HX-TYPE CHAOTIC (HYPERCHAOTIC) SYSTEM BASED ON FUZZY INFERENCE MODELING

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Abstract. In this paper, we investigate HX equations of autonomous system. The right-hand side of n-dimensional HX equations theoretically consists of 2^n terms. HX equations seems to be more complicated than the original system. For autonomous chaotic(hyperchaotic) system with polynomial right-hand side, we obtain its HX equations by termwise modeling. We find not all coefficients are variable in HX equations. Even some HX equations are equal to the original chaotic (hyperchaotic) systems. Under some fuzzy partition, HX-type chaotic (hyperchaotic) system is defined if some of its coefficients is exactly variable. By adjusting fuzzy partition, a family of chaotic (hyperchaotic) systems can be defined. They are different but qualitatively similar. Numerical simulations are provided to verify the existence of HX-type chaotic (hyperchaotic) system.

Keywords: HX equations, termwise modeling, HX-type chaotic system.

1. Introduction

HX equation is a kind of special differential equation with variable coefficients. It is constructed on the fuzzy logic system by using interpolation mechanism. This

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kind of modelling method is called modelling method based on fuzzy inference, proposed by Hongxing Li et al.[1]. Many literatures focus on HX equations, such as marginal linearization method for solving HX equations[2], HX equations of time-varying system[3] and some other literatures published[4, 5, 6, 7, 8, 9, 10].

In this paper, we investigate HX equations of continuous autonomous system. The three-dimensional system is considered as an example to get HX equations. The study indicates that the right-hand side of each HX equation is polynomial with eight terms. Similarly, the right-hand side of each HX equation for a four-dimensional system is polynomial with sixteen terms. For n-dimensional system, the right-hand side of its HX equation is polynomial with 2^n terms. While the typical chaotic(hyperchaotic) system is of more concise and more simple, such as the well-known Lorenz system[11], Rössler system[12], etc. We are interested in the characteristics of HX equations of chaotic(hyperchaotic) system, though few literatures[5, 6] considered it. For the system with polynomial right-hand side, we use termwise modeling method. First we divide it into several differential equations with monomial right-hand side. HX equations of the differential equations with monomial right-hand side are obtained. Then we add them together to get HX equations of the original system. The result suggests that not all coefficients of HX equations are varying. Even some HX equations are equal to the original system, such as Lorenz system, Rössler system, Rössler hyperchaotic system[13], etc.

If HX equations of the chaotic (hyperchaotic) system indeed contains variable coefficients and it is chaotic (hyperchaotic) under some fuzzy partition, we call it HX-type chaotic(hyperchaotic) system. For HX-type chaotic (hyperchaotic) system, the change of the fuzzy partition can give rise to the change of HX-type chaotic (hyperchaotic) system. The study on HX-type chaotic (hyperchaotic) system is very interesting for the chaotic switch may improve the security of secret communication.

The rest of the paper is organized as follows. Section 2 investigates HX equations of the continuous autonomous system. Three-dimensional system is taken as an example. In Section 3, HX equations of chaotic(hyperchaotic) system with polynomial right-hand side is cosidered. Some HX equations of chaotic(hyperchaotic) systems are even equal to the original systems. Section 4 defines HX-type chaotic(hyperchaotic) system. Liu system illustrates the existence of HX-type chaotic system. Finally, conclusions are drawn in Section 5.

2. HX equations of continuous autonomous system

Without loss of generality, we take three dimensional system as example to discuss HX equations of autonomous system. The equations is described as follows:

$$
\begin{cases}
\dot{x}_1 = f_1(x_1, x_2, x_3), \\
\dot{x}_2 = f_2(x_1, x_2, x_3), \\
\dot{x}_3 = f_3(x_1, x_2, x_3),
\end{cases} \tag{1}
$$

where x_1, x_2, x_3 are the states of the system, $f_1, f_2, f_3 : R^3 \to R^3$ are functions.

Following the fuzzy inference modeling method[1] , suppose the universes of $x_1, x_2, x_3,\ \dot{x}_1, \dot{x}_2, \dot{x}_3$ are respectively $X_1, X_2, X_3, \dot{X}_1, \dot{X}_2, \dot{X}_3$, where $X_1 = [\alpha_1, \beta_1], X_2 = [\alpha_2, \beta_2], X_3 = [\alpha_3, \beta_3],\ \dot{X}_1 = [\alpha_4, \beta_4], \dot{X}_2 = [\alpha_5, \beta_5], \dot{X}_3 = [\alpha_6, \beta_6]$.

The fuzzy partitions of $X_1, X_2, X_3, \dot{X}_1, \dot{X}_2, \dot{X}_3$ are
$\mathcal{A} = \{A_i\}_{(1\leqslant i\leqslant p)}$, $\mathcal{B} = \{B_j\}_{(1\leqslant j\leqslant q)}$, $\mathcal{C} = \{C_k\}_{(1\leqslant k\leqslant r)}$,
$\mathcal{D} = \{D_{ijk}\}_{(1\leqslant i\leqslant p, 1\leqslant j\leqslant q, 1\leqslant k\leqslant r)}$, $\mathcal{E} = \{E_{ijk}\}_{(1\leqslant i\leqslant p, 1\leqslant j\leqslant q, 1\leqslant k\leqslant r)}$, and
$\mathcal{F} = \{F_{ijk}\}_{(1\leqslant i\leqslant p, 1\leqslant j\leqslant q, 1\leqslant k\leqslant r)}$, respectively.

