ROUGH SOFT BCK-ALGEBRAS AND THEIR DECISION MAKING

Li Zhang
Xueling Ma
Jianming Zhan*
Department of Mathematics
Hubei University for Nationalities
Enshi, Hubei Province, 445000
China
zjmhbmy@126.com
zhanjianming@hotmail.com

Abstract. Rough sets and soft sets are important tools to deal with uncertainties. In this paper, we apply rough soft set theory to BCK-algebras. The lower and upper rough soft BCK-algebras (ideals) are discussed. Finally, we establish a kind of decision making method for rough soft BCK-algebras.

Keywords: rough soft set, subalgebra (ideals), BCK-algebra, decision making.

1. Introduction

In 1982, Pawlak [23] introduced the concept of rough sets as an important tool to discuss imprecision, vagueness and uncertainties. Since then, this subject has been investigated in many studies, for examples, see [2, 24, 28, 29, 27]. It soon invoked a natural question concerning a possible connection between rough sets and algebraic systems. Biswas [4] introduced the concepts of rough groups and rough subgroups. Kuroki [17] studied the properties of rough ideals in semigroups. In particular, Davvaz [8] dealt with a relationship between rough sets and rings with respect to an ideal of rings.

In 1999, Molodtsov [22] put forward the concept of soft sets as a new mathematical tool for dealing with uncertainties. At present, research on the soft set theory is progressing rapidly. Maji[19] defined some basic operations on soft sets. In 2009, Ali [3] gave some new operations on soft sets. In particular, Çağman and Maji [5, 6, 20] applied soft set theory to decision making. At the same time, some soft algebras were also discussed, such as [1, 14, 16, 18, 25, 26]. Chen [7] presented a new concept of soft set parameterization reduction, and compared this concept with the related concept of attributes reduction in rough set theory. In particular, Feng [10, 11] proposed rough soft sets by combing

∗. Corresponding author

Pawlak rough sets and soft sets, rough sets can be regarded as a collection of rough sets sharing a common Pawlak approximation space.

As is well known, BCK and BCI-algebras [12] are two classes of algebras of logic. They have been extensively investigated by many researchers, for examples, see [15, 26]. Dudek and Jun applied rough set theory to BCK and BCI-algebras [9, 13], respectively. In 2008, Jun [14, 16] applied soft set theory to BCI-algebras.

In the present paper, we apply rough soft set theory to BCK-algebras. Some new basic theory on rough soft BCK-algebras are obtained. Finally, we put forward a kind of decision making for rough soft BCK-algebras.

2. Preliminaries

Recall that a BCK-algebra is an algebra $(X, *, 0)$ of type $(2, 0)$ satisfying the following:

(1) $((x * y) * (x * z)) * (x * y) = 0$,

(2) $(x * (x * y)) * y = 0$,

(3) $x * x = 0$,

(4) $0 * x = 0$,

(5) $x * y = 0$ and $y * x = 0$ imply $x = y$.

For any BCK-algebra X, the relation $\leq$ defined by $x \leq y$ if and only if $x * y = 0$ is a partial order on X.

A non-empty subset S of a BCK-algebra X is called a subalgebra of X if $x * y \in S$ whenever $x, y \in S$. A non-empty subset I of X is called an ideal of X, denoted by $I \lhd X$, if it satisfies: (1) $0 \in I$; (2) $x * y \in I$ and $y \in I$ imply $x \in I$ for all $x, y \in X$. Note that every ideal of a BCK-algebra X is a subalgebra of X.

Throughout this paper, X is always a BCK-algebra.

Definition 2.1 ([22])**.** A pair $\mathfrak{S} = (F, A)$ is called a soft set over U, where $A \subseteq E$ and $F : A \to \mathscr{P}(U)$ is a set-valued mapping.

Definition 2.2 ([14])**.** Let (F, A) be a soft set over X. Then

(1) (F, A) is called a soft BCK-algebra over X if $F(x)$ is a subalgebra of X for all $x \in \mathrm{Supp}(F, A)$.

(2) (F, A) is called an idealistic soft BCK-algebra if $F(x)$ is an ideal of X for all $x \in \mathrm{Supp}(F, A)$, where $\mathrm{Supp}(F, A) = \{x \in A | F(x) \neq \emptyset\}$ is called a soft support of the soft set (F, A).

Definition 2.3 ([3])**.** Let (F, A) and (G, B) be two soft sets over a common universe U.

