CONTRA WEAKLY-θ_I-PRECONTINUOUS FUNCTIONS IN IDEAL TOPOLOGICAL SPACES

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Abstract. The present authors [23] introduced and studied the notion of weakly θ_I-preopen sets in ideal topological spaces. In this paper, we apply this set to introduce and study a new class of functions called contra weakly θ_I-precontinuous functions in ideal topological spaces. Some characterizations and several basic properties of this class of functions are obtained. Further, we introduce the notions of contra-θ_I-precontinuous, contra-θ_I-α-continuous, contra-θ_I-semicontinuous and contra-θ_I-β-continuous functions in ideal topological spaces and also establish relationships among these new classes of functions.
Keywords: weakly θ_I-preopen sets, weakly θ_I-precontinuous, contra weakly-θ_I-precontinuous, contra-θ_I-precon-tinuous, contra-θ_I-α-continuous, contra-θ_I-semicontinuous and contra-θ_I-β-continuous.

1. Introduction

The concept of continuous functions is one of the important and basic topic in the theory of classical point set topology and several branches of mathematics. General Topologists have introduced and investigated many different generalizations of continuous functions. Dontchev [6] introduced a new class of functions called contra-continuous functions. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be contra-

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continuous [6] if the preimage of every open set of Y is closed in X. Subsequently, some generalized forms of this class of functions namely contra-supercontinuous functions [15], *contra-precontinuous* functions [16] etc., in topological spaces and *contra-pre-I*-continuous functions [22] etc. in ideal topological spaces were propounded and studied. An ideal I in a topological space has been considered since 1930 by Kuratowski [13] and Vaidyanathaswamy [24] (c.f.[25]). After several decades, Jankovic and Hamlett [12] investigated the topological ideal which is the generalization of general topology. Jankovic and Hamlett (c.f.[8], [9]) also introduced the notion of *I*-open sets in topological spaces. Abd El-Monsef et al. [2] further investigated *I*-open sets and *I*-continuous functions. Some generalized forms of *I*-open sets are introduced in [3], [7] (see [11]) and other papers. Al-Omari and Noiri [5] introduced the notions of θ_I-preopen, θ_I-semiopen, θ_I-α-*open*, θ_I-β-open sets by using θ_I-*open* sets and derived the decomposition of θ_I-precontinuous functions and θ_I-β-continuous functions in ideal topological spaces. In addition to this, the present authors [23] defined and investigated the notion of weakly θ_I-preopen sets and established the decomposition of weakly θ_I-*precontinuity*.

The purpose of this paper is to give a new class of functions called contra weakly-θ_I-precontinuous functions via weakly θ_I-preopen sets in ideal topological spaces and study their fundamental properties and characterizations. We also devise the concept of contra-θ_I-*precontinuous*, contra-θ_I-α-*continuous*, contra-θ_I-semicontinuous and contra-θ_I-β-continuous functions in ideal topological spaces and establish their interrelationships.

2. Preliminaries

An ideal on a topological space (X, τ) is defined as a non-empty collection I of subsets of X satisfying the following two conditions:

1. $V \in I$ and $U \subset V$ implies $U \in I$,
2. $V \in I$ and $U \in I$ implies $V \cup U \in I$.

The pair (X, τ, I) of a topological space (X, τ) and an ideal I on X is called an ideal topological space or simply an ideal space. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the collection of all subsets of X, then the set operator $(.) : P(X) \to P(X)$ is called the local function [24] of A with respect to τ and ideal I, is defined as follows: For a subset $A \subseteq X$, the set $A^*(I, \tau) = \{x \in X\colon (U \cap A) \notin I$, for every $U \in \tau(x)$ $\}$, where $\tau(x) = \{U \in \tau : x \in U\}$ [12]. We simply write A^* instead of $A^*(I, \tau)$ in case there is no chance of confusion [24] (c.f. [12], [13]). It is well known that $Cl^*(A) = A \cup A^*(I, \tau)$ defines a Kuratowski closure operator for a topology $\tau^*(I, \tau)$ called the $*$-topology which is finer than τ. The topology τ^* is generated by the base $\beta(I, \tau) = \{U \setminus I\colon U \in \tau$ and $I \in I$ $\}$. In general $\beta(I, \tau)$ is not always a topology as shown in [12]. X^* is often a proper subset of X.