The peak points of $A_i, B_j, C_k, D_{ijk}, E_{ijk}, F_{ijk}$ are respectively x_{1i}, x_{2j}, x_{3k}, $\dot{x}_{1ijk}, \dot{x}_{2ijk}, \dot{x}_{3ijk}$, which satisfy the conditions $\alpha_1 \leqslant x_{11} < x_{12} < \cdots < x_{1p} \leqslant \beta_1$, $\alpha_2 \leqslant x_{21} < x_{22} < \cdots < x_{2q} \leqslant \beta_2$, $\alpha_3 \leqslant x_{31} < x_{32} < \cdots < x_{3r} \leqslant \beta_3$. Here $\dot{x}_{1ijk}, \dot{x}_{2ijk}, \dot{x}_{3ijk}$ don't need satisfy order relation. The fuzzy rule is as follows:

$$
\begin{aligned}
&\mathrm{R}^{ijk} : \text{If } x_1 \text{ is } A_i,\ x_2 \text{ is } B_j \text{ and } x_3 \text{ is } C_k, \\
&\text{then } \dot{x}_1 \text{ is } D_{ijk},\ \dot{x}_2 \text{ is } E_{ijk},\ \dot{x}_3 \text{ is } F_{ijk},
\end{aligned} \tag{2}
$$

where $i = 1, 2, \cdots, p$, $j = 1, 2, \cdots, q$, $k = 1, 2, \cdots, r$.

The fuzzy logic system based on fuzzy rules (2) can be represented as the following piecewise interpolation functions:

$$
\begin{cases}
\dot{x}_1 = F_1(x_1, x_2, x_3) \triangleq \displaystyle\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} A_i(x_1)B_j(x_2)C_k(x_3)\dot{x}_{1ijk}, \\
\dot{x}_2 = F_2(x_1, x_2, x_3) \triangleq \displaystyle\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} A_i(x_1)B_j(x_2)C_k(x_3)\dot{x}_{2ijk}, \\
\dot{x}_3 = F_3(x_1, x_2, x_3) \triangleq \displaystyle\sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} A_i(x_1)B_j(x_2)C_k(x_3)\dot{x}_{3ijk}.
\end{cases} \tag{3}
$$

A_i is taken as "triangle wave "membership function:

$$
A_i(x_1) = \begin{cases}
\dfrac{x_1 - x_{1(i-1)}}{x_{1i} - x_{1(i-1)}}, & x_{1(i-1)} \leqslant x_1 \leqslant x_{1i}, \\
\dfrac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}}, & x_{1i} \leqslant x_1 \leqslant x_{1(i+1)}, \\
0, & \text{otherwise},
\end{cases} \tag{4}
$$

where $i = 1, 2, \cdots, p$, and let $x_{10} = x_{11}, x_{1(p+1)} = x_{1p}$. The membership function A_i is drawn as Fig.1.

Similarly, the membership functions of B_j and C_k can be given. We can easily draw the conclusion of theorem 1.

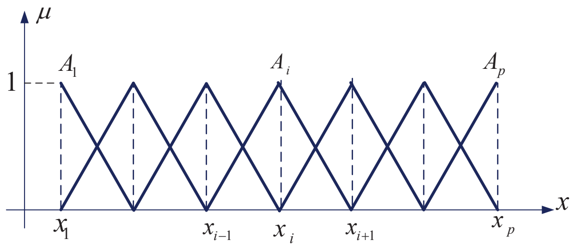

Figure 1: Triangle wave membership function of A_i

Theorem 1. *Based on the above assumptions, the fuzzy model* (3) *of system* (1) *can be described as the following ordinary differential equations with variable-coefficients*

$$
\begin{aligned}
\dot{x}_1 &= F_1(x_1, x_2, x_3) \\
&\triangleq a_0(x_1,x_2,x_3) + a_1(x_1,x_2,x_3)x_1 + a_2(x_1,x_2,x_3)x_2 \\
&\quad + a_3(x_1,x_2,x_3)x_3 + a_{12}(x_1,x_2,x_3)x_1x_2 + a_{13}(x_1,x_2,x_3)x_1x_3 \\
&\quad + a_{23}(x_1,x_2,x_3)x_2x_3 + a_{123}(x_1,x_2,x_3)x_1x_2x_3, \\
\dot{x}_2 &= F_2(x_1, x_2, x_3) \\
&\triangleq b_0(x_1,x_2,x_3) + b_1(x_1,x_2,x_3)x_1 + b_2(x_1,x_2,x_3)x_2 \\
&\quad + b_3(x_1,x_2,x_3)x_3 + b_{12}(x_1,x_2,x_3)x_1x_2 + b_{13}(x_1,x_2,x_3)x_1x_3 \\
&\quad + b_{23}(x_1,x_2,x_3)x_2x_3 + b_{123}(x_1,x_2,x_3)x_1x_2x_3, \\
\dot{x}_3 &= F_3(x_1, x_2, x_3) \\
&\triangleq c_0(x_1,x_2,x_3) + c_1(x_1,x_2,x_3)x_1 + c_2(x_1,x_2,x_3)x_2 \\
&\quad + c_3(x_1,x_2,x_3)x_3 + c_{12}(x_1,x_2,x_3)x_1x_2 + c_{13}(x_1,x_2,x_3)x_1x_3 \\
&\quad + c_{23}(x_1,x_2,x_3)x_2x_3 + c_{123}(x_1,x_2,x_3)x_1x_2x_3.
\end{aligned}
\tag{5}
$$

