A STUDY ON BIJECTIVE SOFT HEMIRINGS

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Abstract. The concept of bijective soft hemirings is firstly proposed and some of related properties are investigated. Especially the basic operations of soft bijective hemirings are discussed and some good examples are also given. We also define a bijective soft ideal and discuss the primary operations of bijective soft h-ideals (k-ideals, strong k-ideals) and idealistic soft hemirings.
Keywords: bijective soft hemirings, bijective soft h-ideals, hemirings.

1. Introduction

The traditional classical models usually fail to conquer the complexities of inconclusive data in many important areas. In 1999, the initial conception of soft sets was firstly proposed by Molodtsov [28]. Molodtsov had undergone great increase and applications in many fields. Maji et al. [26] gave an application of soft set theory in decision making problems through using rough sets, and constructed a theoretical investigation on soft sets in a particular method. Many authors researched on applications of soft sets. Ma et al. [22] gave a servey of decision making methods based on certain hybrid soft set models. Two significant notions of a novel uncertain soft model and a novel soft rough set were given in [36, 39]. Alcantud [4, 5] proposed a novel algorithm for fuzzy soft sets and investigated some formal relationships among soft sets and their extensions. Sun et al. [29, 30] investigated fuzzy rough sets and its many important applications in our life. In 2017, Alcantud et al. [6, 7] researched the problem of collective identity in a fuzzy environment, and put forward the rational fuzzy and sequential fuzzy choice. And then, the study on the soft set theory has been comprehensively learned by a lot of authors. Çağman et al. [3, 13] put forward a rational definition of products of soft sets and uni-int decision method to tackle the uncertain problems. In 2010, a sub soft set of the cartesian product of the soft sets and many associated notions were introduced by Babitha et al. [12]. Xiao et al. [32] come up with the concept of exclusive disjunctive soft sets

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and investigated some of its operations. Gong et al. [16] defined the notion of bijective soft sets, investigated some operations of it, and obtained its several properties. The concept of soft groups and many notions of group theory can be expanded in an elementary way to develop the theory of soft groups were introduced by Aktaş and Çağman [13]. Jun and Park [20] investigated idealistic soft BCK/BCI-algebras and soft ideals. Acar et al. [1] put forward the initial notion of soft rings. Aygünoğlu et al. [11] proposed the notions of fuzzy soft groups and its structural features and properties were researched. Atagun and Sezgín [10] proposed the concepts of soft ideal of a ring, soft sub-ring, and sub fields of a field and researched them. In 2015, Aktaş [2] put forward the definition of bijective soft groups and demonstrated its basic properties with a theoretical theory research, by using Aktaş's definition of soft group and Gong's definition of bijective soft sets. In 2017, Han et al. [17] aimed to find better algorithms for solving parameter reduction problems of soft sets, and gave their potential applications. In theses papers, Aktaş discussed some significant properties of soft groups. Meanwhile, this theory is very important in some different investigate fields such as approximate reasoning, information sciences with intelligent systems, decision making and decision support systems, we can see some examples in [14, 15, 27, 32, 41]. Based on soft sets, many algebraic structures have been introduced such as soft ordered semigroups [19], soft rings [1], soft int-groups [13], soft BCI-algebras [21].

As we know that the hemiring is a special and important algebraic structure, some authors investigated the speciality of hemiring. Torre [31] introduced a definition of h-ideals of hemirings and showed its properties. He established some ring theorems for the hemirings, by using h-ideals. Especially, Jun investigated some properties of hemirings in [18]. Ma and Zhan [23, 40] studied some characteristics of h-hemiregular hemirings. Furthermore, Yin [34, 33] constructed some properties of h-semisimple and h-intra-hemiregular hemirings. Seeing from [9, 23, 24], Allen pointed some generalized fuzzy h-ideals of hemirings.

This article consists of five parts. In section 2, we review some basic knowledge of soft sets, bijective soft groups, hemirings, ideals (h-ideals, k-ideals, strong k-ideals) and so on. In section 3, the concept of bijective soft hemirings is given, its basic properties are investigated and some fundamental applications of bijective soft hemirings are mentioned. In section 4, we give a definition of bijective soft ideals (h-ideals, k-ideals, strong k-ideals) and some primary operations of them.

2. Basic terminologies

Some basic terminologies about soft sets, ideals and hemirings are given. A semiring is an algebrasic system $(S, +, \cdot)$ consisting of a non-empty set S together with two binary operations on S called addition and multiplication (denoted in the usual manner) such that $(S, +)$ and $(S, \cdot)$ are semigroups and the following distributive laws:

For the sake of simplicity, we shall write ab for $a \cdot b$ $(a, b \in S)$,

$a(b+c) = ab + ac$ and $(a+b)c = ac + bc$ are satisfied for all a, b, $c \in S$.

By *zero* of a semiring $(S, +, \cdot)$, we mean an element $0 \in S$ such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$. A semiring with *zero* and a commutative semigroup $(S, +)$ is called a hemiring.