(1) The restricted intersection of (F, A) and (G, B), denoted by $(F, A) \Cap (G, B)$, is defined as the soft set (H, C), where $C = A \cap B$, and $H(c) = F(c) \cap G(c)$ for all $c \in C$,

(2) The extended intersection of (F, A) and (G, B), denoted by $(F, A) \sqcap_{\mathscr{E}} (G, B)$, is defined as the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cap G(e), & \text{if } e \in A \cap B. \end{cases}$$

(3) The restricted union of (F, A) and (G, B), denoted by $(F, A) \cup_{\mathscr{R}} (G, B)$, is defined as the soft set (H, C), where $C = A \cap B$, and $H(c) = F(c) \cup G(c)$ for all $c \in C$,

(4) The extended union of (F, A) and (G, B), denoted by $(F, A) \widetilde{\cup} (G, B)$, is defined as the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.4 ([23])**.** Let ρ be an equivalence relation on the universe X, (X, ρ) be a Pawlak approximation space. A subset $A \subseteq X$ is called definable if $\underline{\rho}(A) = \overline{\rho}(A)$, otherwise, X is a rough set, where

$$\underline{\rho}(A) = \{x \in X : [x]_\rho \subseteq A\},$$

and

$$\overline{\rho}(A) = \{x \in X : [x]_\rho \cap A \neq \emptyset\}.$$

3. Lower and upper soft approximations

Let $I \lhd X$. Define a relation $\equiv_I$ on X as follows:

$x \equiv_I y \Longleftrightarrow x * y \in I$ and $y * x \in I$.

Then $\equiv_I$ is an equivalence on x with respect to (briefly, w.r.t.) I. Moreover, $\equiv_I$ satisfies $x \equiv_I y$ and $u \equiv_I v \Longrightarrow x * u \equiv_I y * v$. Hence $\equiv_I$ is a congruence on X.

Let $[x]_I$ denote the equivalence class of X w.r.t. I and X/I denote the set of all equivalence classes, that is, $X/I = \{[x]_I | x \in X\}$. Define an operation $*$ on X/I by $[x]_I * [y]_I = [x * y]_I$, then it is clear that $(X/I, *, I)$ is a BCK-algebra.

Definition 3.1. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ a soft set over X. The lower and upper soft approximation of $\mathfrak{S} = (F, A)$ w.r.t. I are denoted by:

$\underline{Apr}_I(\mathfrak{S}) = (\underline{F}_I, A)$ and $\overline{Apr}_I(\mathfrak{S}) = (\overline{F}_I, A)$, which are soft sets over X with $\underline{F}_I(x) = \underline{Apr}_I(F(x)) = \{y \in X | [y]_I \subseteq F(x)\}$ and $\overline{F}_I(x) = \overline{Apr}_I(F(x)) = \{y \in S | [y]_I \cap F(x) \neq \emptyset\}$, for all $x \in A$.

(i) $\underline{Apr}_I(\mathfrak{S}) = \overline{Apr}_I(\mathfrak{S})$, $\mathfrak{S}$ is called definable;

(ii) $\underline{Apr}_I(\mathfrak{S}) \neq \overline{Apr}_I(\mathfrak{S})$, $\underline{Apr}_I(\mathfrak{S})$ ($\overline{Apr}_I(\mathfrak{S})$) is called a lower (upper) rough soft set. Moreover, $\mathfrak{S}$ is called a rough soft set.

Therefore, when $U = X$ and ρ is the induced relation by an ideal I, then we use the pair (X, I) instead of the approximation space (U, ρ).

Similar to Theorem 4 in [10], we have

Theorem 3.2. Let $I \lhd X$, (X, I) a Pawlak approximation space and $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ two soft sets over X. Then

