# A COMPARATIVE STUDY ON ACHROMATIC AND $B$-CHROMATIC NUMBER OF CERTAIN GRAPHS

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**Abstract.** In this paper, we find the achromatic number of central graph of Crown graph and we discussed its structural properties. We compare the achromatic and $b$-chromatic number of central graph of Sunlet graph, central graph of web graph. Also we study the structural properties of the central graph of Sunlet graph.

**Keywords:** achromatic number, $b$-chromatic number, central graph, Crown graph, Sunlet graph.

## 1. Introduction

Let $G$ be a finite un directional graph with no loops and multiple edges. The central graph $C(G)$ ([7]) of a graph $G$ is obtained by subdividing each edge of $G$ exactly once and joining all the non adjacent vertices of $G$. An achromatic colouring ([4]) is a proper vertex colouring such that each pair of colours is adjacent by at least one edge. The largest possible number of colours in an achromatic colouring of $G$ is called the achromatic number and it is denoted by $\psi(G)$. The $b$-chromatic number $\varphi(G)$ ([3,5]) of a graph $G$ is the largest integer $k$, such that $G$ admits a proper $k-$ colouring, and every colour class has a representative vertex adjacent at least to one vertex in each other class. This type of colouring is called $b$-colouring. This concept of $b$-chromatic number was introduced by Irwing and & Manlove.

The Crown graph $S_n$ for an integer $n > 2$ is the graph with the vertex set $\{x_1, x_2, \ldots, x_n, y_1, \ldots, y_n\}$ and the edge set $\{(x_i, y_j) : 1 \leq i, j \leq n, i \neq j\}$. The $n-$ Sunlet graph $Sl_n$ is the graph on $2n$ vertices obtained by attaching $n$ pendent edges to the cycle graph $C_n$. The 3- Sunlet graph is also known as the net graph. The Web graph $W_n$ is a graph consisting of concentric copies of the cycle graph $C_n$ with corresponding vertices connected by spokes and adjoining pendant vertex at each node of the outer cycle.

## 2. The structural properties of central graph of crown graph

- The number of vertices in the Crown graph $S_n$ is $p = 2n$.

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- The number of vertices in the central graph of Crown graph $C[S_n]$ is $p = n^2 + n$.
- The number of edges in the Crown graph $S_n$ is $q = n(n-1)$.
- The number of edges in the central graph of Crown graph $C[S_n]$ is $q = 3n^2 - 2n$.
- In the Crown graph all the vertices have the same degree$(n-1)$. So it is a regular graph.

### 2.1 Theorem

For any Crown graph $\psi[C(S_n)] = 2n, n \geq 3$

**Proof.** Let $G$ be the Crown graph with the partition $(X, Y)$of the vertices where $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. In $C(S_n)$, let $v_{i,j}$ be the newly introduced vertex joining $x_i$ and $y_j$ where $i \neq j$ and $i, j = 1, 2, 3, \ldots, n$. Consider the two sets of colours $C = \{C_1, C_2, \ldots, C_n\}$ and $C' = \{C'_1, C'_2, \ldots, C'_n\}$. Assign a proper colouring to G as follows: For $1 \leq i \leq n$, assign $C_i$ to $x_i$ and $C'_i$ to $y_i$. By the definition of central graph each $C_i$ is adjacent to all the other $C_j$'s and to $C'_i$. For a particular $i$, the $n-1$ vertices $x_iy_j$ are allocated the $n-1$ colours $C'_r$,$r \neq j$ in such a way that $x_iy_j$ is given a colour other than $C'_j$. From the construction it is seen that this is the maximal possible colouring. Hence $\psi[C(S_n)] = 2n, n \geq 3$.

**Example.**

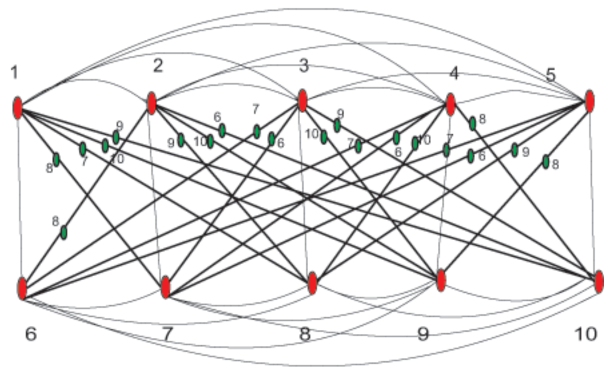


Figure 1: $\psi[C(S_5)] = 10$

### 3. The structural properties of central graph of sunlet graph

- The number of vertices in the Sunlet graph $S_n$ is $p = 2n$.
- The number of vertices in the central graph of Sunlet graph $C[S_n]$ is $p = 4n$.

- The number of edges in the Sunlet graph $S_n$ is $q = 2n$.
- The number of edges in the central graph of Sunlet graph $C[S_n]$ is $q = 2n^2 + n$.

### 3.1 Theorem

Theorem: For any Sunlet Graph $\psi[C(Sl_n)] = 2n, n \geq 3$.

