COMBINED EFFECT OF MAGNETISM AND ROUGHNESS ON A FERROFLUID SQUEEZE FILM IN POROUS TRUNCATED CONICAL PLATES: EFFECT OF VARIABLE BOUNDARY CONDITIONS

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Abstract. This article aims to discuss the performance of a ferrofluid squeeze film between transversely rough porous truncated conical plates resorting to special type of boundary conditions depending on the magnetization parameter. Invoking the stochastic averaging model of Christensen and Tonder regarding the roughness characterization, the associated stochastically averaged Reynolds type equation is solved to get the pressure distribution, in turn, which gives the load carrying capacity. The results affirm that suitable boundary condition may help in scaling down the adverse effect of roughness to a large extent appropriately choosing the magnetization parameter. However, in the case of negatively skewed roughness the situation remains relatively better. It is also found that the absence of flow doesnt deter the bearing system from supporting certain amount of load, which is very much unlikely in the case of conventional lubricant based bearing system.

Keywords: squeeze film, truncated conical plates, porosity, magnetization, roughness.

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1. Introduction

Wu [1] examined the effect of porosity on squeeze film behaviour in annular irrotational disks. The load carrying capacity was found to be reduced due to the porosity. Gupta and Vora [2] analyzed the squeeze film behaviour between curved annular plates. The curvature was found to have considerable influence on the performance of the squeeze film. Lin et. al. [3] extended the configuration of [2] to discuss the magneto hydrodynamic squeeze film characteristics between curved annular plates. Patel and Deheri [4] investigated the configuration of [2] by considering the lower plate as well as the upper plate along the surfaces generated by hyperbolic function. Subsequently, Patel and Deheri [5] modified the approach to consider both plates along the surfaces determined by secant functions. The investigations of [4], [5] confirmed that the magnetization had a significantly positive effect on the squeeze film performance in annular plates. Use of magnetic fluid as a lubricant modifying the performance of the bearing system has been very well recognized. Bhat and Deheri [6], [7] analyzed the performance of a magnetic fluid based squeeze film behaviour between curved annular disks and curved circular plates and found that the performance with the magnetic fluid as lubricant was relatively better than with a conventional lubricant.

It is well known that bearing surfaces particularly, after having some run in and wear develop roughness. Various methods have been proposed to study and analyze the effect of surface roughness of the bearing surfaces on the performance of squeeze film bearings. Several investigators have adopted a stochastic approach to mathematically model the randomness of the roughness. A comprehensive general analysis was presented by Christensen and Tonder [8, 9, 10] for surface roughness (both transverse as well as longitudinal) based on a general probability density function. Later on, this approach of Christensen and Tonder [8, 9, 10] laid down the basis of the analysis to discuss the effect of surface roughness on the performance of the bearing systems in a number of investigations. Ting [11] discussed the engagement behaviour of lubricated porous annular disks by considering the effect of surface roughness on the squeeze film. The roughness significantly affected the performance characteristics. Gupta and Deheri [12] studied the effect of transverse surface roughness on the squeeze film performance in a spherical bearing.

Prakash and Vij [13] investigated the load carrying capacity and time height relation for squeeze films between porous plates. Circular, annular, elliptic, rectangular, conical and truncated conical plates were investigated for the squeeze film performance. Deheri et. al. [14] considered the ferrofluid based squeeze film between rough porous truncated conical plates. The negatively skewed roughness provided a better performance for this type of bearing system. Wierzcholski and Miszczak [15] presented a method of friction calculation in slide conical micro- bearing occurring in hard disk drives computer disks. Andharia and Deheri [16] analyzed the effect of longitudinal surface roughness on the ferrofluid based squeeze film between conical plates. The performance of the bearing system was observed to be little better in this case as compared to the case of transverse surface roughness. Vadher et. al. [17] investigated the effect of transverse surface roughness on the performance of hydromagnetic squeeze film between conducting truncated conical plates. This article confirmed that for suitable values of aspect ratio and conductivities, the magnetization parameter offered some measures to counter the adverse effect of porosity and standard deviation associated with roughness.