(1) $\underline{Apr}_I(\mathfrak{S} \Cap \mathfrak{T}) = \underline{Apr}_I(\mathfrak{S}) \Cap \underline{Apr}_I(\mathfrak{T})$;
(2) $\underline{Apr}_I(\mathfrak{S} \sqcap_{\mathscr{E}} \mathfrak{T}) = \underline{Apr}_I(\mathfrak{S}) \sqcap_{\mathscr{E}} \underline{Apr}_I(\mathfrak{T})$;
(3) $\overline{Apr}_I(\mathfrak{S} \Cap \mathfrak{T}) \subseteq \overline{Apr}_I(\mathfrak{S}) \Cap \overline{Apr}_I(\mathfrak{T})$;
(4) $\overline{Apr}_I(\mathfrak{S} \sqcap_{\mathscr{E}} \mathfrak{T}) \subseteq \overline{Apr}_I(\mathfrak{S}) \sqcap_{\mathscr{E}} \overline{Apr}_I(\mathfrak{T})$;
(5) $\underline{Apr}_I(\mathfrak{S} \cup_{\mathscr{R}} \mathfrak{T}) \supseteq \underline{Apr}_I(\mathfrak{S}) \cup_{\mathscr{R}} \underline{Apr}_I(\mathfrak{T})$;
(6) $\underline{Apr}_I(\mathfrak{S} \widetilde{\cup} \mathfrak{T}) \supseteq \underline{Apr}_I(\mathfrak{S}) \widetilde{\cup} \underline{Apr}_I(\mathfrak{T})$;
(7) $\overline{Apr}_I(\mathfrak{S} \cup_{\mathscr{R}} \mathfrak{T}) = \overline{Apr}_I(\mathfrak{S}) \cup_{\mathscr{R}} \overline{Apr}_I(\mathfrak{T})$;
(8) $\overline{Apr}_I(\mathfrak{S} \widetilde{\cup} \mathfrak{T}) = \overline{Apr}_I(\mathfrak{S}) \widetilde{\cup} \overline{Apr}_I(\mathfrak{T})$;
(9) $\mathfrak{S} \subseteq \mathfrak{T} \Rightarrow \underline{Apr}_I(\mathfrak{S}) \subseteq \underline{Apr}_I(\mathfrak{T}), \overline{Apr}_I(\mathfrak{S}) \subseteq \overline{Apr}_I(\mathfrak{T})$.

Theorem 3.3. Let (X, I) be an approximation space and $\mathfrak{S} = (F, A)$ a soft set over X, If $I = \{0\}$, then $\mathfrak{S}$ is definable.

Proof. For all $x \in A$, we have $[x]_I = \{y \in X | x * y, y * x = 0\} = \{x\}$. Hence $\underline{F}_I(x) = \{y \in X | [y[_I \subseteq F(x)\} = \{y \in X | \{y\} \subseteq F(x)\} = F(x)$ and $\overline{F}_I(x) = \{y \in X | [y]_I \cap F(x) \neq \emptyset\} = \{y \in X | \{y\} \cap F(x) \neq \emptyset\} = F(x)$. Thus, for all $x \in A$, $\underline{F}_I(x) = \overline{F}_I(x)$. Thus means that $\underline{Apr}_I(\mathfrak{S}) = \overline{Apr}_I(\mathfrak{S})$, which implies, $\mathfrak{S}$ is definable. $\square$

Let $A, B \subseteq X$, we denote $A * B = \{x * y | \forall x \in A, y \in B\}$.

Definition 3.4. Let $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ be two soft sets over X, then we denote $\mathfrak{S} * \mathfrak{T}$ by $\mathfrak{S} * \mathfrak{T} = (F, A) * (G, B) = (H, A * B)$, where $H(x, y) = F(x) * G(y)$ for all $x \in A, y \in B$.

Example 3.5. Let $X = \{0, 1, 2, 3, 4\}$ be a BCK-algebra with the following Cayley table:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	1
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

Let $I = \{0, 1, 2\} \lhd X$, then $[0]_I = [1]_I = [2]_I = \{0, 1, 2\}, [3]_I = \{3\}$ and $[4]_I = \{4\}$.

Define two soft sets $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ over X, where $A = \{0, 1, 3\}$ and $B = \{0, 3, 4\}$ by $F(0) = \{0, 2\}, F(1) = \{0, 1\}, F(3) = \{0, 4\}$ and $G(0) = \{0, 1, 4\}, G(3) = \{0, 2\}, G(4) = \{0, 1\}$.

Thus, $H(0, 0) = \{0, 2\}, H(0, 3) = \{0, 2\}, H(0, 4) = \{0, 2\}, H(1, 0) = \{0, 1\}, H(1, 3) = \{0, 1\}, H(1, 4) = \{0, 1\}, H(3, 0) = \{0, 4\}, H(3, 3) = \{0, 4\}, H(3, 4) = \{0, 4\}$.

