THE BORDA RULE COMPREHENSIVE EVALUATION METHOD BASED ON SPEARMAN RANK

Yili Tan*
Xinghuo Wan
Meilin Zhang
College of Science
North China University of Science and technology
Tangshan 063000
China
tan_yil@sina.com

Abstract. In the perspectives of systematization and axiomatization, the improved Borda rule was used to assign the evaluation results to avoid getting the same value when evaluation objects were different. Then the ideal evaluation result which was determined by criteria was summed up. Finally the Spearman rank correlation coefficient of the ideal evaluation result and all commonly used comprehensive evaluation methods was calculated through comparison. According to the size of the correlation coefficient, each method was sorted based on rationality. Finally the rationality of these methods was simulated by numerical values.
Keywords: Borda rule, Criterion, Spearman coefficient of rank correlation.

1. Introduction

Comprehensive evaluation refers to selecting multiple factors or indicators according to different evaluation purposes and using different evaluation forms and then transforming them to information or single-factor indicator that can reflect the general characteristics of evaluated objects.

Today comprehensive evaluation has been widely used, and it appears in different forms. Therefore there are a wide range of comprehensive evaluation methods. We can always get a result when we evaluate a system using different methods. But it is not sure if the result is consistent with the objective practice or reasonable. As we all know, there is nearly no systematical literature concerning these issues. Rationality of comprehensive evaluation methods is the core of decision-making. In daily management, a problem that how to sort or classify a large number of evaluated objects and choose the best one is often encountered. Such a kind of problem reflects the issues that relate to decision-making. For this reason, it is necessary to study methods used in decision-making and the selection process to provide a scientific reference to decision-makers. The rationality of comprehensive evaluation methods is the key to successfully achieve decision-

∗. Corresponding author

making. Since the comprehensive evaluation has been proposed, many scholars have studied it in the aspects of the application of comprehensive evaluation, the classification and the comparison of comprehensive evaluation methods, the problems of the comprehensive evaluation, the research trends, etc. But there were few studies concerning the rationality of comprehensive evaluation methods. Therefore, studying the rationality of comprehensive evaluation methods has important practical significance.

2. Borda criteria and Spearman rank correlation

2.1 Borda criteria

Definition 2.1. Borda count method is a simple sort voting method. Each option obtains its point by ballot order, and the option with the highest point wins.

Voters arrange candidates in order according to the preferences. The candidate ranking the first gets a score, and next is the second candidate. Finally, candidate with the highest score wins.

2.2 Equal treatment of Borda number (improved)

Level-P connection matrix idea in Graph theory was used to deal with the situation when the Borda count score of two objects was the same. A level-1 connection matrix was supposed as follows:

	a	*b*	*c*	*d*	*e*	*f*	Row sum
a	0	4	4	4	4	4	20
b	1	0	5	5	5	5	21
c	1	0	0	5	5	5	16
d	1	0	0	0	5	5	11
e	1	0	0	0	0	5	6
f	1	0	0	0	0	0	1

Matrix (1)

(Considering the number of levels, matrix (2) was obtained)

Suppose level P connectivity matrix as (P_{ij}), then $(P+1)$ level connection matrix $(p+1)$ was:

$$(P+1)_{ij} = (\sum_{k=1}^{n} P_{ik} + \sum_{k=1}^{n} P_{jk})X_{ij}. \tag{1}$$

	a	b	c	d	e	f	Row sum
a	0	4	8	12	16	20	60
b	5	0	5	10	15	20	21
c	4	0	0	5	10	15	16
d	3	0	0	0	5	10	11
e	2	0	0	0	0	5	6
f	1	0	0	0	0	0	1

Matrix (2)

Where i and j refer to the subscripts of the subject being evaluated and X_{ij} is the value in level-1 connection matrix which was at point (i, j). When the row sum of matrix (1) and (2) was the same, the element value in the next level of matrix which was at point (i, j) could be calculated according to formula (1); as a result a new matrix and a new row sum were obtained. This process continued until the row sum could be distinguished. It was assumed that there were five participating objects, i.e., a, b, c, d, e (therefore $i, j = a, b, c, d, e$). The evaluation results are shown in matrix (3) (i.e., level-1 connection matrix): The row sum

	a	b	c	d	e	Row sum
a	0	1	1	0	1	3
b	0	0	0	1	0	1
c	0	1	0	0	1	2
d	1	0	1	0	0	2
e	0	1	0	1	0	2

suggested that c, d and e had the same score; therefore level-2 connection matrix should be calculated, and the step were as follows. Firstly the numerical value on (a, b) was obtained by multiplying the sum of row a and $b(3 + 1 = 4)$ by the numerical value on (a, b) in the level-1 matrix, i.e., $4 * 1 = 4$. According to equation (1), the level-2 connection matrix and the corresponding row sum were as follows: As we can see, the values of players c and e still remained the same. To distinguish the order, it is necessary to calculate the level-3 connection matrix. According to equation (1), the level-3 connection matrix and the corresponding row sum were as follows: According to the row sum, the order was a, d, e, c, b,

	a	*b*	*c*	*d*	*e*	Row sum
a	0	4	5	0	5	14
b	0	0	0	3	0	3
c	0	3	0	0	4	7
d	5	0	4	0	0	9
e	0	3	0	7	0	7

	a	*b*	*c*	*d*	*e*	Row sum
a	0	17	21	0	21	59
b	0	0	0	12	0	12
c	0	10	0	0	14	24
d	23	0	16	0	0	39
e	0	10	0	16	0	26

from excellent to poor.