Proof. When the state $(x_1, x_2, x_3)^T$ is on (ijk)th piece, i.e. $(x_1, x_2, x_3)^T \in [x_{1i}, x_{1(i+1)}] \times [x_{2j}, x_{2(j+1)}] \times [x_{3k}, x_{3(k+1)}]$, from the piecewise interpolation functions (3) and triangle-wave membership function (4), we get

$$
\begin{aligned}
\dot{x}_1 &= F_1(x_1, x_2, x_3) \triangleq \sum_{i=1}^{p}\sum_{j=1}^{q}\sum_{k=1}^{r} A_i(x_1)B_j(x_2)C_k(x_3)\dot{x}_{1ijk} \\
&= A_i(x_1)B_j(x_2)C_k(x_3)\dot{x}_{1ijk} + A_i(x_1)B_j(x_2)C_{k+1}(x_3)\dot{x}_{1ijk+1} \\
&\quad + A_i(x_1)B_{j+1}(x_2)C_k(x_3)\dot{x}_{1ij+1k} + A_i(x_1)B_{j+1}(x_2)C_{k+1}(x_3)\dot{x}_{1ij+1k+1} \\
&\quad + A_{i+1}(x_1)B_j(x_2)C_k(x_3)\dot{x}_{1i+1jk} + A_{i+1}(x_1)B_j(x_2)C_{k+1}(x_3)\dot{x}_{1i+1jk+1} \\
&\quad + A_{i+1}(x_1)B_{j+1}(x_2)C_k(x_3)\dot{x}_{1i+1j+1k} + A_{i+1}(x_1)B_{j+1}(x_2)C_{k+1}(x_3)\dot{x}_{1i+1j+1k+1} \\
&= \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} \dot{x}_{1ijk}
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} \dot{x}_{1ijk+1} \\
&+ \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} \dot{x}_{1ij+1k} \\
&+ \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} \dot{x}_{1ij+1k+1} \\
&+ \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} \dot{x}_{1i+1jk} \\
&+ \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} \dot{x}_{1i+1jk+1} \\
&+ \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} \dot{x}_{1i+1j+1k} \\
&+ \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} \dot{x}_{1i+1j+1k+1} \\
= {}& a_0^{(ijk)} + a_1^{(ijk)} x_1 + a_2^{(ijk)} x_2 + a_3^{(ijk)} x_3 + a_{12}^{(ijk)} x_1 x_2 \\
&+ a_{13}^{(ijk)} x_1 x_3 + a_{23}^{(ijk)} x_2 x_3 + a_{123}^{(ijk)} x_1 x_2 x_3,
\end{aligned}
$$

where

$$
\begin{aligned}
a_0^{(ijk)} = {}& \frac{1}{(x_{1i} - x_{1(i+1)})(x_{2j} - x_{2(j+1)})(x_{3k} - x_{3(k+1)})} \\
&\times (x_{1(i+1)} x_{2(j+1)} (x_{3k} \dot{x}_{1ij(k+1)} - x_{3(k+1)} \dot{x}_{1ijk}) \\
&+ x_{1(i+1)} x_{2j} (x_{3(k+1)} \dot{x}_{1i(j+1)k} - x_{3k} \dot{x}_{1i(j+1)(k+1)}) \\
&+ x_{1i} x_{2(j+1)} (x_{3(k+1)} \dot{x}_{1(i+1)jk} - x_{3k} \dot{x}_{1(i+1)j(k+1)}) \\
&+ x_{1i} x_{2j} (x_{3k} \dot{x}_{1(i+1)(j+1)(k+1)} - x_{3(k+1)} \dot{x}_{1(i+1)(j+1)k})), \\
a_1^{(ijk)} = {}& \frac{1}{(x_{1i} - x_{1(i+1)})(x_{2j} - x_{2(j+1)})(x_{3k} - x_{3(k+1)})} \\
&\times (x_{2(j+1)} x_{3(k+1)} (\dot{x}_{1ijk} - \dot{x}_{1(i+1)jk}) + x_{2(j+1)} x_{3k} (\dot{x}_{1(i+1)j(k+1)} - \dot{x}_{1ij(k+1)}) \\
&+ x_{2j} x_{3(k+1)} (\dot{x}_{1(i+1)(j+1)k} - \dot{x}_{1i(j+1)k}) \\
&+ x_{2j} x_{3k} (\dot{x}_{1i(j+1)(k+1)} - \dot{x}_{1(i+1)(j+1)(k+1)})), \\
a_2^{(ijk)} = {}& \frac{1}{(x_{1i} - x_{1(i+1)})(x_{2j} - x_{2(j+1)})(x_{3k} - x_{3(k+1)})} \\
&\times (x_{1(i+1)} x_{3(k+1)} (\dot{x}_{1ijk} - \dot{x}_{1i(j+1)k}) + x_{1(i+1)} x_{3k} (\dot{x}_{1i(j+1)(k+1)} - \dot{x}_{1ij(k+1)}) \\
&+ x_{1i} x_{3(k+1)} (\dot{x}_{1(i+1)(j+1)k} - \dot{x}_{1(i+1)jk}) \\
&+ x_{1i} x_{3k} (\dot{x}_{1(i+1)j(k+1)} - \dot{x}_{1(i+1)(j+1)(k+1)})),
\end{aligned}
$$