A subhemiring of a hemiring S is a subset A of S closed under addition and multiplication. A subset A of S is called a left (right) ideal of S if A is closed under addition and $SA \subseteq A$ $(AS \subseteq A)$.

A subhemiring (left ideal, right ideal, ideal) of S is called an h-subhemirng (left h-ideal, right h-ideal, h-ideal) of S, respectively, if for any x, $z \in S$, a, $b \in A$, $x + a + z = b + z$ it follows $x \in A$.

The h-closure $\bar{A}$ of a subset A of S is defined as: $\bar{A}=\{x \in S \mid x+a+z = b+z$ for some a, $b \in A$, $z \in S\}$.

From now on, S is a hemiring, U is an initial universe, E is a set of parameters, $P(U)$ is the power set of U and A, B, $C \subseteq E$.

Definition 2.1 ([28])**.** A pair (F, A) is called a soft set over U, where $A \subseteq E$, F is a mapping given by F: $A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $F(\varepsilon)$ may be a considered as the set of ε-approximate.

Definition 2.2 ([31])**.** An ideal I of S is called a k-ideal of S, if $x \in S$, $a, b \in I$, $x + a = b$ implies $x \in I$.

Definition 2.3 ([35])**.** An ideal I of S is called a strong h-ideal, if $x, y, z \in S$, $a, b \in I$ and $x + a + z = y + b + z$ implies $x \in y + I$.

Definition 2.4 ([35])**.** Let (F, A) be a non-null soft set over S. Then:

(1) (F, A) is called a soft hemiring over S if $F(x)$ is a subhemiring of S for all $x \in \mathrm{supp}(F, A)$, where $\mathrm{supp}(F, A) = \{a \in A | F(a) \neq \emptyset\}$,

(2) (F, A) is called an idealistic soft semiring over S if $F(x)$ is an ideal of S for all $x \in \mathrm{supp}(F, A)$, where $\mathrm{supp}(F, A) = \{a \in A | F(a) \neq \emptyset\}$.

The bi-idealistic (k-idealistic, h-idealistic and strong h-idealistic) soft semirings are defined similarly.

Definition 2.5 ([8, 26])**.** Let (F, A) and (G, B) be two soft sets over a common universe U.

(1) The restricted-intersection of (F, A) and (G, B) is defined to the soft set (H, C), where $C = A \cap B$, and $H : c \to P(U)$ is a mapping given by $H(c) = F(c) \cap G(c)$ for all $c \in C$. This is denoted by $(F, A) \tilde{\sqcap} (G, B) = (H, C)$,

(2) (F, A) AND (G, B) denoted by $(F, A) \tilde{\wedge} (G, B) = (H, A \times B)$ where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$,

(3) The extended union of (F, A) and (G, B), denoted by $(F, A)\tilde{\cup}(G, B)$, is defined as the soft set (H, C), where $C = A \cup B$ and $\forall x \in C$,

$$H(x) = \begin{cases} F(x), & \text{if } x \in A - B, \\ G(x), & \text{if } x \in B - A, \\ F(x) \cup G(x), & \text{if } x \in A \cap B. \end{cases}$$

Definition 2.6 ([2])**.** Let (F, A) be a soft group over G, where F is a mapping $F : A \to P(G)$ and A is a nonempty parameter set. We say that (F, A) is a bijective soft group, if (F, A) such that:

(1) $\bigcup_{a \in A} F(a) = G$,

(2) For any two parameters $a_i, a_j \in A, a_i \neq a_j, F(a_i) \cap F(a_j) = \{e\}$.

3. Bijective soft hemirings

In this section, we give a new concept of bijective soft hemirings and investigate some properties of it.

Definition 3.1. Let (F, A) be a soft hemiring over H, where H is a hemiring. Then (F, A) is a bijective soft hemiring, if (F, A) satifies:

(1) $\bigcup_{e \in A} F(e) = H$,

(2) For any two parameters e_i, $e_j \in A$, $e_i \neq e_j$, $F(e_i) \cap F(e_j) = \{0\}$.

Example 3.2. Let $S = \{0, 1, 2, 3\}$ with the following Cayley tables:

Table 2 Table for a hemiring S

+	0	1	2	3
0	0	1	2	3
1	1	1	2	3
2	2	2	2	3
3	3	3	3	2

·	0	1	2	3
0	0	0	0	0
1	0	1	1	1
2	0	1	1	1
3	0	1	1	1

We can obtain that the S is a finite hemiring. Let $A = \{0, 1\}$. Certainly, A is also a hemiring. Now, we define a mapping $G(x) = \{y | y\rho x \Leftrightarrow y = 2xa, a \in A\}$, $\forall x \in A$. From the operations, we can get that $G(0) = \{0\}$, $G(1) = \{0, 1\}$. It is easy to check that (G, A) is a soft hemiring of A. Since $G(0) \cap G(1) = \{0\}$ and $G(0) \cup G(1) = \{0, 1\} = A$. By the Definition 3.1, we can get that (G, A) is a bijective soft hemirings of A.

