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)} \leq \frac{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}+\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}}{2}$$

On adding the above two equations and take summations on both sides, we get

$$\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)} + \sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)$$
$$\leq \sum_{i=1}^{n}\left(\frac{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}+\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}}{2} + \frac{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}+\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}}{2}\right)$$

$$\Rightarrow 0 \leq \frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)} + \sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right) \leq 1$$

$$\Rightarrow \frac{1}{2} \leq \frac{1}{2}\left[1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)} + \sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right] \leq 1$$

$$\Rightarrow 0 \leq -\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)} + \sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right)\right] \leq 1$$

$$\Rightarrow 0 \leq D(A,B) \leq 1.$$

(b) It is obvious that $D(A,B)=0$ if and only if $A=B$.

(c) We have

$$\left[\sqrt{\min\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\frac{\mu_C(x_i)+1-\nu_C(x_i)}{2}\right)}\right.$$
$$\left.-\sqrt{\min\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2},\frac{\mu_C(x_i)+1-\nu_C(x_i)}{2}\right)}\right]^2$$
$$\leq\left(\sqrt{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}}-\sqrt{\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}}\right)^2$$

$$\Rightarrow\left[\begin{array}{c}\min\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\frac{\mu_C(x_i)+1-\nu_C(x_i)}{2}\right)\\+\min\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2},\frac{\mu_C(x_i)+1-\nu_C(x_i)}{2}\right)\\-2\sqrt{\min\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\frac{\mu_C(x_i)+1-\nu_C(x_i)}{2}\right)}\\\times\sqrt{\min\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2},\frac{\mu_C(x_i)+1-\nu_C(x_i)}{2}\right)}\end{array}\right] \tag{3.2}$$
$$\leq\left[\begin{array}{c}\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\\\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\\-2\sqrt{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}}\\\times\sqrt{\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}}\end{array}\right].$$

Similarly, we have

$$\Rightarrow\left[\begin{array}{c}\max\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\frac{\nu_C(x_i)+1-\mu_C(x_i)}{2}\right)\\+\max\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2},\frac{\nu_C(x_i)+1-\mu_C(x_i)}{2}\right)\\-2\sqrt{\max\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\frac{\nu_C(x_i)+1-\mu_C(x_i)}{2}\right)}\\\times\sqrt{\max\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2},\frac{\nu_C(x_i)+1-\mu_C(x_i)}{2}\right)}\end{array}\right]$$

$$\leq \begin{bmatrix} \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2} \\ \frac{\nu_B(x_i)+1-\mu_B(x_i)}{2} \\ -2\sqrt{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}} \\ \times\sqrt{\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}} \end{bmatrix} \tag{3.3}$$

Adding equations (3.2) and (3.3), yields $D(A\cap B, B\cap C) \leq D(A,B)$.

(d) The proof is on similar lines as in part (c).

(e) Consider

$$\begin{aligned} &D(A,B)\\ &= -\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right.\right.\right.\\ &\left.\left.\left.+\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right)\right]\\ &= -\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right.\right.\right.\\ &\left.\left.\left.+\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right)\right)\right] = D(A^C, B^C). \end{aligned}$$

(f) Consider

$$\begin{aligned} &D(A,B^C)\\ &= -\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right.\right.\right.\\ &\left.\left.\left.+\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right)\right)\right]\\ &= -\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right.\right.\right.\\ &\left.\left.\left.+\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right)\right)\right] = D(A^C, B) \end{aligned}$$

(g) Consider $D(A, A\cup B)$

$$= -\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_{A\cup B}(x_i)+1-\nu_{A\cup B}(x_i)}{2}\right)}\right.\right.\right.$$

$$+\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_{A\cup B}(x_i)+1-\mu_{A\cup B}(x_i)}{2}\right)}\Bigg)\Bigg)\Bigg]$$

$$=-\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\min\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right.\right.\right.$$

(3.4)

$$\left.\left.\left.+\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\min\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right)\right]$$

Similarly,$D(A, A\cup B)$

$$=-\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\min\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right.\right.\right.$$

(3.5)

$$\left.\left.\left.+\sqrt{\min\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)\left(\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right)\right]$$

For $A\subseteq B$, (3.4) and (3.5) gives, $D(A, A\cup B) = D(A\cap B, B) = D(A, B)$.