Throughout this paper, for a subset A of a topological space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A, respectively. The family of all weakly θ_I-*preopen* (resp. θ_I-*preopen*, θ_I-semiopen, θ_I-α-open, θ_I-β-open) sets of the space (X, τ, I) will be denoted by $W\theta_I PO(X)$ (resp. $\theta_I PO(X)$, $\theta_I SO(X)$, $\theta_I \alpha O(X)$, $\theta_I \beta O(X)$) .

We start with recalling following definitions and results, which are necessary for this study in the sequel.

Definition 1. A subset A of topological space (X, τ) is said to be preopen [18] (resp. semiopen [14], α-*open* [19] [22], β-open [1]) if $A \subset Int(Cl(A))$ (resp. $A \subset Cl(Int(A))$, $A \subset Int(Cl(Int(A)))$, $A \subset Cl(Int(Cl(A)))$).

Lemma 1 ([12])**.** *Let (X, τ, I) be an ideal topological space and A, B be any two subsets of X. Then the following properties hold:*

1. *If $A \subseteq B$, then $A^* \subseteq B^*$;*
2. *If $A^* = Cl(A^*) \subseteq Cl(A)$;*
3. *$(A^*)^* \subseteq A^*$;*
4. *$(A \cup B)^* = A^* \cup B^*$.*
5. *If $U \in \tau$, then $U \cap A^* \subset (U \cap A)^*$.*

Lemma 2 ([5])**.** *Let (X, τ, I) be an ideal topological space and A be a subset of X. Then the following properties hold.*

1. *If A is open, then $Cl(A) = Cl_{\theta_I}(A) = Cl_\theta(A)$.*
2. *If A is closed, then $Int(A) = Int_{\theta_I}(A) = Int_\theta(A)$.*

Lemma 3 ([10])**.** *For a subset A of a topological space (X, τ), the following properties hold:*

1. *$sCl(A) = A \cup Int(Cl(A))$,*
2. *If A is open then $sCl(A) = Int(Cl(A))$.*

Definition 2. A subset A of an ideal topological space (X, τ, I) is said to be θ_I-*preopen* [5] (resp. θ_I-semiopen [5], θ_I-β-open [5], θ_I-α-open [5], weakly θ_I-preopen [23]) if $A \subseteq Int(Cl_{\theta_I}(A)))$ (resp. $A \subseteq Cl(Int_{\theta_I}(A))$, $A \subseteq Cl(Int(Cl_{\theta_I}(A)))$, $A \subseteq Int(Cl(Int_{\theta_I}(A)))$, $A \subseteq sCl(Int(Cl_{\theta_I}(A)))$).

Definition 3. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) precontinuous [18] if preimage of every open set in Y is preopen in X.

(2) semicontinuous [14] if the inverse image of each open set in Y is semiopen in X.

(3) β-continuous [1] if the inverse image of each open set in Y is β-open in X.

(4) α-continuous [19] if the inverse image of each open set in Y is α-open in X.

Definition 4. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be

(1) weakly *θ_I-precontinuous* [23] if the preimage of every open set in Y is weakly θ_I-preopen in X.

(2) weakly *θ_I-preirresolute* [23] if the preimage of every weakly θ_I-preopen set in Y is a weakly *θ_I-preopen* set in X.

Definition 5. A space (X, τ) is said to be extremally disconnected [26] if the closure of every open set in X is open.

Theorem 1 ([23]). *If a topological space (X, τ) is extremally disconnected and $A \in \theta_I SO(X)$, then $A \in \theta_I \alpha O(X)$.*

Theorem 2 ([23]). *If a topological space (X, τ) is extremally disconnected and $A \in \theta_I \beta O(X)$, then $A \in W\theta_I PO(X)$.*

3. Contra weakly-θ_I-precontinuous functions

Definition 6. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be contra weakly-θ_I-precontinuous if the preimage of every closed set in Y is weakly *θ_I-preopen* in X.