$$
\begin{aligned}
a_3^{(ijk)} = &\frac{1}{(x_{1i}-x_{1(i+1)})(x_{2j}-x_{2(j+1)})(x_{3k}-x_{3(k+1)})}\\
&\times (x_{1(i+1)}x_{2(j+1)}(\dot{x}_{1ijk}-\dot{x}_{1ij(k+1)}) + x_{1(i+1)}x_{2j}(\dot{x}_{1i(j+1)(k+1)}-\dot{x}_{1i(j+1)k})\\
&+ x_{1i}x_{2(j+1)}(\dot{x}_{1(i+1)j(k+1)}-\dot{x}_{1(i+1)jk})\\
&+ x_{1i}x_{2j}(\dot{x}_{1(i+1)(j+1)k}-\dot{x}_{1(i+1)(j+1)(k+1)})),\\
a_{12}^{(ijk)} = &\frac{1}{(x_{1i}-x_{1(i+1)})(x_{2j}-x_{2(j+1)})(x_{3k}-x_{3(k+1)})}\\
&\times (x_{3(k+1)}(\dot{x}_{1i(j+1)k}-\dot{x}_{1ijk}+\dot{x}_{1(i+1)jk}-\dot{x}_{1(i+1)(j+1)k})\\
&+ x_{3k}(\dot{x}_{1i(j+1)(k+1)}-\dot{x}_{1ij(k+1)}+\dot{x}_{1(i+1)j(k+1)}-\dot{x}_{1(i+1)(j+1)(k+1)})),\\
a_{13}^{(ijk)} = &\frac{1}{(x_{1i}-x_{1(i+1)})(x_{2j}-x_{2(j+1)})(x_{3k}-x_{3(k+1)})}\\
&\times (x_{2(j+1)}(\dot{x}_{1ij(k+1)}-\dot{x}_{1ijk}+\dot{x}_{1(i+1)jk}-\dot{x}_{1(i+1)j(k+1)})\\
&+ x_{2j}(\dot{x}_{1i(j+1)k}-\dot{x}_{1i(j+1)(k+1)}+\dot{x}_{1(i+1)(j+1)(k+1)}-\dot{x}_{1(i+1)(j+1)k})),\\
a_{23}^{(ijk)} = &\frac{1}{(x_{1i}-x_{1(i+1)})(x_{2j}-x_{2(j+1)})(x_{3k}-x_{3(k+1)})}\\
&\times (x_{1(i+1)}(\dot{x}_{1ij(k+1)}-\dot{x}_{1ijk}+\dot{x}_{1i(j+1)k}-\dot{x}_{1i(j+1)(k+1)})\\
&+ x_{1i}(\dot{x}_{1(i+1)jk}-\dot{x}_{1(i+1)j(k+1)}+\dot{x}_{1(i+1(j+1)(k+1)}-\dot{x}_{1(i+1)(j+1)k})),\\
a_{123}^{(ijk)} = &\frac{1}{(x_{1i}-x_{1(i+1)})(x_{2j}-x_{2(j+1)})(x_{3k}-x_{3(k+1)})}\\
&\times (\dot{x}_{1ijk}-\dot{x}_{1ij(k+1)}-\dot{x}_{1i(j+1)k}+\dot{x}_{1i(j+1)(k+1)}-\dot{x}_{1(i+1)jk}\\
&+ \dot{x}_{1(i+1)j(k+1)}-\dot{x}_{1(i+1)(j+1)(k+1)}+\dot{x}_{1(i+1)(j+1)k}).
\end{aligned}
$$

When $(x_1,x_2,x_3)^T \notin [x_{1i},x_{1(i+1)}]\times[x_{2j},x_{2(j+1)}]\times[x_{3k},x_{3(k+1)}]$, suppose $a_0^{(ijk)} = a_1^{(ijk)} = a_2^{(ijk)} = a_3^{(ijk)} = a_{12}^{(ijk)} = a_{13}^{(ijk)} = a_{23}^{(ijk)} = a_{123}^{(ijk)} = 0$, the coefficients can be defined on the whole universe $X_1\times X_2\times X_3$, so we have

$$
\begin{aligned}
\dot{x}_1 = &a_0^{(ijk)} + a_1^{(ijk)}x_1 + a_2^{(ijk)}x_2 + a_3^{(ijk)}x_3 + a_{12}^{(ijk)}x_1x_2\\
&+ a_{13}^{(ijk)}x_1x_3 + a_{23}^{(ijk)}x_2x_3 + a_{123}^{(ijk)}x_1x_2x_3,
\end{aligned}
\tag{6}
$$

which is defined on the whole universe.

When i,j,k change, we can get $(p-1)\times(q-1)\times(r-1)$ such equations. Let $a_0(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_0^{(ijk)}$, $a_1(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_1^{(ijk)}$, $a_2(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_2^{(ijk)}$, $a_3(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_3^{(ijk)}$, $a_{12}(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_{12}^{(ijk)}$, $a_{13}(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_{13}^{(ijk)}$, $a_{23}(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_{23}^{(ijk)}$, $a_{123}(x_1,x_2,x_3) = \sum_{i=1}^{p-1}\sum_{j=1}^{q-1}\sum_{k=1}^{r-1} a_{123}^{(ijk)}$. Then we combine the $(p-1)\times(q-1)\times(r-1)$ equations together and obtain

HX equation of the first equation of system (1), that is

$$
\begin{aligned}
\dot{x}_1 = {} & a_0(x_1,x_2,x_3)+a_1(x_1,x_2,x_3)x_1+a_2(x_1,x_2,x_3)x_2+ \\
& a_3(x_1,x_2,x_3)x_3+a_{12}(x_1,x_2,x_3)x_1 \\
& +a_{13}(x_1,x_2,x_3)+a_{23}(x_1,x_2,x_3)x_2x_3+a_{123}(x_1,x_2,x_3)x_1x_2x_3.
\end{aligned} \tag{7}
$$

Similarly, we can get HX equations of the other two equations. □

From the above analysis, we know that the polynomial right-hand side of the HX equation consists of eight terms. Similarly, the polynomial right-hand side of HX equation about four dimension system contains sixteen terms, The right-hand side of HX equation about n-dimension system is composed by 2^n terms. This seems go against the concise form of the typical autonomous system. For example, the structure of the HX equations of Lorenz system may be more complex than the original system.

Fan and Fang [5, 6] investigated the fuzzy inference modeling of chaotic system, but HX equations of Rössler system given by them is theoretically equal to Rössler system. The system error is caused by round-off error in the numerical calculation.

3. HX equations of chaotic (hyperchaotic) system with polynomial right-hand side

Now we investigate the autonomous chaotic(hyperchaotic) system with polynomial right-hand side by termwise modeling method. Suppose we fully understand the structure of the chaotic (hyperchaotic) system considered. There exist a lot of such systems, such as Lorenz system [11], Rössler system[12], hyperchaotic Lorenz system[19, 23, 22], etc. Specifically, f_1, f_2, f_3 in Eq. (1) are all polynomials . The general form of the equation in system (1) is

$$
\dot{w} = \sum_{k_1=0}^{n_1}\sum_{k_2=0}^{n_2}\sum_{k_3=0}^{n_3}\lambda_{k_1k_2k_3}x_1^{k_1}x_2^{k_2}x_3^{k_3}, \tag{8}
$$

where $\dot{w}$ denotes $\dot{x}_1, \dot{x}_2$ or $\dot{x}_3$, $\lambda_{k_1k_2k3}$ is the coefficient of $\lambda_{k_1k_2k_3}x_1^{k_1}x_2^{k_2}x_3^{k_3}$.