Again, for $B\subseteq A$, (3.4) and (3.5) gives, $D(A, A\cup B) = D(A\cap B, B) = 0 \leq D(A, B)$.

(h) Proceeding on similar lines as above, we can prove the required result.

Consider

$D(A\cap B, A\cup B)$

$$=-\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_{A\cap B}(x_i)+1-\nu_{A\cap B}(x_i)}{2}\right)\left(\frac{\mu_{A\cup B}(x_i)+1-\nu_{A\cup B}(x_i)}{2}\right)}\right.\right.\right.$$

$$\left.\left.\left.+\sqrt{\left(\frac{\nu_{A\cap B}(x_i)+1-\mu_{A\cap B}(x_i)}{2}\right)\left(\frac{\nu_{A\cup B}(x_i)+1-\mu_{A\cup B}(x_i)}{2}\right)}\right)\right)\right]$$

$$=-\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\min\left(\begin{array}{c}\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\\ \frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\end{array}\right)\max\left(\begin{array}{c}\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2},\\ \frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\end{array}\right)}\right.\right.\right.$$

$$\left.\left.\left.+\sqrt{\min\left(\begin{array}{c}\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\\ \frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\end{array}\right)\max\left(\begin{array}{c}\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\\ \frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\end{array}\right)}\right)\right)\right]$$

$$=-\log_2\left[\frac{1}{2}\left(1+\frac{1}{n}\sum_{i=1}^{n}\left(\sqrt{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\left(\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}\right)}\right.\right.\right.$$

$$\left.\left.\left.+\sqrt{\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}\right)\left(\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2},\frac{\nu_B(x_i)+1-\mu_B(x_i)}{2}\right)}\right)\right)\right]=D(A,B)$$

Hence the result.

With the study of above properties, we claim that the intuitionistic fuzzy divergence measure proposed in (3.1) is an appropriate measure of divergence. In the next section, we provide the application of the proposed measure to fault diagnosis.

4. Applications of the intuitionistic fuzzy divergence measure to fault-diagnosis of turbine

To apply the proposed model to measure the fault diagnose in steam turbine, we need the following algorithm.

4.1 Fault-diagnosis algorithm

Assume that there exist m fault patterns (knowledge of fault samples), which are represented by IFS $F_i (i = 1, 2, ..., m)$, and there is a testing sample to be recognized which is represented by a IFS F_t. Then diagnosis result F_k should be nearest one to F_t that is

$$D(F_t, F_k) = Min_{1 \leq i \leq m}\{D(F_t, F_i)\}.$$

where $D(F_t, F_i)$ expresses the discrimination degree of the IFS F_t from F_i. The value of $D(F_t, F_i)$ is calculated by using equation (3.1). Then, we decide that testing sample F_t should belong to the fault pattern F_k, where $k = Min_{1 \leq i \leq m}\{D(F_t, F_i)\}$.

The proposed divergence measure of IFSs can realize the classification and identification of the fault by comparing the intuitionistic fuzzy divergence values between a diagnosing sample and the knowledge of system faults. The minimum value of the intuitionistic fuzzy divergence will detect and confirm the type of the fault. The fault-diagnosis process using the intuitionistic fuzzy divergence measure is shown in Fig.-1. The following numerical verification will prove the authenticity of the algorithm mentioned above.

Numerical verification

During our investigation, we have taken the ten kinds of familiar fault types in rotating machines, such as unbalance, offset center, and oil-membrane oscillation, are used as the knowledge of fault samples. The vibration frequency of the turbine is divided into nine different frequency ranges representing the failure patterns known as IFSs as shown in Table-1. (Ye, Qiao, & Wei, 2005). Next, we have established the knowledge database of fault types and then calculated the intuitionistic fuzzy cross entropy between a fault-testing sample and fault knowledge samples. Ten fault knowledge samples in table-1 are expressed by IFSs:

$F_1 = [0.00, 0.00]/x_1 + [0.00, 0.00]/x_2 + [0.00, 0.00]/x_3 + [0.00, 0.00]/x_4$

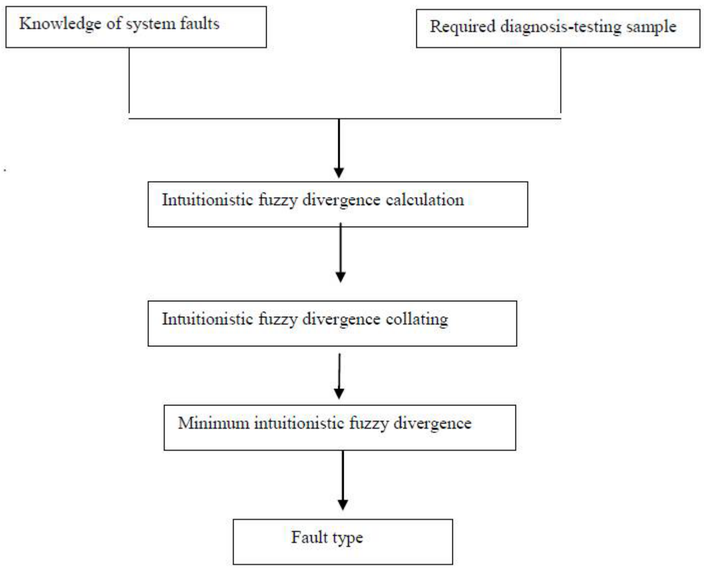

Figure 1: Block diagram of fault diagnosis using the intuitionistic fuzzy cross-entropy

$+[0.85, 1.00]/x_5+[0.04, 0.06]/x_6+[0.04, 0.07]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$
$F_2 = [0.00, 0.00]/x_1 + [0.28, 0.31]/x_2 + [0.09, 0.12]/x_3 + [0.55, 0.70]/x_4$
$+[0.00, 0.00]/x_5+[0.00, 0.00]/x_6+[0.00, 0.00]/x_7+[0.00, 0.00]/x_8+[0.08, 0.13]/x_9$
$F_3 = [0.00, 0.00]/x_1 + [0.00, 0.00]/x_2 + [0.00, 0.00]/x_3 + [0.00, 0.00]/x_4$
$+[0.30, 0.58]/x_5+[0.40, 0.62]/x_6+[0.08, 0.13]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$
$F_4 = [0.09, 0.11]/x_1 + [0.78, 0.82]/x_2 + [0.00, 0.00]/x_3 + [0.08, 0.11]/x_4$
$+[0.00, 0.00]/x_5+[0.00, 0.00]/x_6+[0.00, 0.00]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$
$F_5 = [0.09, 0.12]/x_1 + [0.09, 0.11]/x_2 + [0.08, 0.12]/x_3 + [0.09, 0.12]/x_4$
$+[0.18, 0.21]/x_5+[0.08, 0.13]/x_6+[0.08, 0.13]/x_7+[0.08, 0.12]/x_8+[0.08, 0.12]/x_9$
$F_6 = [0.00, 0.00]/x_1 + [0.00, 0.00]/x_2 + [0.00, 0.00]/x_3 + [0.00, 0.00]/x_4$
$+[0.18, 0.22]/x_5+[0.12, 0.17]/x_6+[0.37, 0.45]/x_7+[0.00, 0.00]/x_8+[0.22, 0.28]/x_9$
$F_7 = [0.00, 0.00]/x_1 + [0.00, 0.00]/x_2 + [0.08, 0.12]/x_3 + [0.86, 0.93]/x_4$
$+[0.00, 0.00]/x_5+[0.00, 0.00]/x_6+[0.00, 0.00]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$
$F_8 = [0.00, 0.00]/x_1 + [0.27, 0.32]/x_2 + [0.08, 0.12]/x_3 + [0.54, 0.62]/x_4$
$+[0.00, 0.00]/x_5+[0.00, 0.00]/x_6+[0.00, 0.00]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$
$F_9 = [0.85, 0.93]/x_1 + [0.00, 0.00]/x_2 + [0.00, 0.00]/x_3 + [0.00, 0.00]/x_4$
$+[0.85, 1.00]/x_5+[0.04, 0.06]/x_6+[0.04, 0.07]/x_7+[0.08, 0.12]/x_8+[0.00, 0.00]/x_9$
$F_{10} = [0.00, 0.00]/x_1 + [0.00, 0.00]/x_2 + [0.00, 0.00]/x_3 + [0.00, 0.00]/x_4$
$+[0.85, 1.00]/x_5+[0.77, 0.83]/x_6+[0.19, 0.23]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$