Example 1. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{b, c\}, \{c, d\}, \{b, c, d\}, \{c\}\}$ and $I = P(X)$ then (X, τ, I) is an ideal topological space. $C(X) = \{X, \phi, \{a, d\}, \{a, b\}, \{a\}, \{a, b, d\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\sigma = \{Y, \phi, \{1, 2\}, \{1, 2, 3\}, \{2\}\}$ and $C(Y) = \{Y, \phi, \{3, 4\}, \{4\}, \{1, 3, 4\}\}$. Then (Y, σ) be a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function defined as $f(a) = 3$, $f(b) = 2$, $f(c) = 4$, $f(d) = 1$. Then f is contra weakly-*θ_I-precontinuous*, since the preimage of every closed set in Y is weakly θ_I-preopen in X.

Definition 7. Let A be any subset of an ideal topological space (X, τ, I).

(1) The intersection of all weakly *θ_I-preclosed* sets containing A is called the weakly *θ_I-pre-closure* of A and it is denoted by $W\theta_I$-$PCl(A)$.

(2) The union of all weakly *θ_I-preopen* sets contained in A is called the weakly *θ_I-pre-interior* of A and it is denoted by $W\theta_I$-$PInt(A)$.

Remark 1. Let A be any subset of an ideal topological space (X, τ, I), then the following properties hold:

(1) A is weakly *θ_I-preclosed* if and only if $W\theta_I$-$PCl(A) = A$.

(2) A is weakly *θ_I-preopen* if and only if $W\theta_I$-$PInt(A) = A$.

Theorem 3. *Let A be any subset of an ideal topological space (X, τ, I) and $x \in X$. Then $x \in W\theta_I$-$PCl(A)$ if and only if $A \cap U \neq \phi$ for each $U \in W\theta_I PO(X, x)$.*

Proof. *Necessity.* Let $x \notin W\theta_I$-$PCl(A)$, then suppose $U = X \setminus W\theta_I$-$PCl(A)$ be a weakly θ_I-*preopen* set containing x in X such that $U \cap W\theta_I$-$PCl(A) = \phi$. Hence the result.

Sufficiency. Let U be a weakly θ_I-*preopen* set containing x in X such that $U \cap W\theta_I$-$PCl(A) = \phi$. Then $X \setminus U$ is a weakly θ_I-*preclosed* set containing A. Since $W\theta_I$-$PCl(A)$ is the smallest weakly θ_I-*preclosed* set containing A, $W\theta_I$-$PCl(A) \subseteq X \setminus U$. Therefore $x \notin W\theta_I$-$PCl(A)$. □

Definition 8. Let A be any subset of a space (X, τ). The set $\bigcap\{U : U \in \tau, A \subset U\}$ is called the kernel of A [20] and is denoted by $ker(A)$. In [17], the kernel of A is called the $\wedge$-set.

Lemma 4 ([15], [17])**.** *The following properties hold for subsets A and B of a space (X, τ) :*

(1) *$x \in ker(A)$ if and only if $A \cap F \neq \phi$ for any $F \in C(X, x)$.*

(2) *$A \subset ker(A)$ and $A = ker(A)$ if A is open in X.*

(3) *$A \subset B$, then $ker(A) \subset ker(B)$.*

Theorem 4. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function from an ideal topological space (X, τ, I) to (Y, σ). Then the following statements are equivalent:*

(1) *f is contra weakly-θ_I-precontinuous,*

(2) *The inverse image of each open set in Y is weakly θ_I-preclosed in X,*

(3) *For each $x \in X$ and each closed subset G of Y containing $f(x)$, there exists a weakly θ_I-preopen set U in X such that $x \in U$ and $f(U) \subset G$.*

(4) *$f(W\theta_I$-$PCl(M)) \subset ker(f(M))$ for every subset M of X,*

(5) *$W\theta_I$-$PCl(f^{-1}(N)) \subset f^{-1}(ker(N))$ for every subset N of Y.*

Proof. (1) $\Leftrightarrow$ (2) is obvious.

(1) $\Rightarrow$ (3). Let $x \in X$ and G be any closed subset of Y containing $f(x)$. Since f is contra *weakly*-θ_I-precontinuous, $f^{-1}(G)$ is weakly θ_I-preopen in X. Let $U = f^{-1}(G)$. Then $x \in U = f^{-1}(G)$. Thus $f(U) \subset G$.