Theorem 3.6. Let $I \lhd X$ and (X, I) a Pawlak approximation space. Suppose that $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ are two soft sets over X. Then

$$\overline{Apr}_I(\mathfrak{S}) * \overline{Apr}_I(\mathfrak{T}) \subseteq \overline{Apr}_I(\mathfrak{S} * \mathfrak{T}).$$

Proof. Let $z \in \underline{F}_I(x) * \overline{G}_I(y)$, then there exist $u \in \underline{F}_I(x)$ and $v \in \overline{G}_I(y)$ such that $z = u * v$, and so $[u]_I \cap F(x) \neq \emptyset$ and $[v]_I \cap G(y) \neq \emptyset$. Thus, there exist $a \in F(x)$ and $b \in G(y)$ such that $a \in [u]_I, b \in [v]_I$, and so $a * b \in [u]_I * [v]_I = [u * v]_I$, which implies $[u * v]_I \cap (F(x) * G(y)) \neq \emptyset$. This means that $z \in \overline{Apr}_I(\mathfrak{S} * \mathfrak{T})$. Therefore, $\overline{Apr}_I(\mathfrak{S}) * \overline{Apr}_I(\mathfrak{T}) \subseteq \overline{Apr}_I(\mathfrak{S} * \mathfrak{T})$. □

The following example shows that the inclusion symbol "$\subseteq$" in above theorem may not be replaced by an equal sign.

Example 3.7. Let $X = \{0, 1, 2, 3, 4\}$ be a BCK-algebra with the following Cayley table:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	0
3	3	3	3	0	0
4	4	3	4	1	0

Let $I = \{0, 1, 2\} \lhd X$, then $[0]_I = [1]_I = [2]_I = \{0, 1, 2\}$ and $[3]_I = [4]_I = \{3, 4\}$.

Define two soft sets $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ over X, where $A = \{1\}$ and $B = \{3\}$ by $F(1) = \{4\}$ and $G(3) = \{0, 4\}$. By calculation, we have $\overline{F}_I(1) = \{3, 4\}$ and $\overline{G}_I(3) = \{0, 1, 2, 3, 4\}$. Thus, $\overline{F}_I(1) \cdot \overline{G}_I(3) = \{0, 1, 3, 4\}$ and $\overline{Apr}_I(F(1) \cdot G(3)) = \overline{Apr}_I(\{0, 4\}) = \{0, 1, 2, 3, 4\}$, which implies, $\overline{Apr}_I(\mathfrak{S}) * \overline{Apr}_I(\mathfrak{T}) \subsetneq \overline{Apr}_I(\mathfrak{S} * \mathfrak{T})$.

4. Rough soft ideals

In this section, we introduce the concepts of rough soft subalgebras (ideals) of BCK-algebras and obtain some related properties.

Definition 4.1. Let $I \lhd X$, (X, I) a Pawlak approximation space and $\mathfrak{S} = (F, A)$ a soft set over X. Then $\underline{Apr}_I(\mathfrak{S})$ ($\overline{Apr}_I(\mathfrak{S})$) is called a lower (upper) rough soft BCK-algebras (resp., ideal) w.r.t. I over X if $\underline{F}_I(x)$ ($\overline{F}_I(x)$) is a subalgebra (resp., ideal) of X for all $x \in \mathrm{Supp}(F, A)$. Moreover, $\mathfrak{S}$ is called a rough soft BCK-algebra (resp., rough soft ideal) w.r.t. I over X if $\underline{F}_I(x)$ and $\overline{F}_I(x)$ are subalgebras (resp., ideals) of X for all $x \in \mathrm{Supp}(F, A)$.

Example 4.2. Let $X = \{0, 1, 2, 3\}$ be a BCK-algebra with the Cayley table as follows:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	2
3	3	3	3	0

Let $I = \{0,1\} \lhd X$, then $[0]_I = [1]_I = \{0,1\}, [2]_I = \{2\}$ and $[3]_I = \{3\}$.

Let $A = X$ and $F : A \to \mathscr{P}(X)$ be a set-valued function define by $F(0) = F(1) = X, F(2) = \{0,1,3\}$ and $F(3) = \{0,1,2\}$.

By calculations, $\underline{F}_I(0) = \underline{F}_I(1) = X$, $\underline{F}_I(2) = \underline{F}_I(3) = \{0,1\} \lhd X$ and $\overline{F}_I(0) = \overline{F}_I(1) = X$, $\overline{F}_I(2) = \{0,1,3\} \lhd X$ and $\overline{F}_I(3) = \{0,1,2\} \lhd X$.