Definition 3.3. (1) (F, A) is said to be an identity bijective soft hemiring over H, if $F(x) = \{0\}$ for all $x \in A$, where $\{0\}$ is the identity element of hemiring.

(2) (F, A) is said to be an absolute bijective soft hemiring over H, if $F(x) = H$ for all $x \in A$.

Theorem 3.4. Let (F,A) and (G,B) be two bijective soft hemirings over S with $A \cap B \neq \emptyset$. Then their intersection $(F,A)\tilde{\sqcap}(G,B)$ is still a bijective soft hemiring over S.

Proof. By Definition 2.5 (1), $(F,A)\tilde{\sqcap}(G,B) = (H,C)$, where $H(c) = F(c) \cap G(c)$ for all $c \in C$ where $C = A \cap B \neq \emptyset$. Since (F,A) and (G,B) are two bijective soft hemirings over common S. Then, we can obtain that every $H(c)$ is a hemiring of S, for all $c \in C$, where $C = A \cap B$. That is $(F,A)\tilde{\sqcap}(G,B)$ is a soft hemiring of S. By Definition 3.1, we have that $\bigcup_{x\in A} F(x) = S$, and $F(x_i) \cap F(x_j) = \{0\}$, for any two parameters x_i, $x_j \in A$, $x_i \neq x_j$; $\bigcup_{y\in B} G(y) = S$, $G(y_i) \cap G(y_j) = \{0\}$ for any two parameters y_i, $y_j \in B$, $y_i \neq y_j$. Then $\bigcup_{x\in C=A\cap B} H(x) = \bigcup_{x\in C}(F(c)\cap G(c)) = (\bigcup_{x\in C} F(x)) \cap (\bigcup_{x\in C} G(x)) = S \cap S = S$. Let $\forall a \in C$, $b \in C$, $a \neq b$ where $C = A \cap B$, since $H(a) \cap H(b) = (F(a)\cap G(a))\cap(F(b)\cap G(b)) = (F(a)\cap F(b))\cap(G(a)\cap G(b)) = \{0\}\cap\{0\} = \{0\}$. Thus, $(F,A)\tilde{\sqcap}(G,B)$ is a bijective soft hemiring over S. □

Theorem 3.5. If (F,A) and (G,B) are two bijective soft hemirings over S, then (F,A) AND (G,B) is also a bijective soft hemiring.

Proof. Since (F,A) and (G,B) are two bijective soft hemirings, then $F(x)$ and $G(y)$ are two subhemirings of S. Thus, we can get that $H(x,y) = F(x) \cap G(y)$ is also a subhemiring of S, for all $(x,y) \in A \times B$. Then, $(F,A)\tilde{\wedge}(G,B)$ is a soft hemiring of S. Because the $\bigcup_{(x,y)\in A\times B} H(x,y) = \bigcup_{(x,y)\in A\times B} F(x) \cap G(y) = (\bigcup_{x\in A} F(x)) \cap (\bigcup_{y\in B} G(y)) = S \cap S = S$. Suppose that $(a_i,a_j) \in A \times B$ where $a_i = (a_1,b_1)$, $a_j = (a_2,b_2)$ and $a_i \neq a_j$. That is $a_1 \neq a_2$ or $b_1 \neq b_2$, $a_1 \neq a_2$ and $b_1 \neq b_2$. When $a_1 \neq a_2$ or $b_1 \neq b_2$, we have

$$\begin{aligned} H(a_i)\cap H(a_j) &= (F(a_1)\cap G(b_1))\cap(F(a_2)\cap H(b_2)) \\ &= (F(a_1)\cap F(a_2))\cap(H(b_1)\cap H(b_2)) \\ &= \{0\}\cap(H(b_1)\cap H(b_2)) \\ &= \{0\}. \end{aligned}$$

or

$$\begin{aligned} H(a_i)\cap H(a_j) &= (F(a_1)\cap G(b_1))\cap((Fa_2)\cap H(b_2)) \\ &= (F(a_1)\cap F(a_2))\cap(H(b_1)\cap H(b_2)) \\ &= (F(a_1)\cap F(a_2))\cap\{0\} \\ &= \{0\}. \end{aligned}$$

When $a_1 \neq a_2$ and $b_1 \neq b_2$, we obtain that

$$\begin{aligned} H(a_i)\cap H(a_j) &= (F(a_1)\cap G(b_1))\cap(F(a_2)\cap H(b_2)) \\ &= (F(a_1)\cap F(a_2))\cap(H(b_1)\cap H(b_2)) \\ &= \{0\}\cap\{0\} \\ &= \{0\}. \end{aligned}$$

That is $H(a_i) \cap H(b_j) = \{0\}$. Thus, we prove that $(F,A)\tilde{\wedge}(G,B)$ is a bijective soft hemiring over S. □

Theorem 3.6. Let (F, A) be a bijective soft hemiring over S and (H, B) be an identity soft hemiring, then $(F, A)\tilde{\cup}(H, B)$ is a bijective soft hemiring.