(3) $\Rightarrow$ (4). Let M be any subset of X. Let $y \in Y$ be any element such that $y \notin ker(f(M))$. Then by Lemma 4, there exists a closed set V containing y in Y such that $f(M) \cap V = \phi$; hence $M \cap f^{-1}(V) = \phi$. By (3) for any $x \in f^{-1}(\{y\})$, there exists a weakly θ_I-*preopen* set W containing 'x' such that $f(W) \subset V$. Therefore $f(M \cap W) \subset f(M) \cap f(W) \subset f(M) \cap V = \phi$ and hence $M \cap W = \phi$. Thus by Theorem 3, $x \notin W\theta_I$-$PCl(M)$ for any $x \in f^{-1}(\{y\})$. Thus $f^{-1}(\{y\}) \cap W\theta_I$-$PCl(M) = \phi$ and hence $y \notin f(W\theta_I$-$PCl(M))$. Hence we

obtain $f(W\theta_I - PCl(M)) \subset ker(f(M)$ for every subset M of X.

(4) $\Rightarrow$ (5). Let N be any subset of Y, then $f^{-1}(N)$ is a subset of X. By (4) and by Lemma 4, we have $f(W\theta_I\text{-}PCl(f^{-1}(N))) \subset ker(f(f^{-1}(N)) \subset ker(N)$. Therefore $W\theta_I\text{-}PCl(f^{-1}(N)) \subset f^{-1}(ker(N))$.

(5) $\Rightarrow$ (1). Let U be an open subset of Y, then by Lemma 4, $ker(U) = U$. By (6), $W\theta_I\text{-}PCl(f^{-1}(U)) \subset f^{-1}(ker(U)) = f^{-1}(U)$ and thus we get $W\theta_I\text{-}PCl(f^{-1}(U)) \subset f^{-1}(U)$. We already have $f^{-1}(U) \subset W\theta_I\text{-}PCl(f^{-1}(U))$. Therefore we get $f^{-1}(U) = W\theta_I\text{-}PCl(f^{-1}(U))$. Thus U is weakly θ_I-*preclosed* in X. Hence f is contra weakly-θ_I-precontinuous. □

Remark 2. The concepts of weakly θ_I-*precontinuous* functions and contra-weakly θ_I-*precontinuous* functions are independent.

Example 2. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}, \{b\}\}$ and $I = P(X)$ then (X, τ, I) is an ideal topological space. Let $Y = \{p, q, r\}$ and $\sigma = \{Y, \phi, \{q, r\}\}$. Then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function defined as $f(a) = q$, $f(b) = r$, $f(c) = f(d) = p$. Then f is weakly θ_I-*precontinuous*. But it is not contra weakly-θ_I-*precontinuous*, since the preimage of a closed set $\{p\}$ in Y is $\{\{c\}, \{d\}\}$, which is not weakly θ_I-*preopen* in X.

Example 3. Let $X = \{1, 2, 3, 4\}$, $\tau = \{X, \phi, \{1\}, \{1, 2\}, \{1, 2, 4\}\}$ and $I = P(X)$ then (X, τ, I) is an ideal topological space. Let $Y = \{a, b, c, d\}$, $\sigma = \{Y, \phi, \{b\}, \{b, c\}, \{b, c, d\}\}$. Then (Y, σ) is a topological space. Suppose a function $f : (X, \tau, I) \to (Y, \sigma)$ is defined as $f(1) = a$, $f(2) = c$, $f(3) = d$, $f(4) = b$. Then it is contra weakly-θ_I-*precontinuous* but it is not weakly θ_I-precontinuous, since the preimage of an open set $\{b, c\}$ in Y is $\{2, 4\}$, which is not weakly θ_I-preopen in X.

Remark 3. The following example shows that composition of any contra weakly-θ_I-*precontinuous* functions need not be contra weakly-θ_I-*precontinuous* in general.