This means that, $\mathfrak{S}$ is a rough soft ideal w.r.t. I over X.

Since any ideal of a BCK-algebra is a subalgebra of X [21], we can obtain the following:

Proposition 4.3. Any rough soft ideal of X is a rough soft BCK-algebra.

The converse of the above proposition may not be true as shown in the following example:

Example 4.4. Let $X = \{0,1,2,3,4\}$ be a BCK-algebra with the following Cayley table:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	4	4	0

Let $I = \{0,2\} \lhd X$, then $[0]_I = [2]_I = \{0,2\}, [1]_I = \{1\}, [3]_I = \{3\}$ and $[4]_I = \{4\}$.

Define a soft set $\mathfrak{S} = (F,A)$ over X, where $A = \{2\}$ by $F(2) = \{0,3\}$. By calculations, $\underline{F}_I(2) = \emptyset$ and $\overline{F}_I(2) = \{0,2,3\}$. Then $\mathfrak{S}$ is a rough soft BCK-algebra w.r.t. I over X, but it is not a rough soft ideal w.r.t. I over X since $\overline{F}_I(2) = \{0,2,3\}$ is not an ideal of X.

Now, we give some operations of rough soft ideals.

Theorem 4.5. Let $I \lhd X$ and (X,I) a Pawlak approximation space. Assume that $\underline{Apr}_I(\mathfrak{S}) = (\underline{F}_I, A)$ and $\underline{Apr}_I(\mathfrak{T}) = (\underline{G}_I, B)$ are lower rough soft ideals w.r.t. I over X. If $\mathfrak{S} \Cap \mathfrak{T}$ is a non-null soft set, then $\underline{Apr}_I(\mathfrak{S} \Cap \mathfrak{T})$ and $\underline{Apr}_I(\mathfrak{S} \sqcap_{\mathscr{E}} \mathfrak{T})$ are lower rough soft ideals over X.

Proof. By the hypothesis, $\forall x \in \mathrm{Supp}(F,A)$, $y \in \mathrm{Supp}(G,B)$, $\underline{F}_I(x)$ and $\underline{G}_I(y)$ are ideals of X. Since $\mathfrak{S} \Cap \mathfrak{T}$ is non-empty, $\forall x' \in A \cap B$, $\underline{F}_I(x') \cap \underline{G}_I(x')$ is an ideal of X. By Theorem 3.2, $\underline{Apr}_I(\mathfrak{S} \Cap \mathfrak{T})$ is a lower rough soft ideal over X. Similarly, we can prove $\underline{Apr}_I(\mathfrak{S} \sqcap_{\mathscr{E}} \mathfrak{T})$ is also a lower rough soft ideal over X. $\square$

Remark 4.6. In general, $\overline{Apr}_I(\mathfrak{S} \Cap \mathfrak{T})$ and $\overline{Apr}_I(\mathfrak{S} \sqcap_{\mathscr{E}} \mathfrak{T})$ are not upper rough soft ideals over X if $\overline{Apr}_I(\mathfrak{S}) = (\overline{F}_I, A)$ and $\overline{Apr}_I(\mathfrak{T}) = (\overline{G}_I, A)$ are both upper rough soft ideals over X as shown in the following example:

Example 4.7. Let $X = \{0, 1, 2, 3, 4\}$ be a BCK-algebra with the Cayley table as follows:

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	2	0	2	0
3	3	3	3	0	0
4	4	4	4	4	0

Let $I = \{0, 1, 2\} \lhd X$, then $[0]_I = [1]_I = [2]_I = \{0, 1, 2\}$, $[3]_I = \{3\}$ and $[4]_I = \{4\}$. Define two soft sets $\mathfrak{S} = (F, A)$ and $\mathfrak{T} = (G, B)$ over X, where $A = \{0, 1, 3\}$ and $B = \{3, 4\}$ by $F(0) = \{0, 2\}$, $F(1) = \{0, 1\}$, $F(3) = \{0, 3\}$ and $G(3) = \{0, 3, 4\}$, $G(4) = \{0, 4\}$.

By calculations, $\overline{F}_I(3) = \{0, 1, 2, 3\} \lhd X$ and $\overline{G}_I(3) = \{0, 1, 2, 3, 4\} \lhd X$.