Suppose that the IFSs of fault-testing samples are as follows:
$F_{t1} = [0.00, 0.00]/x_1 + [0.00, 0.00]/x_2 + [0.10, 0.10]/x_3 + [0.90, 0.90]/x_4$
$+[0.00, 0.00]/x_5+[0.77, 0.83]/x_6+[0.21, 0.35]/x_7+[0.00, 0.00]/x_8+[0.00, 0.00]/x_9$

$F_{t2} = [0.39, 0.39]/x_1 + [0.07, 0.07]/x_2 + [0.10, 0.10]/x_3 + [0.06, 0.06]/x_4$
$+[0.00, 0.00]/x_5 + [0.13, 0.13]/x_6 + [0.00, 0.00]/x_7 + [0.00, 0.00]/x_8 + [0.35, 0.35]/x_9$
The cross-entropy values of vague sets are calculated by use of equation (3.1) as follows:
$D(F_{t1}, F_1) = 0.003200, D(F_{t1}, F_2) = 0.082967, D(F_{t1}, F_3) = 0.001733, D(F_{t1}, F_4) = 0.000288, D(F_{t1}, F_5) = 0.000205, D(F_{t1}, F_6) = 0.000117, D(F_{t1}, F_7) = 0.000279,$
$D(F_{t1}, F_8) = 0.000328, D(F_{t1}, F_9) = 0.000548, D(F_{t1}, F_{10}) = 0.082573;$
$D(F_{t2}, F_1) = 0.003081, D(F_{t2}, F_2) = 0.082835, D(F_{t2}, F_3) = 0.001582, D(F_{t2}, F_4) = 0.000048, D(F_{t2}, F_5) = 0.000132, D(F_{t2}, F_6) = 0.000155, D(F_{t2}, F_7) = 0.00018,$
$D(F_{t2}, F_8) = 0.000131, D(F_{t2}, F_9) = 0.000102, D(F_{t2}, F_{10}) = 0.000082.$

Diagnosis result 1: The fault-diagnosis result is as follows:
$F_6 \rightarrow F_3 \rightarrow F_5 \rightarrow F_7 \rightarrow F_4 \rightarrow F_8 \rightarrow F_9 \rightarrow F_1 \rightarrow F_{10} \rightarrow F_2.$

From the above diagnosis order, we observe the sequence of vibration fault of turbine. This sequence has discovered that firstly antithrust bearing is damage, next offset center, and then oil membrane oscillation, and so on, which results in the vibration of turbine.

Diagnosis result 2: The fault-diagnosis result is as follows:
$F_4 \rightarrow F_{10} \rightarrow F_9 \rightarrow F_8 \rightarrow F_5 \rightarrow F_6 \rightarrow F_7 \rightarrow F_3 \rightarrow F_1 \rightarrow F_2.$

From the above sequence our observations proves that the fault causing the vibration of turbine has been resulted firstly from radial impact friction of rotor, next non uniform bearing stiffness, and then looseness of bearing block, and so on.

From the above diagnostic scheme, we claim that the proposed diagnosis method is effective and provides reasonable information in detecting a fault during vibration of turbine.

Conclusions

In this paper, we have proposed a new intuitionistic fuzzy divergence measure and proved some elegant properties of the intuitionistic fuzzy divergence measure in detail. Furthermore, based on the proposed divergence measure for IFSs, an algorithm to deal with fault diagnosis of turbine under fuzzy environments is described. On the basis of diagnosis results for the turbine, the proposed method cannot only diagnose the main fault of the turbine; it can also detect useful information for multi-fault analysis. Finally, a numerical example is provided to illustrate the fault diagnosis of turbine and proves the authenticity of our model providing useful information in detecting the fault under consideration. This work can be extended to Interval valued intuitionistic fuzzy divergence measures.