Proof. Since (F, A) is a bijective soft hemiring over S and (H, B) is an identity soft hemiring, by Definition 2.5 (3), for $\forall x \in C$, where $C = A \cup B$. We have:

$$H(x) = \begin{cases} F(x), & \textit{if } x \in A - B, \\ 0, & \textit{if } x \in B - A, \\ F(x) \cup G(x), & \textit{if } x \in A \cap B. \end{cases}$$

If $A \cap B = \emptyset$, then $H(x) = F(x)$ or $\{0\}$. If $H(x) = F(x)$ for $x \in A - B$. Since (F, A) is a bijective soft hemiring over S, so is (H, C). If $H(x) = \{0\}$ for $x \in B - A$, thus (H, C) is a trival bijective soft hemiring over S. If $A \cap B \neq \emptyset$, then $H(x) = F(x) \cup \{0\} = F(x)$. It is easy to check that (H, C) is a bijective soft hemiring. Above all, thus $(F, A)\tilde{\cup}(G, B)$ is a bijective soft hemiring. □

Definition 3.7. Let (F, A) and (H, K) be two bijective soft hemirings over S. Then (H, K) is a bijective soft subhemiring of (F, A), written $(H, K)\tilde{<}(F, A)$, if:

(1) $K \subseteq A$,

(2) $H(x) \subseteq F(x)$ for all $x \in K$.

Example 3.8. Consider that $S = \{0, a, b, c\}$ is set with a multiplication operation an addition $(+, \cdot)$ as follow:

Table 2 Table for a hemiring S

$+$	0	a	b	c
0	0	a	b	c
a	a	0	b	c
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Let $A = S$, $K = \{a, b, c\}$, $K \subset A$. It is obvious that S is a hemiring. We give a mapping, where $F(x) = \{y \epsilon S \ x\rho y \Leftrightarrow y = x^n$ for some $n \in N\}$ for all $x \in A$. Here $x^0 = 0$. Then $F(0) = \{0\}$, $F(a) = \{0, a\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$, which are subhemirings of S. Since $F(e_i) \cap F(e_j) = \{0\}$, for $\forall e_i$, $e_i \neq e_j$, $e_j \in A$, and $F(0) \cup F(a) \cup F(b) \cup F(c) = S$. Hence, it is easy to check that (F, A) is a bijective soft hemiring. Since $K \subset A$, $F(a) \cup F(b) \cup F(c) = S$ and $F(e_i) \cap F(e_j) = \{0\}$, for $\forall e_i$, $e_i \neq e_j$, $e_j \in K$. So, we obtain that (F, K) is a bijective soft hemiring of S. It is easy to check that $F(x) \subseteq F(x)$ for all $x \in K$. Then (F, K) is a bijective soft subhemiring of (F, A). That is $(F, K)\tilde{<}(F, A)$.

Definition 3.9. Let (F, A) be a bijective soft hemiring over S and (H, B) be a soft subhemiring of (F, A). Then (H, B) is called a bijective soft hemirng of (F, A), if it satisfies :

(1) $\bigcup_{a\in B} H(a)$ is a subhemiring of S,

(2) For any two parameters $a_i, a_j \in B, a_i \neq a_j, H(a_i) \cap H(a_j) = \{0\}$.

If $(H, B) = \{S\}$ and $(H, B) = \{0\}$, then (H, B) is a bijective soft subhemiring of (F, A). In this case, (H, B) is called trivial bijective soft hemiring.

Example 3.10. Let (H, B) be a soft set over Z_6, where $A{=}B{=}S{=}\{\bar{0}, \bar{1}\}$ and $H : B \to P(Z_6)$ is defined by $H(x) = \{y \in Z_6 | x\rho y \iff xy = \bar{0}\}$ for $x \in B$. And suppose that (F, A) is a absolute bijective soft hemiring of S, that is $F(x) = S$ for all $x \in B$. Then $H(\bar{0}) = Z_6$ is a subhemiring of $F(\bar{0})$, $H(\bar{1}) = \{\bar{0}\}$ is a subhemiring of $F(\bar{1})$, although $\bar{0} \neq \bar{1}$, $H(\bar{1}) \cap H(\bar{0}) = \{0\}$, $H(\bar{1}) \cup H(\bar{0}) = \{0, 1\} = S$. Hence, by the Definition 3.7, we have that (H, B) is a bijective soft subhemiring of (F, A).