Let $\dot{w}_{k_1k_2k_3} = x_1^{k_1}x_2^{k_2}x_3^{k_3}$,

$$
\dot{w} = \sum_{k_1=0}^{n_1}\sum_{k_2=0}^{n_2}\sum_{k_3=0}^{n_3}\lambda_{k_1k_2k_3}\dot{w}_{k_1k_2k_3}. \tag{9}
$$

The equation (3) is divided into several differential equations with monomial right-hand side. From the linear relation, if we get HX equation $\dot{w}^{HX}_{k_1k_2k_3}$ of each monomial differential equation $\dot{w}_{k_1k_2k_3}$, and then add them together, we can get HX equation of $\dot{w}$.

$$\dot{w}^{HX} = \sum_{k_1=0}^{n_1} \sum_{k_2=0}^{n_2} \sum_{k_3=0}^{n_3} \lambda_{k_1 k_2 k_3} \dot{w}^{HX}_{k_1 k_2 k_3}. \tag{10}$$

Now our aim is to obtain HX equations $\dot{w}^{HX}_{k_1 k_2 k_3}$ of $\dot{w}_{k_1 k_2 k_3}$. Following the method of Li[1], on the piece $[x_{1i}, x_{1(i+1)}] \times [x_{2j}, x_{2(j+1)}] \times [x_{3k}, x_{3(k+1)}]$, we have

$$\begin{aligned}
\dot{w}^{HX(ijk)}_{k_1 k_2 k_3} &= \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} x^{k_1}_{1(i+1)} x^{k_2}_{2(j+1)} x^{k_3}_{3(k+1)} \\
&\quad + \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} x^{k_1}_{1(i+1)} x^{k_2}_{2(j+1)} x^{k_3}_{3k} \\
&\quad + \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} x^{k_1}_{1(i+1)} x^{k_2}_{2j} x^{k_3}_{3(k+1)} \\
&\quad + \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} x^{k_1}_{1(i+1)} x^{k_2}_{2j} x^{k_3}_{3k} \\
&\quad + \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} x^{k_1}_{1i} x^{k_2}_{2(j+1)} x^{k_3}_{3(k+1)} \\
&\quad + \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} x^{k_1}_{1i} x^{k_2}_{2(j+1)} x^{k_3}_{3k} \\
&\quad + \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} x^{k_1}_{1i} x^{k_2}_{2j} x^{k_3}_{3(k+1)} \\
&\quad + \frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} x^{k_1}_{1i} x^{k_2}_{2j} x^{k_3}_{3k} \\
&= \left(\frac{x_1 - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} x^{k_1}_{1i} + \frac{x_1 - x_{1i}}{x_{1(i+1)} - x_{1i}} x^{k_1}_{1(i+1)} \right) \\
&\quad \times \left(\frac{x_2 - x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} x^{k_2}_{2j} + \frac{x_2 - x_{2j}}{x_{2(j+1)} - x_{2j}} x^{k_2}_{2(j+1)} \right) \\
&\quad \times \left(\frac{x_3 - x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} x^{k_3}_{3k} + \frac{x_3 - x_{3k}}{x_{3(k+1)} - x_{3k}} x^{k_3}_{3(k+1)} \right),
\end{aligned}$$

that is,

$$\begin{aligned}
\dot{w}^{HX(ijk)}_{k_1 k_2 k_3} &= \left(\frac{x^{k_1}_{1i} - x^{k_1}_{1(i+1)}}{x_{1i} - x_{1(i+1)}} x_1 + \frac{x_{1i} x^{k_1}_{1(i+1)} - x^{k_1}_{1i} x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} \right) \\
&\quad \times \left(\frac{x^{k_2}_{2j} - x^{k_2}_{2(j+1)}}{x_{2j} - x_{2(j+1)}} x_2 + \frac{x_{2j} x^{k_2}_{2(j+1)} - x^{k_2}_{2j} x_{2(j+1)}}{x_{2j} - x_{2(j+1)}} \right) \\
&\quad \times \left(\frac{x^{k_3}_{3k} - x^{k_3}_{3(k+1)}}{x_{3k} - x_{3(k+1)}} x_3 + \frac{x_{3k} x^{k_3}_{3(k+1)} - x^{k_3}_{3k} x_{3(k+1)}}{x_{3k} - x_{3(k+1)}} \right).
\end{aligned} \tag{11}$$

It is easy to see that the content in three parenthesis of equation (11) are all of first degree. We need only to analyze the content of first one. Similar conclusions can be drawn about the other two.

When $k_1 = 0$, that is to say that $\dot{w}_{k_1k_2k_3}$ doesn't contain x_1

$$\begin{aligned}&\frac{x_{1i}^{k_1} - x_{1(i+1)}^{k_1}}{x_{1i} - x_{1(i+1)}}x_1 + \frac{x_{1i}x_{1(i+1)}^{k_1} - x_{1i}^{k_1}x_{1(i+1)}}{x_{1i} - x_{1(i+1)}}\\ &= \frac{x_{1i}^{0} - x_{1(i+1)}^{0}}{x_{1i} - x_{1(i+1)}}x_1 + \frac{x_{1i}x_{1(i+1)}^{0} - x_{1i}^{0}x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} = 0 \times x_1 + 1 = 1.\end{aligned} \tag{12}$$

This suggests $\dot{w}_{k_1k_2k_3}^{HX(ijk)}$ don't contain x_1. In another word, $\dot{w}_{k_1k_2k_3}^{HX(ijk)}$ is irrelevant to the fuzzy partition of the universe of x_1.