Fault samples	Frequency	range	(f:	operating	frequency)				
	0.01-0.39f	0.40-0.49f	0.50f	0.51-0.99f	f	2f	3-5f	Odd times of f	High freq $> 5f$
Pneumatic force couple	(0.00, 0.00)	(0.28, 0.31)	(0.09, 0.12)	(0.55, 0.70)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.08, 0.13)
Unbalance	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.85, 1.00)	(0.04, 0.06)	(0.04, 0.07)	(0.00, 0.00)	(0.00, 0.00)
Offset center	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.30, 0.58)	(0.40, 0.62)	(0.08, 0.13)	(0.00, 0.00)	(0.00, 0.00)
Radial impact friction of rotor	(0.09, 0.12)	(0.09, 0.11)	(0.08, 0.12)	(0.09, 0.12)	(0.18, 0.21)	(0.08, 0.13)	(0.08, 0.13)	(0.08, 0.12)	(0.08, 0.12)
Oil-membrane oscillation	(0.09, 0.11)	(0.78, 0.82)	(0.00, 0.00)	(0.08, 0.11)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)
Damage of antithrust bearing	(0.00, 0.00)	(0.00, 0.00)	(0.08, 0.12)	(0.86, 0.93)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)
Symbiosis looseness	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.18, 0.22)	(0.12, 0.17)	(0.37, 0.45)	(0.00, 0.00)	(0.22, 0.28)
Surge	(0.00, 0.00)	(0.27, 0.32)	(0.08, 0.12)	(0.54, 0.62)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)
Looseness of bearing block	(0.85, 0.93)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.08, 0.12)	(0.00, 0.00)
Non-uniform bearing stiffness	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.00, 0.00)	(0.77, 0.83)	(0.19, 0.23)	(0.00, 0.00)	(0.00, 0.00)

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References

[1] K.T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, 20 (1986), 87-96.

[2] D. Bhandari, N.R. Pal, *Some new information measure for fuzzy sets*, Information Science, 67 (1993), 209-228.

[3] A. Dey, A. Pal, *Prim's algorithm for solving minimum spanning tree problem in fuzzy environment*, Annals of Fuzzy Mathematics and Informatics, 12 (2016), 419-430.

[4] A. Dey, A. Pal, T. Pal, *Interval type 2 fuzzy set in fuzzy shortest path problem*, Mathematics, 4 (2016), 62.

[5] A. Dey, R. Pradhan, A. Pal, T. Pal, *The fuzzy robust graph coloring problem*. In Proceedings of the 3rd International Conference on Frontiers of Intelligent Computing: Theory and Applications (FICTA), pp. 805-813, Springer, 2015.

[6] W. Fei, D.F. Li, *Bilinear programming approach to solve interval bimatrix games in tourism planning management*, International Journal of Fuzzy Systems, 18 (2016), 504-510.

[7] Y. He, Z. He, *Extensions of Atanassov's intuitionistic fuzzy interaction Bonferroni means and Their Application to Multiple attribute decision making*, IEEE Transactions on Fuzzy Systems, 24 (2016), 558-573.

[8] Y. He, Z. He, H. Chen, *Intuitionistic Fuzzy Interaction Bonferroni Means and Its Application to Multiple Attribute Decision Making*, IEEE Transactions on Cybernetics, 45 (2015), 116-128.

[9] Y. He, Z. He, D.H. Lee, K.J. Kim, L. Zhang, X. Yang, *Robust Fuzzy Programming Method for MRO Problems Considering Location Effect*, Dispersion Effect and Model Uncertainty, Computers Industrial Engineering, 105 (2017), 76-83.

[10] Y. He, Z. He, G. Wang, H. Chen, *Hesitant Fuzzy Power Bonferroni Means and their Application to Multiple Attribute Decision Making*, IEEE Transactions on Fuzzy Systems, 23 (2015), 1655-1668.