Proposition 3.11. Let (F, A) be a bijective soft hemirirng over S and $\{(H_i, (K_i)|i \in I\}$

(1) If $\bigcap_{i\in I} K_i \neq \emptyset$, then $\tilde{\bigcap}_{i\in I}(H_i, K_i)$ is a bijective soft subhemiring of (F, A),

(2) $\tilde{\bigwedge}_{i\in I}(H_i, K_i)$ is a bijective soft subhemiring of $\tilde{\bigwedge}_{i\in I}(F, A)$,

(3) If $K_i \cap K_j = \emptyset$ for all $i, j\epsilon I$ with $i \neq j$, then $\tilde{\cup}_{i\in I}(H_i, K_i)$ is a bijective soft subhemiring of (F, A),

Proof. It is straightforward. □

Definition 3.12. Let (F, A) and (H, B) be two bijective soft hemirings over H and K, respectively, and define $f : H \to K$ and $g : A \to B$ as two functions. Then we say (f, g) is a bijective soft homomorphism of soft hemiring to (H, B), denoted by $(F, A) \sim (H, B)$, if the following conditions are satisfied:

(1) f is a homomorphism from G to K,

(2) g is a mapping from A to B,

(3) $f(F(x)) = H(g(x))$ for all $x \in A$.

Proposition 3.13. Let (F, A) be a bijective soft hemiring over S, (H, B) be a soft hemiring over K and (f, g) be a soft hemiring homomorphism from (F, A) to (H, B). If f is a one to one function, then (H, B) is a bijective soft hemiring over K.

Proof. Assume that (f, g) is a soft homomorphism, f is a one to one function and $g(x)$, $g(y) \in B$ such that $g(x) \neq g(y)$. Then we have $H(g(x)) \cap H(g(y)) = f(F(x)) \cap f(F(y)) = f(F(x) \cap F(y)) = f(\{0\}) = \{0\}$. Since (F, A) is a bijective soft hemiring. We have $\bigcup_{g(x)\in B} H(g(x)) = \bigcup_{x\in A} f(F(x)) = f(\bigcup_{x\in A}) = f(S) = K$. This is completed the proof. □

Proposition 3.14. Let (F, B) and (H, B) two be bijective soft hemirings of two hemirings S and K, respectively, over S. If (H, B) is a bijective soft subhemirng of (F, A) and f is a homomorphism over S to another hemiring K, then $(f(F), B)$ is a bijective soft subhemiring of $(f(F), A)$.

Proof. Since f is a homomorphism from S to K, $f(F(x))$ and $f(H(y))$ are bijective soft hemirings of K for any $x \in A$, and $y \in B$. Then $(f(F), A)$ and $(f(H), B)$ are both bijective soft hemirngs over K. If (H, B) is a bijective soft subheming of (F, A), then $H(y)$ is a bijective soft subhemiring of $F(y)$ and $f(H(y))$ is a subhemiring of $f(F(y))$ for any $y \in B$. $f(H(y_1)) \cap f(H(y_2)) = f(H(y_1) \cap H(y_2)) = f(0) = \{0\}$ and $\bigcup_{x\in B} f(H(x)) = f(\bigcup_{x\in B} H(x)) = f(S) = K$. By Definition 3.6, we obtain immediately that $(f(H), B)$ is a bijective soft subhemiring of $(f(F), A)$. □

4. Bijective soft ideals

In this section, we put forward a new notion of bijective soft ideals and research some important properties of it. And some basic operations of these ideals are also studied.

Definition 4.1. Let (F, A) be a bijective soft hemiring over H. A non-null soft set (I, B) over S is called a bijective soft ideal (h-ideal, k-ideal, k-ideal, strong k-ideal) of (F, A), if it satisfies the following conditions:

(1) $B \subseteq A$,

(2) $I(x)$ is an ideal (h-ideal, k-ideal, k-ideal, strong k-ideal) of $F(x)$ and $\bigcup_{x\in B} I(B) = H$, $I(x_i) \cap I(x_j) = \{0\}$, $x_i \neq x_j$, x_i, $x_j \in B$, for all $x \in \mathrm{Supp}(I, B)$.

Example 4.2. Let $H = \{0, a, b\}$ be a set with a multiplication operation and an addition $(+, \cdot)$ as follow:

Table 3 Table for a hemiming H

$+$	0	a	b
0	0	a	b
a	a	0	b
b	b	b	b

$\cdot$	0	a	b
0	0	0	0
a	0	0	0
b	0	0	0

Let $B = \{0, a, b\}$, $A = \{a, b\}$. From the table, we can see that A and B are two hemirings. Two functions are given: $I(x) = F(x) = \{y \in S\ x\rho y \Leftrightarrow y = x^n$ for some $n \in N\}$ for all $x \in A$. Here $x^0 = 0$. Then we can get $F(0) = \{0\}$, $I(a) = F(a) = \{0, a\}$, $I(b) = F(b) = \{0, b\}$. It is proved that $IF \subseteq I$, and $FI \subseteq I$. Besides that $\bigcup_{x_i\in A} I(x_i) = H$, and $I(x_i) \cap I(x_j) = \{0\}$, $x_i \neq x_j$, x_i, $x_j \in A$. By the Definition 4.1, we obtain that (I, B) is a bijective ideal of (F, A).

Theorem 4.3. Let (F, A) and (G, B) be two bijective soft h-ideals (strong h-ideals, k-ideals) over a hemiring H. Then $(F, A)\tilde{\sqcap}(G, B)$ is a bijective soft h-ideal (k-ideal, strong k-ideal) over H if it is non-null.