Deheri et. al. [18] dealt with the behavior of a ferrofluid squeeze film in porous rough conical plates. A suitable combination of magnetization parameter and semi vertical angle of the cone presented a better performance in the case of negatively skewed roughness. Shimpi and Deheri [19] discussed the combined effect of slip velocity and bearing deformation on the behavior of a ferrofluid based squeeze film in rough porous truncated conical plates. For an overall improved performance this article confirmed that the slip velocity was required to be kept at minimum. Same was the case for bearing deformation. Hsu et. al. [20] presented a theoretical study of non- Newtonian effects in conical squeeze film plates that was based on the Rabinowitsch fluid modal. The non-Newtonian effect provided better load carrying capacity and lengthened response time.

Here, it has been proposed to analyze the performance characteristics of a transversely rough ferrofluid squeeze film in porous truncated conical plates, taking recourse to a new set of boundary conditions.

2. Analysis

The configuration of the bearing system is presented below

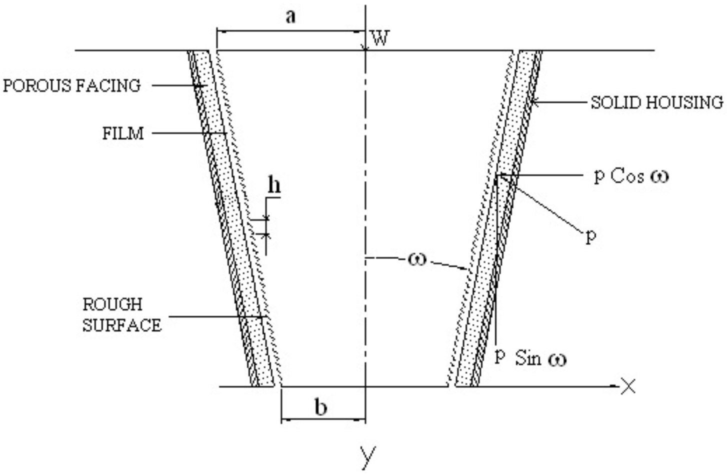

Figure-I. Geometry and configuration of bearing system

The lower plate having porous face is fixed. The upper plate moves towards the lower plate along the normal with a angular velocity $\dot{h} = (\frac{dh}{dt})$. Both the plates are considered to be electrically conducting and an electrically conducting lubricant fills the clearance space. A uniform transverse magnetic field is applied between the plates. The transverse surface roughness of the bearing surface is characterized by a random variable with non zero mean, variance and skewness. Following the discussions of Christensen and Tonder [8, 9, 10], the film thickness $h(x)$ is considered as

$$h(x) = \overline{h}(x) + h_s(x),$$

where $\overline{h}_s(x)$ is the mean film thickness and $h_s(x)$ is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. $h_s(x)$ is described by a probability density function $f(h_s)$, defined by

$$f(h_s) = \begin{cases} \frac{35}{32c^7}(C^2 - h_s^2)^3, & \text{if } -C \leq h_s \leq C \\ 0, & \text{elsewhere} \end{cases}$$

C being the maximum deviation from the mean film thickness. The mean α, the standard deviation σ and the parameter ε, which is the measure of symmetry, of random variable h_s, are defined by the relationships

$$\alpha = E(h_s), \;\; \sigma^2 = E[(h_s - \alpha)^2]$$

and

$$\varepsilon = E[(h_s - \alpha)^3]$$

where E denotes the expected value defined by

$$E(R) = \int_{-C}^{C} Rf(h_s)dh_s.$$

The details regarding the roughness and characterization can be seen from Christensen and Tonder [8, 9, 10].

A modified form of Darcys law (Prajapati [21]) governs the flow in the porous medium while in the film region the equation of the hydromagnetic lubrication theory holds. Under the traditional assumptions of hydrodynamic lubrication the modified stochastically averaged Reynolds equation for the lubricant film pressure (Prajapati [21]) is found to be

$$\frac{1}{x}\frac{d}{dx}\left\{x\frac{d}{dx}(p - 0.5\mu_0\overline{\mu}H^2)\right\} = \frac{12\mu\dot{h}\sin\omega}{A} \tag{1}$$

where

$$A = h^3\sin^3\omega + 3\sigma^2 h\sin\omega + 3\alpha^2 h\sin\omega + 3\alpha h^2\sin^2\omega + 3\sigma^2\alpha + \alpha^3 + \varepsilon + 12\phi H_0$$

and

$$H^2 = (a\mathrm{cosec}\omega - x)(x - b\mathrm{cosec}\omega)$$

μ_0 is permeability of the free space, $\overline{\mu}$ is the magnetic permeability, μ is the viscosity of the fluid, ϕ is permeability of porous facing, ω is the semi vertical angle of cone and H_0 is the thickness of porous facing.