Example 4. Let $X = Y = Z = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}, \{b\}, \{a, b, c\}\}$ and $I_1 = \{\phi, \{b\}\}$, $\sigma = \{Y, \phi, \{b\}\}$, $I_2 = P(X)$ then (X, τ, I_1) and (Y, σ, I_2) are ideal topological spaces. Let $\eta = \{Z, \phi, \{a\}, \{a, b, c\}\}$, then (Z, η) is a topological space. Let $f : (X, \tau, I_1) \to (Y, \sigma, I_2)$ be a function defined as $f(a) = c$, $f(b) = d$, $f(c) = b$ and $f(d) = a$. Then f is contra weakly-θ_I-*precontinuous*. Let $g : (Y, \sigma, I_2) \to (Z, \eta)$ be a function defined as $g(a) = b$, $g(b) = c$, $g(c) = d$ and $g(d) = a$. Then g is contra weakly-θ_I-*precontinuous*. But their composition $g \circ f$: $(X, \tau, I_1) \to (Z, \eta)$ is not contra weakly-θ_I-*precontinuous*, since the preimage of a closed set $\{b, c, d\}$ in (Z, η) i.e. $(g \circ f)^{-1}(\{b, c, d\}) = f^{-1}(g^{-1}(\{b, c, d\})) = f^{-1}(\{a, b, c\}) = \{a, c, d\}$, which is not weakly θ_I-*preopen* in (X, τ, I_1).

Theorem 5. *If a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra weakly θ_I-precontinuous and Y is a regular space, then f is weakly θ_I-precontinuous.*

Proof. Let (X, τ, I) be any ideal topological space and let $x \in X$ be any point. Suppose P is an open set in Y with $f(x) \in P$. Since Y is regular, then there exists an open set Q in Y with $f(x) \in Q$ such that $Cl(Q) \subset P$. Since f is contra weakly θ_I*-precontinuous*, then by Theorem 4, there exists a weakly θ_I-preopen set M containing 'x' such that $f(M) \subset Cl(Q)$. Then $f(M) \subset Cl(Q) \subset P$. It follows from Theorem 18 of [23] that f is weakly θ_I*-precontinuous*. □

Definition 9. An ideal topological space (X, τ, I) is said to be weakly θ_I-preconnected if X is not the union of two disjoint non-empty weakly θ_I-preopen subsets of X.

Theorem 6. *If a function $f : (X, \tau, I) \to (Y, \sigma)$ is a surjection and contra weakly-θ_I-precontinuous and X is weakly θ_I-preconnected, then Y is connected.*

Proof. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a contra weakly-θ_I-precontinuous function from a weakly θ_I*-preconnected* space X onto Y. On contrary, assume that space Y is disconnected. For, let $Y = A \cup B$ be the separation of Y into two disjoint non-empty clopen subsets of Y. Since f is onto contra weakly-θ_I-precontinuous function, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty weakly θ_I*-preopen* sets in X and also $f^{-1}(A) \cap f^{-1}(B)$ $= f^{-1}(A \cap B) = f^{-1}(\phi) = \phi$. Thus $f^{-1}(A)$ and $f^{-1}(B)$ forms a separation of X, which is contrary to the assumption that X is weakly θ_I*-preconnected*. Hence Y is connected. □

Theorem 7. *Let (X, τ, I) be any ideal topological space, which is weakly θ_I-preconnected and (Y, σ) be a T_1-space. If $f : (X, \tau, I) \to (Y, \sigma)$ is a contra weakly θ_I-precontinuous function, then f is constant.*

Proof. Suppose the space X is weakly θ_I*-preconnected*. Since Y is a T_1-space, $\lambda = \{f^{-1}(\{y\}) : y \in Y\}$ is the disjoint weakly θ_I*-preopen* partition of X. If $|\lambda| \geq 2$ (where $|\lambda|$ denotes the cardinality of the set λ), then X is the union of two disjoint non-empty weakly θ_I-preopen sets. But X is weakly θ_I*-preconnected* and we have $|\lambda| = 1$. It shows that the function f is constant. □

Definition 10 ([20])**.** A space (X, τ) is said to be locally indiscrete if every open set in X is closed in X.

Theorem 8. *If a function $f : (X, \tau, I) \to (Y, \sigma)$ is weakly θ_I-precontinuous and the space (Y, σ) is locally indiscrete, then f is contra weakly-θ_I-precontinuous.*

Proof. Let V be any open set in (Y, σ). Since Y is locally indiscrete, V is closed in Y. Since f is weakly θ_I-precontinuous, $f^{-1}(V)$ is weakly θ_I-preclosed in (X, τ, I). Hence, by Theorem 4 f is contra weakly-θ_I-precontinuous. □

Definition 11. An ideal topological space (X, τ, I) is said to be weakly θ_I*-pre-T_2* if for each pair of distinct points 'x' and 'y' in X, there exist weakly θ_I*-preopen* sets U and V containing 'x' and 'y', respectively, such that $U \cap V = \phi$.