Proof. By Definition 2.5 (1), we can write $(F, A)\tilde{\sqcap}(G, B) = (\gamma, C)$, where $C = A\cap B$ and $\gamma(c) = F(c)\cap G(c)$ for all $c \in C$. Suppose that (γ, C) is a non-null soft set over H. If $c \in C$, then $\gamma(c) = F(c) \cap G(c) \neq \emptyset$. Since (F, A) and (G, B) are two bijective soft h-ideals, thus the nonempty sets $F(c)$ and $G(c)$ are h-ideals of H. It follows that $\gamma(x)$ is a h-ideal of H, for all $c \in I$.

In the following, we will prove that $\bigcup_{x\in I} \gamma(x) = H$, and $I(x_i) \cap I(x_j) = \{0\}$ for every $x_i \neq x_j$, x_i, $x_j \in I$.

(1) $\bigcup_{x\in I} \gamma(x) = \bigcup_{x\in A\times B} \gamma(x) = \bigcup_{a\in A} \bigcup_{b\in B}(F(a) \cap G(b)) = (\bigcup_{a\in A} F(a)) \cap (\bigcup_{b\in B} G(b)) = H \cap H = H$.

(2) Suppose that $(a_i, a_j) \in A \times B$, where $a_i = (a_1, b_1)$ and $a_j = (a_2, b_2)$ and $a_i \neq a_j$. Then $\gamma(a_i) \cap \gamma(a_j) = (F(a_1) \cap G(b_1)) \cap (F(a_2) \cap G(b_2)) = (F(a_1) \cap F(a_2)) \cap (G(b_1) \cap G(b_2)) = \{0\} \cap \{0\} = \{0\}$.

Above all, thus$(F, A)\tilde{\sqcap}(G, B)$ is a bijective soft $h-$ideal over H. The k-ideals and strong k-ideals have the similar properties, and the proofs are also similar. □

Theorem 4.4. Let (F, A) and (G, B) be two bijective soft h-ideals (k-ideals, strong k-ideals) over H. If A and B are disjiont, then $(F, A)\tilde{\wedge}(G, B)$ is also a bijective soft h-ideal over H.

Proof. The proof is similar to Theorem 4.3. □

Theorem 4.5. Let (I_1, A_1) and (I_2, A_2) be two bijective soft h-ideals of two bijective soft hemirings, (F, A) and (G, B) over H respectively.
Then $(I_1, A_1)\tilde{\sqcap}(I_2, A_2)$ is a bijective soft h-ideal of $(F, A)\tilde{\sqcap}(G, B)$.

Proof. The proof is easyly omited. □

Theorem 4.6. Let $(F_i, A_i)_{i\in I}$ be a non-empty family of bijective soft h-ideals of a bijective soft hemiring (F, A) over H. Then:

(1) $\bigcap_{i\in I}(F_i, A_i)$ is a bijective soft $h-$ideal of (F, A), if it is non-null.

(2) $\bigwedge_{i\in I}(F_i, A_i)$ is a bijective soft $h-$ideal of (F, A), if it is non-null.

(3) $\bigcup_{i\in I}(F_i, A_i)$ is a bijective soft $h-$ideal of (F, A), if $\{A_i, i \in I\}$ are pair wise disjoint, whenever it is non-null.

Proof. It is straightforward. □

Proposition 4.7. Let (F, A) be a bijective h-idealistic (k-idealistic) soft hemiring over H. If $B \subset A$, then (F, B) is an h-idealistic (k-idealistic) soft hemiring over H, if $B = A - \{x_i\}$, where $F(x_i) = \{0\}$ and it is non-null.

Proof. Since (F, A) is an h-idealistic bijective soft hemiring over H, then $F(x)$ is an h-ideal of H, for all $x \in \text{supp}(F, A)$. Since $B = A-\{x_i\}$, $F(x)$ is also an h-ideal of H for all $x \in \text{supp}(F, B)$. Because (F, A) is a bijective soft hemiring over H and $F(x_i) = \{0\}$, we can get $\bigcup_{x\in A} F(x) = \bigcup_{x\in B} F(x)\cup F(x_i) = \bigcup_{x\in B} F(x) =$

H, $F(x_j) \cap F(x_k) = \{0\}$, $x_j \neq x_k$ for all x_j, $x_k \in A$. It is easy to check that $\bigcup_{x \in B} F(x) = H$ and $F(\acute{x_i}) \cap F(\acute{x_j}) = \{0\}$ for $\acute{x_i} \neq \acute{x_j}$, $\acute{x_i}$, $\acute{x_j} \in B$. Thus (F, B) is an $h-$idealistic soft hemiring over H. □

Example 4.8. Let $A = \{0, a, b, c\}$, $B = \{a, b, c\}$, we define a mapping F, where $F(0) = \{0\}$, $F(a) = \{0, a\}$, $F(b) = \{0, b\}$, $F(c) = \{0, c\}$. Then we can easyly to check that (F, A) is a bijective h-idealistic soft hemiring.