When $k_1 = 1$, the degree of x_1 is 1 in $\dot{w}_{k_1k_2k_3}$

$$\begin{aligned}\frac{x_{1i}^{k_1} - x_{1(i+1)}^{k_1}}{x_{1i} - x_{1(i+1)}}x_1 + \frac{x_{1i}x_{1(i+1)}^{k_1} - x_{1i}^{k_1}x_{1(i+1)}}{x_{1i} - x_{1(i+1)}} &= \frac{x_{1i} - x_{1(i+1)}}{x_{1i} - x_{1(i+1)}}x_1 + \frac{x_{1i}x_{1(i+1)} - x_{1i}x_{1(i+1)}}{x_{1i} - x_{1(i+1)}}\\ &= 1 \times x_1 + 0 = x_1.\end{aligned} \tag{13}$$

This suggests the degree of x_1 is still 1 in $\dot{w}_{k_1k_2k_3}^{HX(ijk)}$. In another word $\dot{w}_{k_1k_2k_3}^{HX(ijk)}$ is irrelevant to the fuzzy partition of the universe of x_1.

When $k_1 \geqslant 2$

$$\begin{aligned}&\frac{x_{1i}^{k_1} - x_{1(i+1)}^{k_1}}{x_{1i} - x_{1(i+1)}}x_1 + \frac{x_{1i}x_{1(i+1)}^{k_1} - x_{1i}^{k_1}x_{1(i+1)}}{x_{1i} - x_{1(i+1)}}\\ &= \left(\sum_{s=0}^{k_1-1} x_{1i}^{s}x_{1(i+1)}^{k_1-1-s}\right)x_1 - x_{1i}x_{1(i+1)}\sum_{t=0}^{k_1-2} x_{1i}^{t}x_{1(i+1)}^{k_1-2-t}.\end{aligned} \tag{14}$$

Obviously, the conclusion is relevant to the fuzzy partition of the universe of x_1.

Similar conclusions can be obtained for the other two expressions with parenthesis in (11). So we can get

Theorem 2. *If $\dot{w}_{k_1k_2k_3} = x_1^{k_1}x_2^{k_2}x_3^{k_3}$, HX equation have the following features:*
1) If $0 \leqslant k_i \leqslant 1$, $(i = 1, 2, 3)$, HX equation $\dot{w}_{k_1k_2k_3}^{HX_1} = \dot{w}_{k_1k_2k_3}$ of $\dot{w}_{k_1k_2k_3} = x_1^{k_1}x_2^{k_2}x_3^{k_3}$;
2) If $0 \leqslant k_i \leqslant 1$ $(i = 1, 2, 3)$ for some k_i, then the coefficients of the HX equation of $\dot{w}_{k_1k_2k_3} = x_1^{k_1}x_2^{k_2}x_3^{k_3}$ is irrelated to the fuzzy partition of the ith universe, and HX equation's exponent of the ith variable is still k_i;
3) If $k_i \geqslant 2$, $(i = 1, 2, 3)$ for some variable, then in the piece $[x_{1i}, x_{1(i+1)}] \times [x_{2j}, x_{2(j+1)}] \times [x_{3k}, x_{3(k+1)}]$, HX equation of $\dot{w}_{k_1k_2k_3}$ is related to the fuzzy partition of the ith universe;
4) If $k_i \geqslant 2$, $(i = 1, 2, 3)$ for all variables, then in the piece $[x_{1i}, x_{1(i+1)}] \times [x_{2j}, x_{2(j+1)}] \times [x_{3k}, x_{3(k+1)}]$, HX equation of $\dot{w}_{k_1k_2k_3}$ is related to the fuzzy partition of every universe.

Proof. From the equations (12) and (13), we know when $0 \leqslant k_1 \leqslant 1$, the coefficient of local HX equation on the (ijk)th piece of $\dot{w}_{k_1k_2k_3}$ is still 1, the exponent is still k_1. In a similar method, when $0 \leqslant k_2 \leqslant 1, 0 \leqslant k_3 \leqslant 1$, the coefficients of local HX equation on the (ijk)th piece of exponents of $\dot{w}_{k_1k_2k_3}$ is still 1, the exponents is still k_2, k_3. So on the (ijk)th partition, the local HX equation $\dot{w}^{HX(ijk)}_{k_1k_2k_3} = x_1^{k_1}x_2^{k_2}x_3^{k_3}$. Meanwhile, if $0 \leqslant k_i \leqslant 1$ for k_i, then the coefficients of HX equation is irrelated to the fuzzy partition of the ith universe.

From equation (14), if $k_1 \geqslant 2$ HX equation is relevant to the fuzzy partition of the universe of x_1. Similarly, if $k_i \geqslant 2$ for some k_i, on the piece $[x_{1i}, x_{1(i+1)}] \times [x_{2j}, x_{2(j+1)}] \times [x_{3k}, x_{3(k+1)}]$, HX equation is relevant to the fuzzy partition of the universe x_i. While $k_i \geqslant 2$ for all variables, on the piece $[x_{1i}, x_{1(i+1)}] \times [x_{2j}, x_{2(j+1)}] \times [x_{3k}, x_{3(k+1)}]$,

$$\begin{aligned}\dot{w}^{HX(ijk)}_{k_1k_2k_3} &= \left(\sum_{s_1=0}^{k_1-1} x_{1i}^{s_1}x_{1(i+1)}^{k_1-1-s_1}x_1 - x_{1i}x_{1(i+1)}\sum_{t_1=0}^{k_1-2} x_{1i}^{t_1}x_{1(i+1)}^{k_1-2-t_1}\right)\\ &\times \left(\sum_{s_2=0}^{k_2-1} x_{2j}^{s_2}x_{2(j+1)}^{k_2-1-s_2}x_2 - x_{2j}x_{2(j+1)}\sum_{t_2=0}^{k_2-2} x_{2j}^{t_2}x_{2(j+1)}^{k_2-2-t_2}\right)\\ &\times \left(\sum_{s_3=0}^{k_3-1} x_{3k}^{s_3}x_{3(k+1)}^{k_3-1-s_3}x_3 - x_{3k}x_{3(k+1)}\sum_{t_3=0}^{k_3-2} x_{3k}^{t_3}x_{3(k+1)}^{k_3-2-t_3}\right).\end{aligned}$$

This suggests HX equation relevant to fuzzy partitions of all variables. □

Theorem 2 is not only applicable to three-dimensional differential equation with monomial right-hand side, but also it is can be used to four-dimensional or more high dimensional systems. As discussed above, we can combine HX equations obtained together to get HX equations $\dot{w}^{HX}$ of system . So the following result can be easily obtained.