Since $A \cap B = \{3\}$ and $F(3) \cap G(3) = \{3\}$, but $\overline{Apr}_I(\mathfrak{S} \Cap \mathfrak{T}) = \overline{Apr}_I(\{3\}) = \{3\} \ntriangleleft X$.

Theorem 4.8. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ a soft BCK-algebra over X. If $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset$, then $\mathfrak{S}$ is a lower rough soft BCK-algebra w.r.t. I over X.

Proof. $\forall a, b \in \underline{F}_I(x)$, then $[a]_I \subseteq F(x)$ and $[b]_I \subseteq F(x)$. Since $F(x)$ is a subalgebra of X for all $x \in A$, then $F(x) * F(x) \subseteq F(x)$, $[a * b]_I = [a]_I * [b]_I \subseteq F(x) * F(x) \subseteq F(x)$, which implies, $a * b \in \underline{F}_I(x)$. Thus, $\mathfrak{S}$ is a lower rough soft BCK-algebra w.r.t. I over X. □

Theorem 4.9. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ a soft BCK-algebra over X, then $\mathfrak{S}$ is an upper rough soft BCK-algebra w.r.t. I over X.

Proof. $\forall a, b \in \overline{F}_I(x)$, then $[a]_I \cap F(x) \neq \emptyset$ and $[b]_I \cap F(x) \neq \emptyset$, and so there exist $y, z \in F(x)$ such that $y \in [a]_I$ and $z \in [b]_I$. Since $F(x)$ is a subalgebra of X, then $y * z \in F(x)$. Moreover, $y * z \in [a]_I * [b]_I = [a * b]_I$. This implies that $y * z \in F(x) \cap [a * b]_I$, that is, $F(x) \cap [a * b]_I \neq \emptyset$. Thus, $a * b \in \overline{F}_I(x)$, and so $\mathfrak{S}$ is an upper rough soft BCK-algebra w.r.t. I over X. □

Corollary 4.10. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ a soft BCK-algebra over X. If $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset$, then $\mathfrak{S}$ is a rough soft BCK-algebra w.r.t. I over X.

Theorem 4.11. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ an idealistic soft BCK-algebra over X. If $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset$, then $\mathfrak{S}$ is a lower rough soft ideal w.r.t. I over X.

Proof. Since $\mathfrak{S}$ is an idealistic soft BCK-algebra over X, then for all $x \in A$, $F(x)$ is an ideal of X. Let $y \in [0]_I$, then $y = y * 0 \in I \subseteq F(x)$, which implies, $[0]_I \subseteq F(x)$, that is, $0 \in \underline{F}_I(x)$. Now, let $a, b \in X$ be such that $b \in \underline{F}_I(x)$ and

$a*b \in \underline{F}_I(x)$, then $[b]_I \subseteq F(x)$ and $[a]_I * [b]_I = [a*b]_I \subseteq F(x)$. Suppose $y \in [a]_I$ and $z \in [b]_I$, then $y * z \in [a]_I * [b]_I = [a * b]_I \subseteq F(x)$. Since $z \in [b]_I \subseteq F(x)$ and $F(x)$ is an ideal of X, $y \in F(x)$, that is, $[a]_I \subseteq F(x)$. This means that $a \in \underline{F}_I(x)$, which implies, $\underline{F}_I(x)$ is an ideal of X. Hence $\mathfrak{S}$ is a lower rough soft ideal w.r.t. I over X. □

In general, we need to add a condition on upper rough soft ideals as shown in the following:

Theorem 4.12. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ an idealistic soft BCK-algebra over X with $I \subseteq F(x)$ for all $x \in A$, then $\mathfrak{S}$ is an upper rough soft ideal w.r.t. I over X.

Proof. Clearly, $0 \in \overline{F}_I(x)$. Let $y, z \in X$ be such that $z \in \overline{F}_I(x)$ and $y * z \in \overline{F}_I(x)$, then $[z]_I \cap F(x) \neq \emptyset$ and $[y * z]_I \cap F(x) \neq \emptyset$, and so there exist $a, b \in F(x)$ such that $a \in [z]_I$ and $b \in [y * z]_I$. Thus, $z * a \in I \subseteq F(x)$ and $(y*z)*b \in I \subseteq F(x)$. Since $F(x)$ is an ideal of X, then $z \in F(x)$ and $y*z \in F(x)$, and so $y \in F(x)$. This means that $[y]_I \cap F(x) \neq \emptyset$, and so, $y \in \overline{F}_I(x)$. Hence $\overline{F}_I(x)$ is an ideal of X. Thus, $\mathfrak{S}$ is an upper rough soft ideal w.r.t. I over X. □

Corollary 4.13. Let $I \lhd X$ and $\mathfrak{S} = (F, A)$ an idealistic soft BCK-algebra over X with $I \subseteq F(x)$ for all $x \in A$. If $\underline{Apr}_I(\mathfrak{S}) \neq \emptyset$, then $\mathfrak{S}$ is a rough soft ideal w.r.t. I over X.