Note - A closed neighbourhood of a point 'x' is a closed set that contains an open set containing the point 'x'.

Definition 12 ([4])**.** A Urysohn space (also called T-$2\frac{1}{2}$-space or T_e-space) is a space in which any two distinct points can be separated by closed neighbourhoods.

Theorem 9. *Let (X, τ, I) be any ideal topological space. For each pair of points x_1, $x_2 \in X$ with $x_1 \neq x_2$, there exists a function f from space X into an Urysohn space Y such that $f(x_1) \neq f(x_2)$ and function f is contra weakly-θ_I-precontinuous at x_1 and x_2, then the space X is weakly θ_I-pre-T_2.*

Proof. Let x_1, $x_2 \in X$ with $x_1 \neq x_2$ be any pair of points and suppose $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Then by assumption, $y_1 \neq y_2$. Since Y is an Urysohn space, there exist two open sets U and V in Y such that $y_1 \in U$, $y_2 \in V$ and $Cl(U) \cap Cl(V) = \phi$. Since f is contra weakly θ_I-*precontinuous*, by Theorem 4, there exist two weakly θ_I-*preopen* sets P and Q containing x_1 and x_2, respectively, such that $f(P) \subset Cl(U)$ and $f(Q) \subset Cl(V)$. Since $Cl(U) \cap Cl(V) = \phi$, $P \cap Q = \phi$. Hence X is weakly θ_I-*pre*-T_2. □

Corollary 1. *If $f : (X, \tau, I) \to (Y, \sigma)$ is a contra weakly-θ_I-precontinuous injection and (Y, σ) is Urysohn, then (X, τ, I) is weakly θ_I-pre-T_2.*

Theorem 10. *If $f : (X, \tau, I) \to (Y, \sigma, J)$ and $g : (Y, \sigma, J) \to (Z, \eta)$ are any two functions, then following hold:*

1. *If f is a contra weakly-θ_I-precontinuous function and g is a continuous function, then their composition $g \circ f$ is contra weakly-θ_I-precontinuous.*
2. *If f is weakly-θ_I-preirresolute and g is weakly-θ_I-precontinuous, then their composition $g \circ f$ is contra weakly-θ_I-precontinuous.*

Proof. 1. Let F be any closed set in (Z, η). Since g is continuous, $g^{-1}(F)$ is closed in (Y, σ, J). Since f is contra weakly-θ_I-*precontinuous*, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is weakly-θ_I-*preopen* in (X, τ, I). Therefore their composition $g \circ f$ is contra weakly-θ_I-*precontinuous*.

2. Let F be any closed set in (Z, η). Since g is weakly-θ_I-precontinuous, $g^{-1}(F)$ is weakly-θ_I-*preopen* in (Y, σ, J). Since f is weakly-θ_I-*preirresolute*, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is weakly-θ_I-*preopen* in (X, τ, I). Therefore their composition $g \circ f$ is contra weakly-θ_I-*precontinuous*. □

4. Contra-θ_I-precontinuous, contra-θ_I-α-continuous, contra-θ_I-semicontinuous and contra-θ_I-β-continuous functions

Here, we introduce the notions of contra-θ_I-precontinuous, contra-θ_I-α-*continuous*, contra-θ_I-*semicontinuous* and contra-θ_I-β-*continuous* functions and elaborate their relationship with contra weakly-θ_I-precontinuous functions and their interrelationships.

Definition 13. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be θ_I-α-continuous if the preimage of every open set in Y is θ_I-α-open in X.

Definition 14. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be θ_I-semicontinuous if the preimage of every open set in Y is θ_I-semiopen in X.

Definition 15. A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be contra-θ_I-precontinuous ((resp. contra-θ_I-*α-continuous*, contra-θ_I-*semicontinuous*, contra-θ_I-*β-continuous*) if the preimage of every closed set in Y is θ_I-*preopen* (resp. θ_I-*α-open*, θ_I-*semiopen*, θ_I-*β-open*) in X.