Theorem 4.9. Let (I_1, A_1) and (I_2, A_2) be bijective soft $h-$ideal of a bijective soft hemiring (F, A) over H. Then :

(1) $(I_1, A_1)\tilde{\sqcap}(I_2, A_2)$ is a bijective soft $h-$ideal of (F, A) if it is non-null.

(2) $A_1 \cap A_2 = \emptyset$, then $(I_1, A_1)\tilde{\cup}(I_2, A_2)$ is a bijective soft h-ideal of (F, A).

Proof. (1) Let $(I_1, A_1)\tilde{\sqcap}(I_2, A_2) = (I, B)$, where $B = A_1 \cap A_2 \subset A$. We can prove $I_1(x)$ is a bijective soft h-ideal of $F(x)$ for all $x \in A_1$, so is $I_2(x)$. Therefore $I(x) = I_1(x) \cap I_2(x) \neq \emptyset$. Since (I, B) is non-null. We can obtain $I(x)$ is a bijective h-ideal of $F(x)$, for all $x \in B$. That is (I, B) is a bijective soft h-ideal of (F, A).

(2) Let $(I_1, A_1)\tilde{\cup}(I_2, A_2) = (I, C)$, where $C = A_1 \cup A_2$ and $\forall x \in C \subset A$. By Definition 2.5 (3), $I(x) = I_1(x)$ is a non-empty h-ideal of $F(x)$, $\forall x \in \mathrm{supp}(I, C)$. Also $I(x) = I_1(x)$ is a non-empty h-ideal of $F(x)$, $\forall x \in \mathrm{supp}(I, C)$. Therefore $I(x)$ is a non-empty $h-$ideal of $F(x)$, $\forall x \in$supp(I, C). Hence (I, C) is a bijective soft $h-$ideal of (F, A). □

5. Conclusion

In [16], Gong et al. firstly defined the concept of bijective soft sets, then Aktas [2] put forth a concept of bijective soft groups based on the bijective soft sets. Now, based on the bijective soft groups, the concept of bijective soft hemirings is proposed. But, it is different from the bijective soft groups. For example, some conditions of the two notions are different. For any two parameters e_i, $e_j \in A$, $e_i \neq e_j$, $F(e_i) \cap F(e_j) = \{e\}$ in the bijective soft groups, but $F(e_i) \cap F(e_j) = \{0\}$ in the bijective soft hemirings. Also, the definition of dependent and independent element of bijective soft groups cannot suit the bijective soft hemirings. They are two different structures, so the operations and characterizations are also different. The groups are closed under the multiplication but the hemirings are both closed under the addition and multiplication. We hope that our works can invoke some research topics for algebraic systems and provide more applications in many important fields.

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References

[1] U. Acar, F. Koyuncu, B. Tanay, *Soft sets and soft rings*, Comput. Math. Appl., 59 (2010), 3458-3463.

[2] H. Aktaş, *Some algebraic applications of soft sets*, Appl. Soft Comput., 28 (2015), 327-331.

[3] H. Aktaş, N. Çağman, *Soft sets and soft groups*, Inform. Sci., 177 (2007), 2726-2735.

[4] J.C.R. Alcantud, *A novel algorithm for fuzzy soft set based decision making from multiobserver input parameter data set*, Inform. Fusion, 29 (2016), 142-148.

[5] J.C.R. Alcantud, *Some formal relationships among soft sets, fuzzy sets, and their extensions,* Int. J. Approx. Reason., 68 (2016), 45-53.

[6] J.C.R. Alcantud, R. de Andrés, *The problem of collective identity in a fuzzy environment*, Fuzzy Set. Syst., 315 (2017), 57-75.

[7] J.C.R. Alcantud, S. Días, *Rational fuzzy and sequential fuzzy choice*, Fuzzy Set. Syst., 315 (2017), 76-98.

[8] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, *On some new operations in soft set theory*, Comput. Math. Appl., 57 (2009), 1547-1533.

[9] P.J. Allen, H.S. Kim, J. Neggers, *Ideal theory in graded semirings*, Appl. Math. Inform. Sci., 7 (2013), 87-91.

[10] A.O. Atagün, A. Sezgín, *Soft substructures of rings, fields and modules*, Comput. Math. Appl., 61 (2011), 592-601.

[11] A. Aygünoğlu, H. Aygün, *Introduction to fuzzy soft groups*, Comput. Math. Appl., 58 (2009), 1279-1286.

[12] K.V. Babitha, J.J. Sunil, *Soft set relations and functions*, Comput. Math. Appl., 60 (2010), 1840-1849.

[13] N. Çağman, F. Çitak, H. Aktaş, *Soft int-group and its applications to group theory*, Neural Comput. Appl., 21 (2012), 151-158.