5. Applications of rough soft BCK-algebras in decision making

In this section, we illustrate a kind of new decision making method for rough soft sets on BCK-algebras.

Decision making method:

We will put forward the new method to find which is the best parameter e of a given soft set $\mathfrak{S} = (F, A)$. In other words, $F(e)$ is the nearest accurate BCK-algebra on $\mathfrak{S}$ w.r.t. an ideal of BCK-algebra.

Let X be a BCK-algebra and E a set of related parameters. Let $A = \{e_1, e_2, \cdots, e_m\} \subseteq E$ and $\mathfrak{S} = (F, A)$ be an original description soft set over X. Let $I \lhd X$ and (X, I) be a Pawlak approximation space. Then we present the decision algorithm for rough soft BCK-algebras as follows:

Step 1. Input the original description BCK-algebra X, soft set $\mathfrak{S}$ and Pawlak approximation space (X, I), where $I \lhd X$.

Step 2. Compute the lower and upper rough soft approximation operators $\underline{Apr}_I(\mathfrak{S})$ and $\overline{Apr}_I(\mathfrak{S})$ on $\mathfrak{S}$, respectively.

Step 3. Compute the different values of $\|F(e_i)\|$, where

$$\|F(e_i)\| = \frac{|\overline{F}_I(e_i)| - |\underline{F}_I(e_i)|}{|F(e_i)|}.$$

Step 4. Find the minimum value $\|F(e_k)\|$ of $\|F(e_i)\|$, where $\|F(e_k)\| = \min_i \|F(e_i)\|$.

Step 5. The decision is $F(e_k)$.

Example 5.1. Assume that we want to find the nearest accurate BCK-algebra on a soft set $\mathfrak{S}$. Let a BCK-algebra as in Example 4.7, $I = \{0,1,2\} \lhd X$. Define a soft set $\mathfrak{S} = (F, A)$ over X, where $A = \{e_1, e_2, e_3, e_4\}$. The tabular representation of the soft set $\mathfrak{S}$ is given in Table 1.

Table 1 table for soft set $\mathfrak{S}$

	0	1	2	3	4
e_1	0	1	0	1	0
e_2	0	0	1	0	1
e_3	1	1	0	1	1
e_4	1	0	1	0	0

Now, the tabular representations of two soft sets $\underline{Apr}_I(\mathfrak{S})$ and $\overline{Apr}_I(\mathfrak{S})$ over X are given by Tables 2 and 3, respectively.

Table 2 table for soft set $\underline{Apr}_I(\mathfrak{S})$

	0	1	2	3	4
e_1	0	0	0	1	0
e_2	0	0	1	0	1
e_3	0	0	0	1	1
e_4	0	0	0	0	0

Table 3 table for soft set $\overline{Apr}_I(\mathfrak{S})$

	0	1	2	3	4
e_1	1	1	1	1	0
e_2	1	1	1	0	1
e_3	1	1	1	1	1
e_4	1	1	1	0	0

Then, we can calculate $\|F(e_1)\| = 1.5$, $\|F(e_2)\| = 1$, $\|F(e_3)\| = 0.75$, $\|F(e_4)\| = 1.5$. This means the minimum value for $\|F(e_i)\|$ is $\|F(e_3)\| = 0.75$. That is $F(e_3)$ is the closest accurate BCK-algebra on $\mathfrak{S}$.

6. Conclusion

Recently, some researchers established some decision making methods based on soft sets [20, 5] and fuzzy soft sets. In the present paper, we first put forward a kind of new decision making method based on rough soft sets. We apply rough soft set theory to BCK-algebras and investigate some related results. We hope it would be served as a foundation of rough soft sets and other decision making methods in different areas, such as theoretical computation sciences, information sciences and intelligent systems, and so on.

Acknowledgements

This research is partially supported by a grant of National Natural Science Foundation of China (11461025).