Theorem 11. (1) *Every contra-θ_I-precontinuous function is contra weakly-θ_I-precontinuous.*

(2) *Every contra-precontinuous function is contra-θ_I-precontinuous and therefore contra weakly-θ_I-precontinuous.*

(3) *Every contra-θ_I-α-continuous function is contra-θ_I-precontinuous and contra-θ_I-semicontinuous.*

(4) *Every contra weakly-θ_I-precontinuous function is contra-θ_I-β-continuous.*

(5) *Every contra-θ_I-semicontinuous function is contra-θ_I-β-continuous.*

Proof. (1) Suppose a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra-θ_I-*precontinuous* and A is any closed set in Y, then $f^{-1}(A)$ is θ_I-preopen in X and $f^{-1}(A)$ is also weakly θ_I-*preopen* in X and therefore the function f is contra weakly-θ_I-*precontinuous*, since every θ_I-*preopen* set is weakly θ_I-*preopen*. For, if A is θ_I-*preopen* set in X. By using the definition of a θ_I-*preopen* set, we have $A \subseteq Int(Cl_{\theta_I}(A)) \subseteq sCl(Int(Cl_{\theta_I}(A)))$. This shows that A is weakly θ_I-*preopen*. This shows that A is weakly θ_I-*preopen*.

(2) Suppose a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra-precontinuous and A is any closed set in Y, then $f^{-1}(A)$ is preopen in X and therefore $f^{-1}(A)$ is θ_I-*preopen* in X, it follows that the function f is contra-θ_I-*precontinuous*, therefore f is contra weakly-θ_I-*precontinuous*, since every preopen set is θ_I-*preopen* and therefore weakly θ_I-*preopen*. For, if A is any preopen set in X, $A \subseteq Int(Cl(A)) \subseteq Int(Cl_{\theta_I}(A)))$ and therefore A is θ_I-*preopen* and hence A is weakly θ_I-*preopen*.

(3) Suppose a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra-θ_I-*α-continuous* and A is any closed set in Y, then $f^{-1}(A)$ is θ_I-*α-open* in X and also $f^{-1}(A)$ is θ_I-*preopen*, consequently function f is contra-θ_I-*precontinuous*. For, if A is any θ_I-*α-open* set in X, $A \subseteq Int(Cl(Int_{\theta_I}(A)))$. By using Lemma 3, $A \subseteq Int(Cl(Int_{\theta_I}(A))) \subseteq Int(Cl(Int(A))) \subseteq Int(Cl(Int(Cl_{\theta_I}(A)))) = sCl(Int(Cl_{\theta_I}(A)))$. Hence A is weakly θ_I-*preopen*.

(4) Suppose $f : (X, \tau, I) \to (Y, \sigma)$ is any contra weakly-θ_I-*precontinuous* function and A is any closed set in Y, then $f^{-1}(A)$ is weaklyθ_I-*preopen* in X and $f^{-1}(A)$ is also θ_I-β-*open* in X, since every weaklyθ_I-*preopen* set is θ_I-β-open. For, if A is weaklyθ_I-preopen then we have $A \subseteq sCl(Int(Cl_{\theta_I}(A))) \subseteq Cl(Int(Cl_{\theta_I}(A))))$. This implies that A is θ_I-β-*open*. Hence f is contra-θ_I-β-*continuous*.

(5) Suppose a function $f : (X, \tau, I) \to (Y, \sigma)$ is contra-θ_I-*semicontinuous* and A is any closed set in Y, then $f^{-1}(A)$ is θ_I-*semiopen* in X and $f^{-1}(A)$ is also θ_I-β-*open* in X, since every θ_I-*semiopen* set is θ_I-β-open. For, if A be any θ_I-*semiopen* set in X, $A \subseteq Cl(Int_{\theta_I}(A))$. Let us assume that $Int_{\theta_I}(A) \subseteq Int(A) \subseteq Int(Cl_{\theta_I}(A)))$, $A \subseteq Cl(Int_{\theta_I}(A)) \subseteq Cl(Int(Cl_{\theta_I}(A)))$, which implies that A is θ_I-β-open. Thus f is contra-θ_I-β-*continuous*.

□

Remark 4. The converses (1)-(5) of Theorem 12 are not true, as shown by the following examples.