The following dimensionless terms are introduced:

$$\sigma^* = \frac{\sigma}{h},\ \ \alpha^* = \frac{\alpha}{h},\ \ \varepsilon^* = \frac{\varepsilon}{h^3},\ \ \psi = \frac{\phi H_0}{h^3},\ \ k = \frac{a}{b}$$

$$\overline{A} = \sin^3\omega + 3(\sigma^*)^2\sin\omega + 3(\alpha^*)\sin\omega + 3\alpha^*\sin^2\omega + 3(\sigma^*)^2\alpha^* + (\alpha^*)^3 + 12\psi$$

and

$$\mu^* = -\frac{h^3\mu_0\overline{\mu}}{\mu\dot{h}},\ \overline{p} = -\frac{h^3 p}{\pi\mu\dot{h}(a^2-b^2)\text{cosec}\omega},\ W = -\frac{h^3 w}{\pi\mu\dot{h}(a^2-b^2)^2\text{cosec}^2\omega}.$$

Solution of equation (1) by making use of boundary conditions

$$\frac{d\overline{p}}{d\overline{x}_1} = \frac{\mu^*}{2} \text{ at } \overline{x}_1 = 1$$

and

$$\frac{d\overline{p}}{d\overline{x}_2} = -\frac{\mu^*}{2} \text{ at } \overline{x}_2 = 1$$

where

$$\overline{x}_1 = \frac{x}{a\text{cosec}\omega},\ \ \overline{x}_2 = \frac{x}{b\text{cosec}\omega},\ \ b < x < a \tag{2}$$

determines the pressure distribution as

$$\begin{aligned} p = {} & 0.5\mu_0\overline{\mu}H^2 + \frac{3\mu\dot{h}\sin\omega}{A}(x^2 - a^2\text{cosec}^2\omega) \\ & + b\text{cosec}\omega\left\{\frac{\mu\dot{h}(a^2-b^2)\mu^*}{2h^3b} - 05\mu_0\overline{\mu}(a-b)\text{cosec}\omega - \frac{6\mu\dot{h}b}{A}\right\}\ln\left(\frac{x}{\text{cosec}\omega}\right) \\ & - \frac{\mu\dot{h}\pi(a^2-b^2)\mu^*\text{cosec}\omega}{2h^3}. \end{aligned} \tag{3}$$

Then, the load carrying capacity given by

$$\begin{aligned} w &= 2\pi\int_{b\text{cosec}\omega}^{a\text{cosec}\omega} ap(x)dx \\ &= 2\pi\left[\left\{\frac{0.5\mu_0\overline{\mu}(a^2-b^2)(a-b)^2}{12} - \frac{3\mu\dot{h}\sin\omega(a^2-b^2)}{4A}\right\}\text{cosec}^4\omega\right. \\ & \quad \left\{\frac{\mu\dot{h}\pi(a^2-b^2)\mu^*}{2h^3b} - 0.5\mu_0\overline{\mu}(a-b)\text{cosec}\omega - \frac{6\mu\dot{h}b}{A}\right\} \\ & \quad \left. \cdot + \left\{\frac{b^2-a^2}{4} + \frac{b^2}{a^2}\ln\left(\frac{a}{b}\right)\right\} b\text{cosec}^3\omega - \frac{\mu\dot{h}\pi(a^2-b^2)^2\mu^*\text{cosec}^3\omega}{4h^3}\right] \end{aligned} \tag{4}$$

is expressed in dimensionless form as

$$
\begin{aligned}
W = {} & \frac{\mu^{*}\mathrm{cosec}^{2}\omega}{2}\left\{\frac{(k-1)}{6\pi(k+1)}+\frac{3}{2}\sin\omega-\frac{\ln k}{k^{2}-1}\sin\omega\right.\\
& \left.+\frac{1}{2\pi(k+1)}-\frac{\ln k}{\pi(k-1)(k+1)^{2}}\right\}\\
& +\frac{3\mathrm{cosec}\omega}{2\pi\overline{A}(k^{2}-1)^{2}}\{(k^{2}-1)(k^{2}-3)+4\ln k\}.
\end{aligned}
\tag{5}
$$