References

[1] H. Aktaş, N. Çağman, *Soft sets and soft groups*, Inform. Sci., 177 (2007), 2726-2735.

[2] M.I. Ali, B. Davvaz, M. Shabir, *Some properties of generalized rough soft sets*, Inform. Sci., 224 (2013), 170-179.

[3] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl., 57 (2009), 1547-1553.

[4] R. Biswas, S. Nanda, *Rough groups and rough subgroups*, Bull. Polish. Acad. Sci. Math., 42 (1994), 251-254.

[5] N. Çağman, S. Enginoğlu, *Soft matrix theory and its decision making*, Comput. Math. Appl., 59 (2010), 3308-3314.

[6] N. Çağman, S. Enginoğlu, *Soft set theory and uni-int decision making*, Eur. J. Oper. Res., 207 (2010), 848-855.

[7] D. Chen, C. Wang, Q. Hu, *A new approach to attribute reduction of consistent and inconsistent covering decision systems with covering rough sets*, Inform. Sci., 177 (2007), 3500-3518.

[8] B. Davvaz, *Roughness in rings*, Inform. Sci., 164 (2004), 147-163.

[9] W.A. Dudek, Y.B. Jun, H.S. Kim, *Rough set theory to BCI-algebras*, Quasigroups Related Systems, 9 (2002), 45-54.

[10] F. Feng, C. Li, B. Davvaz, M.I. Ali, *Soft sets combined with fuzzy sets and rough sets: A tentative approach*, Soft Comput., 14 (2010), 899-911.

[11] F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Y. B. Jun, *Soft sets and soft rough sets*, Inform. Sci., 181 (2011), 1125-1137.

[12] K. Iseki, S. Tanaka, *An introduction to the theory of BCK-algebras*, Math. Japon., 23 (1978), 1-26.

[13] Y.B. Jun, *Roughness of ideals in BCK-algebras*, Sci. Math. Japon., 7 (2003), 165-169.

[14] Y.B. Jun, *Soft BCK/BCI-algebras*, Comput. Math. Appl., 56 (2008), 1408-1413.

[15] Y.B. Jun, H.S. Kim, C.H. Park, *Positive implicative ideals of BCK-algebras based on a soft set theory*, Bull. Malays. Math. Sci. Soc., (2) 34 (2011), 345-354.

[16] Y.B. Jun, C.H. Park, *Applications of soft set in ideal theory of BCK/BCI-algebras*, Inform. Sci., 178 (2008), 2466-2475.

[17] N. Kuroki, *Rough ideals in semigroups*, Inform. Sci., 100 (1997), 139-163.

[18] X. Ma, J. Zhan, Y.B. Jun, *Generalized fuzzy h-ideals of hemirings*, Bull. Malays. Math. Sci. Soc., (2) 34 (2011), 561-574.

[19] P.K. Maji, R. Bismas, A.R. Roy, *Soft set theory*, Comput. Math. Appl., 45 (2003), 555-562.

[20] P.K. Maji, A.R. Roy, R. Bismas, *An application of soft sets in a decision making*, Comput. Math. Appl., 44 (2002), 1077-1083.

[21] J. Meng, Y.B. Jun, *BCK-algebras*, Kyungmoon Sa Co., Seoul, 1994.

[22] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., 37 (1999), 19-31.

[23] Z. Pawlak, *Rough sets*, Int. J. Inform. Comp. Sci., 11 (1982), 341-356.

[24] Y.Y. Yao, *Constructive and algebraic methods of the theory of rough sets*, Inform. Sci., 109 (1998), 21-47.

[25] J. Zhan, N. Cagman, A. Singin, *Applications of soft union sets to hemirings via $SU-h$-ideals*, J. Intell. Fuzzy Systems, 26 (2014), 1363-1370.

[26] J. Zhan, Y.B. Jun, *Generalized fuzzy ideals of BCI-algebras*, Bull. Malays. Math. Sci. Soc., (2) 32 (2009), 119-130.

[27] X.H. Zhang, J. Dai, Y. Yu, *On the union and intersection operations of rough sets based on various approximation spaces*, Inform. Sci., 292 (2015), 214-229.

[28] W. Zhu, *Generalized rough sets based on relations*, Inform. Sci., 177 (2007), 4997-5011.

[29] W. Zhu, S. Wang, *Rough metroids based on relations*, Inform. Sci., 232 (2013), 241-252.

Accepted: 1.11.2017