2.3 Spearman rank correlation

Definition 2.3. Spearman rank correlation is mainly used to solve the problem which concerns nominal data and sequential data. It is applied to two-column variable and has rank variable properties and materials with rank variable property and linear relationship.

Spearman rank correlation formula was:

$$\rho_i = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n^3 - n}.$$

Where n refers to the number of ranks and d refers to the rank difference of two-column paired variates.

3. The evaluation criteria of the rationality of the comprehensive evaluation methods

To further discuss and analyze the comprehensive evaluation methods, each evaluation method was regarded as a voter participating in Borda count. Borda

count was performed according to the evaluation results of each method; then a comprehensive result was obtained. Then the result obtained by Borda count method was regarded as a reasonable and accurate result. The correlation of the obtained comprehensive result and the evaluation results of the other methods was calculated using Spearman rank correlation theories. Then the methods were ranked based on the correlation. It was concluded that the comprehensive evaluation method with a larger correlation was more reasonable than that with a smaller correlation.

Specific steps were as follows.

Step one: Suppose that there were n evaluation objects in the evaluation system, which were denoted as $D_1, D_2, \ldots, D_n$, there were m comprehensive evaluation methods (equivalent to m voters), which were denoted as $P_1, P_2, \ldots, P_m$. The comprehensive evaluation results are as follows.

Evaluation methods/Evaluation objects	D_1	D_2	.....	D_n
P_1	a_{11}	a_{12}	.....	a_{1n}
P_2	a_{21}	a_{22}	.....	a_{2n}
:	:	:	.....	:
P_m	a_{m1}	a_{m2}	.....	a_{mn}

$a_{i1}, a_{i2}, \Lambda a_{in}$ was a sequence of $1, 2, \Lambda, n$.

Step two: The sum of evaluation results on $D_i(i = 1, 2, Ł, n)$ using different comprehensive evaluation methods ($D_i(i = 1, 2, Ł, n)$) were calculated.

- The sum of evaluation results on D_1 using different comprehensive evaluation methods was $\sum_{i=1}^{m} a_{i1}$;
- The sum of evaluation results on D_2 using different comprehensive evaluation methods was $\sum_{i=1}^{m} a_{i2}$;
- The sum of evaluation results on D_n using different comprehensive evaluation methods was $\sum_{i=1}^{m} a_{im}$.

Step three: $\sum_{i=1}^{m} a_{i1}, \sum_{i=1}^{m} a_{i2}, \ldots, \sum_{i=1}^{m} a_{in}$ were compared.

1. If $\sum_{i=1}^{m} a_{i1}, \sum_{i=1}^{m} a_{i2}, \ldots, \sum_{i=1}^{m} a_{in}$ were completely different (absence of two equal values), then there was a sequence of $1, 2, Ł, n$. It was determined as the ideal evaluation result by the criteria, which was denoted as P_L.

2. If there were two equal values among $\sum_{i=1}^{m} a_{i1}, \sum_{i=1}^{m} a_{i2}, \ldots, \sum_{i=1}^{m} a_{in}$, the evaluation results in Table 1 were assigned by Borda count method, until they were completely different (absence of two equal values). Then there was a sequence of $1, 2, Ł, n$. It was determined as the ideal evaluation result by the criteria, which was denoted as P_L.

Step Four: Spearman rank correlation coefficients of the comprehensive evaluation results of $P_1, P_2, Ł, P_m$ and the ideal evaluation result P_L were calculated. The size of the correlation coefficient was regarded as the rationality of each comprehensive evaluation method.

4. Numerical simulation on rationality evaluation criteria for comprehensive evaluation

There were ten objects to be evaluated in the evaluation system, which were denoted as $D_1, D_2, Ł, D_{10}$. Six comprehensive evaluation methods were used, which were denoted as $P_1, P_2, Ł, P_6$ (the weighted average method P_1, the fuzzy comprehensive evaluation method P_2, analytic hierarchy process P_3, principal component analysis; P_4, comprehensive index method P_5 and the efficacy coefficient method P_6). The rationality of the evaluation criterion was simulated, and the following numerical values were randomly generated. The comprehensive evaluation results are as follows. Due to the equal comprehensive evaluation

Methods	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
P_1	1	2	3	4	5	6	7	8	9	10
P_2	5	2	7	1	4	10	3	6	9	8
P_4	8	6	7	3	4	9	2	1	10	5
P_5	9	5	10	4	7	2	1	8	6	3
P_6	8	7	6	3	2	1	5	4	10	9

results, $D_1 > D_4 > D_5 > D_2 > D_8 > D_1 > D_6 > D_3 > D_{10} > D_9$ was obtained after assignment with Borda count method. According to the criteria, the result was P_L.

It was found that $\rho_2 > \rho_4 > \rho_6 > \rho_3 > \rho_5 > \rho_1$ based on the size of the correlation coefficient. Therefore, the fuzzy comprehensive evaluation method was more reasonable than the others and the rationality of weighted average method was the worst.

In the perspectives of systematization and axiomatization, the results of several commonly used comprehensive evaluation methods were analyzed, and the improved Borda rule was used to assign the results to avoid the same values. Then the ideal evaluation result which was determined by the criteria

was obtained. Through comparing it with the commonly used comprehensive evaluation methods, the Spearman rank correlation coefficients of them were obtained. According to the size of the correlation coefficient, the rationality of the various methods was sorted. In this study, the quantified characteristics of comprehensive evaluation methods and process selected in decision-making were investigated to provide adaptive strategies for decision-makers.

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