Example 5. In Example 1, $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{b, c\}, \{c, d\}, \{b, c, d\}, \{c\}\}$ and $I = P(X)$ then (X, τ, I) is an ideal topological space. Let $Y = \{1, 2, 3, 4\}$, $\sigma = \{Y, \phi, \{1, 2\}, \{1, 2, 3\}, \{2\}\}$. Then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ is a function defined as $f(a) = 3$, $f(b) = 2$, $f(c) = 4$, $f(d) = 1$. Then f is contra weakly θ_I-precontinuous, since the preimage of every closed set in Y is weakly θ_I-*preopen* in X. But f is not contra θ_I-*precontinuous*, since the preimage of a closed set $\{3, 4\}$ in Y is $\{a, c\}$, which is not θ_I-*preopen* in X.

Example 6. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $I = \{\phi, \{a\}\}$ then (X, τ, I) is an ideal topological space. Let $Y = \{1, 2, 3, 4\}$, $\sigma = \{Y, \phi, \{1, 3\}, \{1, 3, 4\}\}$. Then (Y, σ) is a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be a function defined as $f(a) = 2$, $f(b) = 3$, $f(c) = 1$, $f(d) = 4$. Then f is contra weakly-θ_I-*precontinuous*, since the preimage of every closed set in Y is weakly-θ_I-*preopen* in X. But f is neither contra θ_I-α-*continuous* nor *contra-precontinuous*, since the preimage of a closed set $\{2, 4\}$ in Y is $\{a, b\}$, which is neither θ_I-α-*open* nor preopen in X.

Example 7. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{d\}, \{b\}, \{b, d\}\}$ and $I = \{\phi, \{b\}, \{d\}, \{b, d\}\}$ then (X, τ, I) is an ideal topological space. Let $Y = \{p, q, r, s\}$, $\sigma = \{Y, \phi, \{p\}, \{p, q, r\}\}$. Then (Y, σ) be a topological space. Let $f : (X, \tau, I) \to (Y, \sigma)$ be function defined as $f(a) = q$, $f(b) = p$, $f(c) = r$, $f(d) = s$. Then f is contra-θ_I-β-*continuous*, since the preimage of every closed set in Y is θ_I-β-*open* in X. But it is not contra weakly-θ_I-*precontinuous*, since the preimage of a closed set $\{q, r, s\}$ in Y is $\{a, c, d\}$, which is not weakly θ_I-*preopen* in X.

Example 8. Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}, \{b, c\}, \{a, b, c\}, \{b\}\}$ and $I = \{\phi, \{b, c\}\}$ then (X, τ, I) is an ideal topological space. Let $Y = \{p, q, r, s\}$, $\sigma = \{Y, \phi, \{p\}, \{p, q, r\}, \{p, r\}\}$. Then (Y, σ) is a topological space. Let f :

$(X, \tau, I) \to (Y, \sigma)$ be a function defined as $f(a) = r$, $f(b) = q$, $f(c) = s$, $f(d) = p$. Then f is contra- θ_I-β-*continuous*, since the preimage of every closed set in Y is θ_I-β-open in X. But it is not contra θ_I-semicontinuous, since the preimage of a closed set $\{s\}$ in Y is $\{c\}$, which is not θ_I-semiopen in X.

Theorem 12. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be any contra θ_I-semicontinuous function from an ideal topological space (X, τ, I) to any space (Y, σ). If the space (X, τ, I) is extremally disconnected, then f is contra θ_I-α-continuous.*

Proof. Let $f : (X, \tau, I) \to (Y, \sigma)$ be any contra-θ_I-semicontinuous function from the space X to Y. If F is any closed set in Y, then $f^{-1}(F)$ is θ_I-semiopen in X. Since X is extremally disconnected, by Theorem 1, $f^{-1}(F)$ is θ_I-α-*open* in X. Hence f is contra-θ_I-α-*continuous*. □

Theorem 13. *Let $f : (X, \tau, I) \to (Y, \sigma)$ be any contra-θ_I-β-continuous function from an ideal topological space (X, τ, I) to any space (Y, σ). If the space (X, τ, I) is extremally disconnected, then f is contra weakly-θ_I-precontinuous.*

Proof. This follows directly from Theorem 2. □

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