[14] F. Feng, Y.B. Jun, X. Liu, L. Li, *An adjustable approach to fuzzy soft set based decision making*, J. Comput. Appl. Math., 234 (2010), 10-20.

[15] F. Feng, X.Y. Liu, V. Leoreanu-Fotea, Y.B. Jun, *Soft sets and soft rough sets*, Inform. Sci., 181 (2011), 1125-1137.

[16] K. Gong, Z. Xiao, X. Zhang, *The bijective soft set with its operations*, Comput. Math. Appl., 60 (2010), 2270-2278.

[17] B. Han, Y. Li, S. Geng, *0-1 Linear programming methods for optimal normal and pseudo parameter reductions of soft sets*, Appl. Soft Comput., 54 (2017), 467-484.

[18] Y.B. Jun, K.J. Lee, A. Khan, *Soft ordered semigroups*, Math Logic. Q., 56 (2010), 42-50.

[19] Y.B. Jun, M.A. Ozturk, S.Z. Song, *On fuzzy h-ideals in hemirings*, Inform. Sci., 162 (2004), 211-226.

[20] Y.B. Jun, C.H. Park, *Applications of soft set in ideal theory of BCK/ BCI-algebras*, Comput. Sci., 178 (2008), 2466-2475.

[21] O. Kazanci, S. Yilmaz, S. Yamak, *Soft sets and soft BCH-algebras*, Hacet. J. Math. Stat., 39 (2010), 205-217.

[22] X. Ma, Q. Liu, J. Zhan, *A survey of decision making methods based on certain hybrid soft set models*, Artif. Intell. Rev., 47 (2017), 507-530.

[23] X. Ma, Y. Yin, J. Zhan, *Characterizations of h-intra and h-quasi-hemiregular hemirings*, Comput. Math. Appl., 63 (2017), 783-793.

[24] X. Ma, J. Zhan, *Generalized fuzzy h-bi-ideals and h-quasi-hemiregular hemirings*, Inform. Sci., 179 (2009), 1249-1268.

[25] X. Ma, J. Zhan, *Applications of a new soft set to h-hemiregular via (M,N)-SI-h-ideals*, J. Intell. Fuzzy Syst., 26 (2014), 2515-2525.

[26] P.K. Maji, R. Biswas, A.R. Roy, *Soft set theory*, Comput. Math. Appl., 45 (2003), 555-562.

[27] P.K. Maji, A.R. Roy, R. Biswas, *An aplication of soft sets in a decision making problem*, Comput. Math. Appl., 44 (2002), 1077-1083.

[28] D. Molodtsov, *Soft set theory-first results*, Comput. Math. Appl., 37 (1999), 19-31.

[29] B. Sun, W. Ma, X. Chen, *Fuzzy rough set on probabilistic approximation space over two universes and its application to emergency decision making*, Expert Syst., 32 (2015), 507-521.

[30] B. Sun, W. Ma, X. Xiao, *Three-way group decision making based on multigranulation fuzzy decision-theoretic rough set over two universes*, Int. J. Approx. Reason., 81 (2017), 87-102.

[31] D.R. La Torre, *On h-ideals and k-ideals in hemirings*, Pulb. Math. Debrecen, 12 (1965), 219-226.

[32] Z. Xiao, K. Gong, S. Xia, Y. Zou, *Exclusive disjunctive soft sets*, Comput. Math. Appl., 59 (2010), 2128-2137.

[33] Y. Yin, X.K. Huang, D.H. Xu, H.J. Li, *The characterizations of h-semisimple hemirings*, Int. J. Fuzzy Syst., 11 (2009), 116-122.

[34] Y. Yin, H.X. Li, *The characterizations of h-hemiregular hemirings and h-intra-hemiregular hemirings*, Inform. Sci., 178 (2008), 3451-3464.

[35] Y. Yin, J. Wang, *Fuzzy Hemirings*, Science Press, 2010.

[36] J. Zhan, M.I. Ali, N. Mehmood, *On a novel uncertain soft set model: Z-soft fuzzy rough set model and corresponding decision making methods*, Appl. Soft Comput., 56 (2017), 446-457.

[37] J. Zhan, N. Çağman, A. Sezgín Sezer, *Appplications of soft union sets to hemirings via SU-h-ideals*, J. Intell. Fuzzy Syst., 26 (2014), 1363-1370.

[38] J. Zhan, W.A. Dudek, *Fuzzy h-ideals of hemirings*, Inform Sci., 177 (2007), 876-886.

[39] J. Zhan, Q. Liu, T. Herawan, *A novel soft rough set: soft rough hemirings and its multicriteria group decision making*, Appl. Soft Comput., 54 (2017), 393-402.

[40] J. Zhan, P.K. Maji, *Applications of soft union sets to h-semisimple and h-quasi-hemiregular hemirings*, Hacet. J. Math. Stat., 43 (2014), 815-825.

[41] Y. Zou, Z. Xiao, *Data analysis approaches of soft sets under incomplete information*, Knowl-Based Syst., 21 (2008), 941-945.

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