Results and discussions:

Equation (5) makes it clear that the increase in load carrying capacity due to the magnetization comes out to be

$$
\begin{aligned}
& \frac{\mu^{*}\mathrm{cosec}^{2}\omega}{2}\left\{\frac{(k-1)}{6\pi(k+1)}+\frac{3}{2}\sin\omega-\frac{\ln k}{k^{2}-1}\sin\omega\right.\\
& \left.+\frac{1}{2\pi(k+1)}-\frac{\ln k}{\pi(k-1)(k+1)^{2}}\right\}
\end{aligned}
$$

in comparison with the conventional lubricant based bearing system.

The variation of load carrying capacity with respect to the magnetization parameter is presented in Figures: 1-5. It is seen that the load carrying capacity increases sharply with increasing magnetization.

The effect of standard deviation on the distribution of load carrying capacity is shown in Figures: 6-8. These figures indicate that the standard deviation has an adverse effect on the bearing performance as the load carrying capacity is considerably reduced.

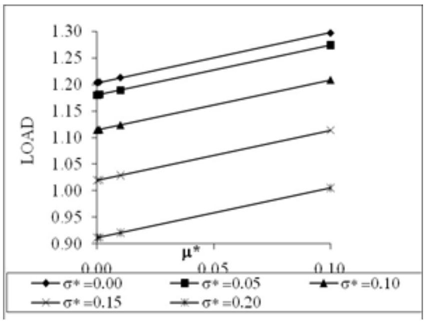

Figure 1: Variation of load carrying capacity with respect to μ^{*} and σ^{*}