[11] K.C. Hung, *Medical pattern recognition: applying an improved intuitionistic fuzzy cross-entropy approach*, Advances in fuzzy systems, Article ID, 863549, 2012.

[12] M. Junjun, Y. Dengbao, W. Cuicui, *A novel cross-entropy and entropy measures of IFSs and their applications*, Knowledge-Based Systems, 48 (2013), 37-45.

[13] S. Kullback, R.A. Leibler, *On information and sufficiency*, The Annals Mathematical Statistics, 22 (1951), 79-86.

[14] A. Kyriazis, K. Mathioudakis, *Gas turbine fault diagnosis using fuzzy-based decision fusion*, J. Propulsion Power, 25 (2009), 335-343.

[15] D.F. Li, *Decision and Game Theory in Management with Intuitionistic Fuzzy Sets*, Germany: Springer, Heidelberg, 2014.

[16] D.F. Li, *Linear Programming Models and Methods of Matrix Games with Payoffs of Triangular Fuzzy Numbers*, Berlin, Springer, Heidelberg, 2016.

[17] D.F. Li, J.C. Liu, *A parameterized non-linear programming approach to solve matrix games with payoffs of I-fuzzy numbers*, IEEE Transactions on Fuzzy Systems, 23 (2015), 885-896.

[18] D.F. Li, H.P. Ren, *Multi-attribute decision making method considering the amount and reliability of intuitionistic fuzzy information*, Journal of Intelligent Fuzzy Systems, 28 (2015), 1877-1883.

[19] I. Montes, N.R. Pal, V. Janis, S. Montes, *Divergence measures for intuitionistic fuzzy sets*, IEEE Transactions on Fuzzy Systems, 23 (2015), 444-456.

[20] O. Parkash, R. Kumar, *New parametric and non - parametric measures of cross entropy*, Canadian Journal of Pure Applied Sciences, 10 (2016), 3921-3925.

[21] O. Parkash, P.K. Sharma, *Some new measures of fuzzy directed divergence and their generalization*, Journal of the Korean Society of Mathematical Education Series B 12 (2005), 307-315.

[22] K. Salahshoor, M.S. Khoshro, M. Kordestani, *Fault detection and diagnosis of an industrial steam turbine using a distributive configuration of adaptive neuro-fuzzy inference systems*, Simulation Modell. Pract. Theory, 19 (2011), 1280-1293.

[23] R. Verma Sharma, B.D. Sharma, *On generalized intuitionistic fuzzy divergence (relative information) and their properties*, Journal of Uncertain Systems, 6 (2012), 308-320.

[24] K. Vlachos, G.D. Sergiadis, *Intuitionistic fuzzy information-applications to pattern recognition*, Pattern Recognition Letters, 28 (2007), 197-206.

[25] P. Wei, J. Ye, *Improved intuitionistic fuzzy cross-entropy and its application to pattern recognition,* International Conference on Intelligent Systems and Knowledge Engineering, 114-116, 2010.

[26] W. Yang, K.Y. Lee, S.T. Junker, H. Ghezel-Ayagh, *Fuzzy fault diagnosis and accommodation system for hybrid fuel-cell/gas-turbine power plant*, IEEE Trans. Energy Convers., 25 (2010), 1187-1194.

[27] J. Ye, X.L. Qiao, H.L. Wei, *Fault diagnosis of turbine on similarity measures between vague sets*, In Proceedings of the Asia pacific symposium on safety (Part B, pp. 1358-1362), Shaoxing, Zhejiang, China, 2005.

[28] J. Ye, *Fault diagnosis of turbine based on fuzzy cross entropy of vague sets*, Expert Systems with Applications, 36 (2009), 8103-8106.

[29] L.A. Zadeh, *Fuzzy sets*, Information and Control, 8 (1965), 338-353.

[30] Q.S. Zhang, S.Y. Jiang, *A note on information entropy measures for vague sets and its applications*, Information Sciences, 178 (2008), 4184-4191.

[31] X. Zhao, Y. Liu, X. He, *Fault diagnosis of gas turbine based on fuzzy matrix and the principle of maximum membership degree*, Energy Proc., 16 (2012), 1448-1454.

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