Theorem 3. *For a chaotic (hyperchaotic) system, if the exponent of some variable is no more than* 1, *then the exponent of the variable in its HX equations is unchanged; if the exponent of all variables no more than* 1, *then HX equations is the same as the original one.*

Proof. From 1), 2) of theorem 2, combining with (10), the result easily obtained. □

In conclusion, before solving HX equations of chaotic (hyperchaotic) system with polynomial right-hand side, we analyze the characteristic of the system, carry on fuzzy inference modelling term by term, make linear additivity, and get HX equations of the original system.

4. HX-type chaotic (hyperchaotic) system

From Section 3, we know that HX equations of chaotic (hyperchaotic) system with exponent of variables less than 2 is identical to the original system. Lorenz system and Rössler system et al. are such type systems. For the system which exponent of some variable greater than or equal to 2, some coefficients of HX equations of the system is exactly variable. There are many such systems, such as Liu system [14], hyperchaotic Liu system [15], hyperchaotic Fang system proposed by Fang [16], chaotic system proposed by Li [17], chaotic system proposed by Xie[18], which cotain nonlinear terms with exponent 2 of some variable. Chen proposed four-dimension Lorenz-type hyperchaotic system with equilibrium curve[20], which contain nonlinear terms with exponent 3 of some variable.

Definition 1. *If some coefficient of HX equations about chaotic (hyperchaotic) system is variable under some fuzzy partition and the HX equations is chaotic (hyperchaotic), it is called HX-type chaotic (hyperchaotic) system.*

Now we investigate HX equations of Liu system

Example 1. Liu system is described as follows:

$$\begin{cases} \dot{x_1} = a(x_2 - x_1), \\ \dot{x_2} = bx_1 - kx_1x_3, \\ \dot{x_3} = hx_1^2 - cx_3, \end{cases} \tag{15}$$

where $a = 10, b = 40, c = 2.5, k = 1, h = 1$. The Lyapunov exponents are $\lambda_1 = 1.6535, \lambda_2 = 0, \lambda_3 = -14.1446$. The phase diagram of Liu system is shown in Fig.2.

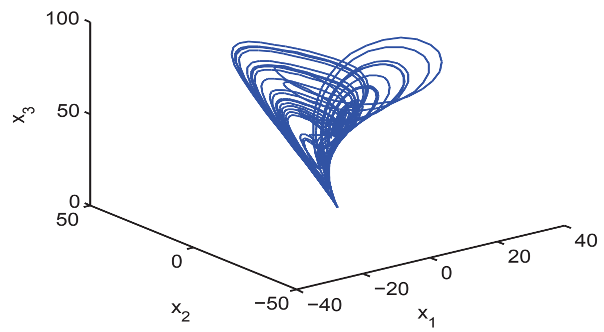

Figure 2: The phase diagram of Liu system

By numerical calculation, we know $x_1 \in [-35.5815, 35.2084]$, $x_2 \in [-60.9790, 62.7936]$, $x_3 \in [6.5037, 108.6809]$. From theorem 2, we know that the first and second equations of system (15) is exact fuzzy modelling. In the third equation, only the quadratic component is not exact modelling, that is to say, HX equations of Liu system is only related to the fuzzy partition of the universe of x_1. The universe of x_1 is partitioned by triangular wave membership function. For convenience, suppose the distances of two peak points are same. Avoiding the data overflow the universe, we added a redundancy to the universe. Let $-35.5815-35.5815\times0.1=x_{11}<x_{12}<\cdots<x_{1p}=35.2084+35.2084\times0.1$. When $x_1\in[x_{1i},x_{1(i+1)}]$, local HX equations is

$$\begin{cases} \dot{x_1} = a(x_2 - x_1), \\ \dot{x_2} = bx_1 - kx_1x_3, \\ \dot{x_3} = h(x_{1i} + x_{1(i+1)})x_1 - cx_3 - hx_{1i}x_{1(i+1)}. \end{cases} \tag{16}$$

Now the system's divergence is $\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = -a+0-c = -10-2.5 = -12.5 < 0$. This suggests that each local HX equation is dissipative. So HX equations of Liu system is dissipative. If the largest Lyapunov exponent of HX equations greater than zero, HX-type chaotic system is achieved. By using Wolf's method [24], we calculate the Lyapunov exponents of HX equations, as shown in Fig.3, where the peak point number is from $p = 2$ to $p = 100$, i.e. the fuzzy set number is from 2 to 100.