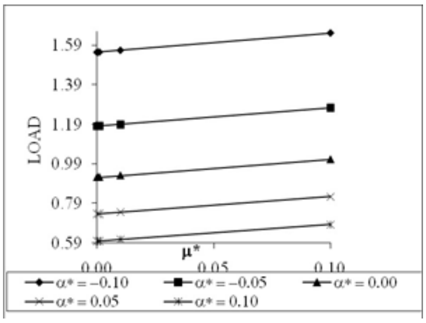

Figure 2: Variation of load carrying capacity with respect to μ^* and α^*

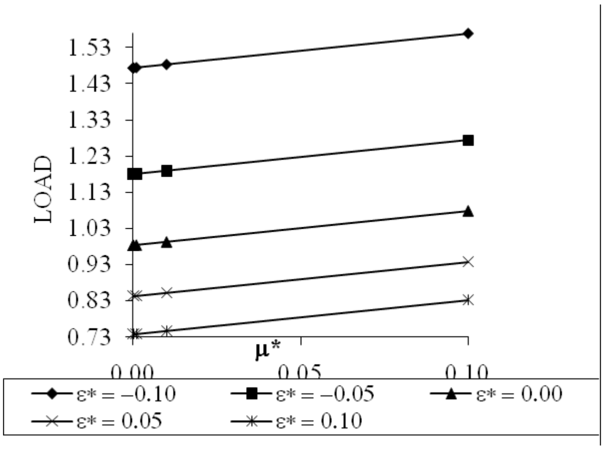

Figure 3: Variation of load carrying capacity with respect to μ^* and ε^*

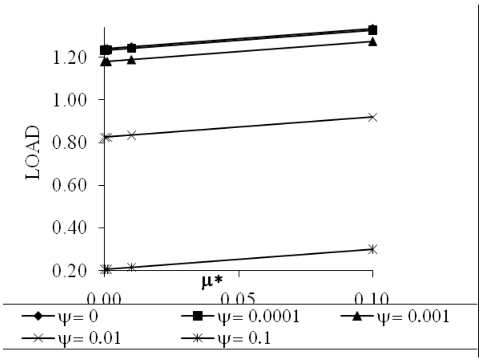

Figure 4: Variation of load carrying capacity with respect to μ^* and ψ

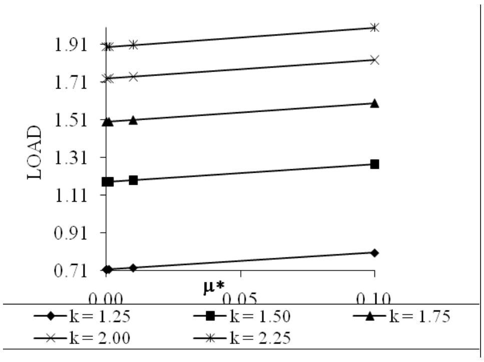

Figure 5: Variation of load carrying capacity with respect to μ^* and k

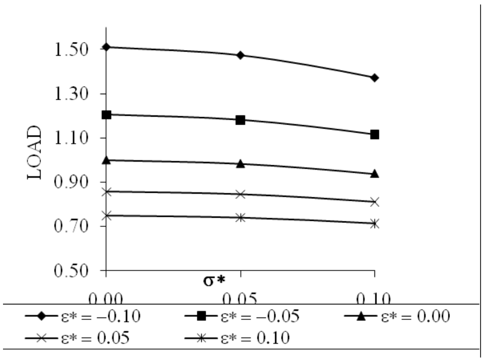

Figure 6: Variation of load carrying capacity with respect to σ^* and ε^*

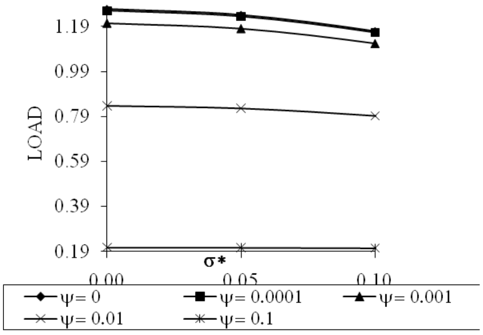

Figure 7: Variation of load carrying capacity with respect to σ^* and ψ

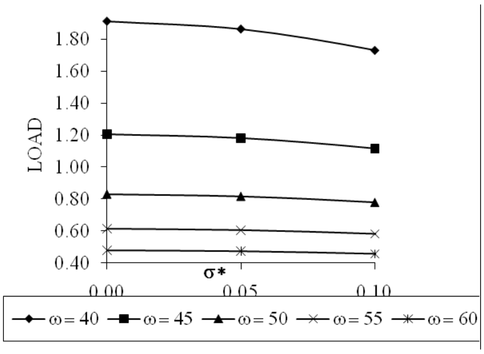

Figure 8: Variation of load carrying capacity with respect to σ^* and ω

The effect of variance depicted in Figures: 9-11 indicates that the variance (+ve) causes reduced load while the load carrying capacity increases sharply owing to variance(- ve).

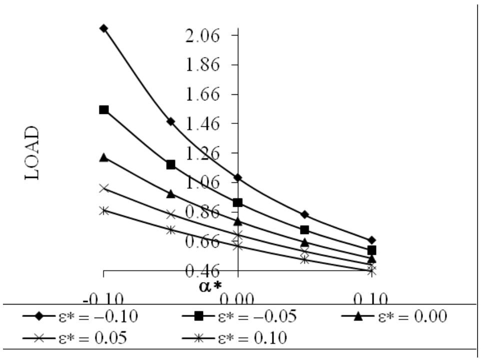

Figure 9: Variation of load carrying capacity with respect to α^* and ε^*

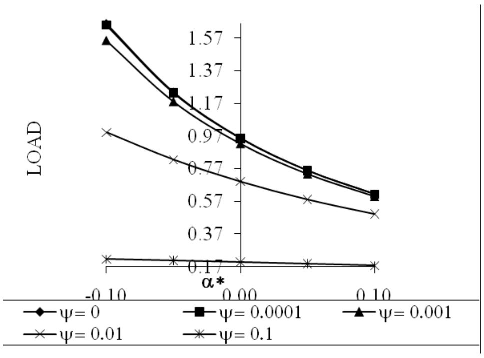

Figure 10: Variation of load carrying capacity with respect to α^* and ψ

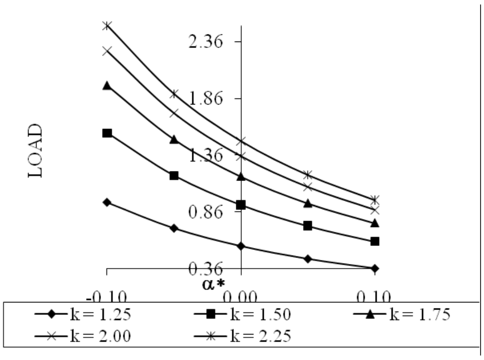

Figure 11: Variation of load carrying capacity with respect to α^* and k

The fact that the skewness goes along the path of variance so far as the trends of load carrying capacity are concerned, is manifest in Figures: 12-13.

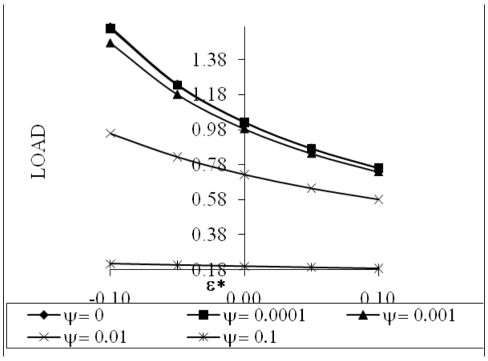

Figure 12: Variation of load carrying capacity with respect to ε^* and ψ

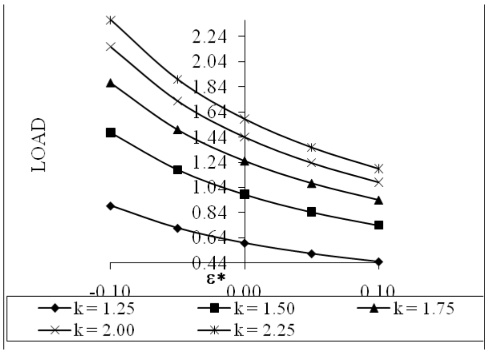

Figure 13: Variation of load carrying capacity with respect to ε^* and k

As usual porosity brings down the load carrying capacity. This can be seen from Figure:-14.

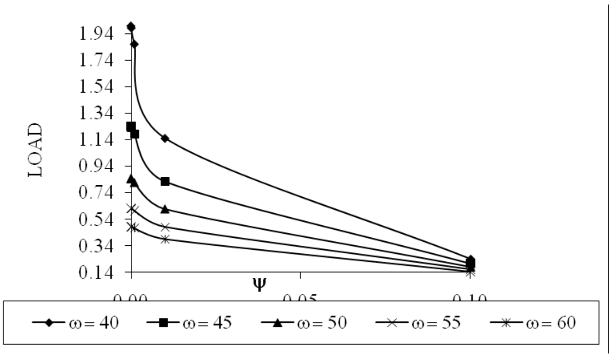

Figure 14: Variation of load carrying capacity with respect to ψ and ω

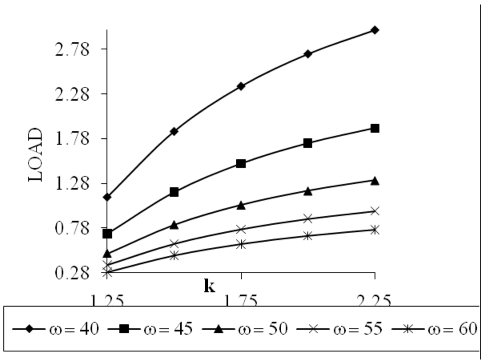

Figure 15: Variation of load carrying capacity with respect to k and ω

The graphical representations suggest that the combined positive effect of negatively skewed roughness and variance (- ve) can be channelized to improve the bearing performance. The magnetic fluid may aid to this positive effect in overcoming the adverse effect of porosity and standard deviation. However, here the semi vertical angle of the cone may play a crucial role. Equally important becomes the role of the aspect ratio (Figure-15), for augmenting the bearing performance.

Conclusions

This investigation reveals that the roughness aspect must be duly addressed while designing this type of bearing system, even if suitable magnetic strength is in place. Also, it can be seen that judiciously choosing the boundary conditions, the performance of the bearing system can be improved by picking up suitable magnetic strength in spite of the fact that the roughness has an adverse effect in general.

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