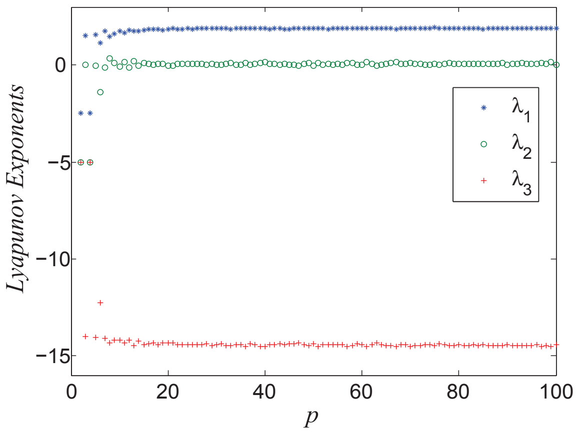

Figure 3: The diagram of Lyapunov exponents as p changing from 2 to 100

From Fig.3, in addition to $p = 2$ and $p = 4$, HX equations of Liu system is chaotic, called HX-type Liu system. When the initial value $\boldsymbol{x}_0 = (20, 2, 3)^T$, if the peak point number $p = 3$, the phase diagram of HX-type Liu system is shown as Fig. 4, if the peak point number $p = 10$, the phase diagram of HX-type Liu system is shown as Fig. 5, if the peak point number $p = 20$, the phase diagram of HX-type Liu system is shown as Fig.6, if the peak point number $p = 30$, the phase diagram of HX-type Liu system is shown as Fig.7.

From the phase diagram of HX-type Liu system, we know, as the increase of peak points, the phase diagram of HX-type Liu system is more like the phase diagram of Liu system.

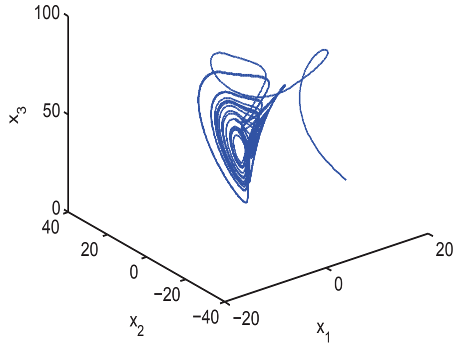

Figure 4: The phase portrait of HX-Type Liu system as $p = 3$

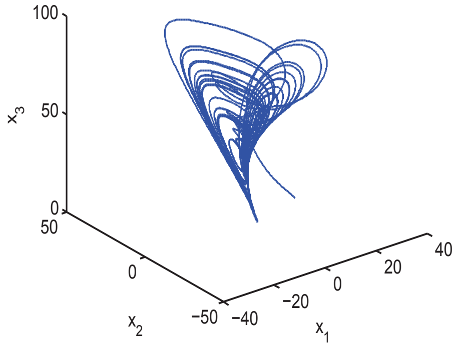

Figure 5: The phase portrait of HX-Type Liu system as $p = 10$

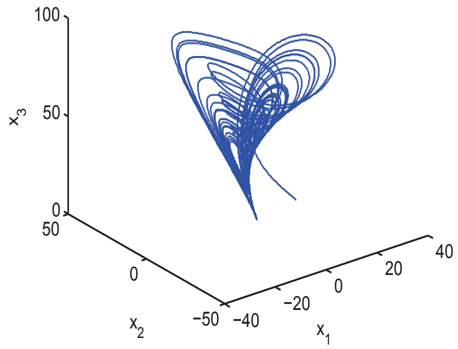

Figure 6: The phase portrait of HX-Type Liu system as $p = 20$

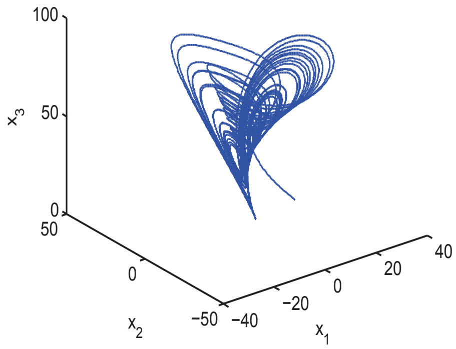

Figure 7: The phase portrait of HX-Type Liu system as $p = 30$

The equilibrium points of the HX equations (16) are obtained by solving the equations

$$\begin{cases} a(x_2 - x_1) = 0, \\ bx_1 - kx_1x_3 = 0, \\ h(x_{1i} + x_{1(i+1)})x_1 - cx_3 - hx_{1i}x_{1(i+1)} = 0. \end{cases} \tag{17}$$

From equations (17), we see that the HX equations (16) has two equilibrium points

$$E_1\left(0, 0, -\frac{hx_{1i}x_{1(i+1)}}{c}\right), E_2\left(\frac{bc + khx_{1i}x_{1(i+1)}}{kh(x_{1i} + x_{1(i+1)})}, \frac{bc + khx_{1i}x_{1(i+1)}}{kh(x_{1i} + x_{1(i+1)})}, \frac{b}{k}\right)$$

as $x_1 \in [x_{1i}, x_{1(i+1)}]$.

Remark 1. As we all know, Liu system (15) has three equilibriums:

$$E_1(0,0,0), E_2(\sqrt{\frac{bc}{kh}}, \sqrt{\frac{bc}{kh}}, \frac{b}{k}), E_3(-\sqrt{\frac{bc}{kh}}, -\sqrt{\frac{bc}{kh}}, \frac{b}{k}),$$

while the HX equations has a family of equilibrium points under the fuzzy partition.

If the fuzzy partition is given, HX-type Liu system is definite. That is to say an appropriate fuzzy partition defined a HX-type Liu system. In the secret communication, changing fuzzy partion can give rise to switch chaotic system, then the security is enhanced.

5. Conclusion

In this paper, we gave HX equations of continuous autonomous system and found that HX equations of high dimensional system is more complicated than the original system. For chaotic (hyperchaotic) system with polynomial right-hand side, we divided it into differential equtions with monomial right-hand side and obtained HX equation of them. By using linear addition, we got HX equations of chaotic (hyperchaotic) system. We discovered that not all coefficients in HX equations are variable. Even HX equations of some chaotic (hyperchatic) system is equal to the original system. Under some fuzzy partition, if HX equations with variable coefficient is chaotic (hyperchaotic), we defined it HX-type chaotic (hyperchaotic) system. The equilibrium points of HX-type chaotic systems were also analyzed. Numerical simulations verified the existence of HX-type chaotic (hyperchaotic) system.

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