**Proof.** Let $C_n$ be any cycle graph of length $n$ with vertices $v_1, v_2, \ldots, v_n$ named in the cyclic order. Name the attached pendant vertices $u_1, u_2, \ldots, u_n$ in the same cyclic order. Now in $C(Sl_n)$, let $v_{i,j}$ be the newly introduced vertex on the edge joined $v_i$ and $v_j$ and $uv_i$ represent the newly introduced vertex on the edge joining $v_i$ and $u_i$. For $1 \leqslant \ i \leqslant \ n$,the vertex $v_i$ is not adjacent with $u_i$. For $1 \leqslant \ i \leqslant \ n-1$, the vertex $v_i$ is not adjacent with $v_{i+1}$ and $v_1$ is not adjacent with $v_n$. To make the colouring as achromatic, assign the following colouring procedure: Consider the two set of colours $C = \{C_1, C_2, \ldots, C_n\}$ and $C' = \{C'_1, C'_2, \ldots, C'_n\}$.

- For $1 \leq i \leq n$, assign $C_i$ to $v_i$.
- For $1 \leq i \leq n$, assign $C'_i$ to $u_i$.
- For $1 \leq i \leq n-1$, assign $C'_i$ to $v_{i,i+1}$ and $C'_n$ to $v_{1,n}$.
- For $1 \leq i \leq n-1$, assign $C_{i+1}$ to $uv_i$ and assign the colour $C_1$ to the vertex $uv_n$.

By this assignment any pair in the colour set is adjacent by at least one edge. Thus it is complete and it is a maximal one. Hence $\psi[C(Sl_n)] = n + n = 2n, n \geqslant \ 3$.

**Example.**

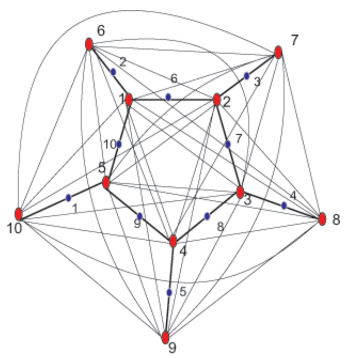


Figure 2: $\psi[C(Sl_5)] = 10$

**3.2 Theorem**

For any Sunlet Graph $\varphi[C(Sl_n)] = \begin{cases} \frac{3n}{2}, & n = even \\ n + \lfloor \frac{n}{2} \rfloor, & n = odd \end{cases}$

**Proof.** Consider the Sunlet graph which is obtained by attaching $n$ pendent edges to the cycle $C_n$. Consider the two vertex sets $X = \{x_1, x_2, \ldots, x_n\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. First assign the name $x_1, x_2, \ldots, x_n$ to the pendant vertices in the cyclic order. Then assign the names $y_1, y_2, \ldots, y_n$ to the vertices of the cycle graph in the same cyclic order. Now in $C[S_n]$, let $y_{i,j}$ be the newly introduced vertex on the edge joining $y_i$ and $y_j$. And let $xy_i$ be the newly introduced vertex on the edge joining $x_i$ and $y_i$. Here we can observe that

- For $1 \leq i \leq n$, the vertex $x_i$ is not adjacent to $y_i$.
- For $2 \leq i \leq n-1$, the vertex $y_i$ is not adjacent to $y_{i+1}$ and $y_{i-1}$.
- $y_1$ is not adjacent to $y_2$ and $y_n$ also $y_n$ is not adjacent with $y_1$ and $y_{n-1}$.

**Case 1.** When $n$ =even Consider the two set of colours
$C = \{C_1, D_1, C_2, D_2, C_{\frac{n}{2}}, D_{\frac{n}{2}}\}$ and $C' = \{C'_1, C'_2, \ldots, C'_{\frac{n}{2}}\}$. When i=odd, assign $C_{\frac{i+1}{2}}$ to $x_i$, when $i$ =even, assign $D_{\frac{i}{2}}$ to $x_i$. Also assign $C'_i$ to $y_{2i-1}, y_{2i}$.

- For $1 \leq i \leq \frac{n}{2}$, assign the colour $C_i$ to the vertex $y_{2i-1,2i}$.
- For $1 \leq i \leq \frac{n}{2} - 1$, assign the colour $D_i$ to the vertex $y_{2i,2i+1}$. And assign the colour $D_{\frac{n}{2}}$ to the vertex $y_{1,n}$.
- For $1 \leq i \leq \frac{n}{2}$, assign the colour $D_{\frac{i+1}{2}}$to the vertex $xy_i$, when $i$ is odd.
- For $1 \leq i \leq \frac{n}{2}$, assign the colour $C_{\frac{i}{2}}$ to the vertex $xy_i$,when $i$ is even.

**Case 2.** When $n$ =odd Consider the two set of colours $C = \{C_1, C_2, \ldots, C_{\frac{n+1}{2}}, D_1, D_2, \ldots, D_{\frac{n-1}{2}}\}$ and $C' = \{C'_1, C'_2, \ldots, C'_{\frac{n}{2}}\}$. When $i$=odd, assign $C_{\frac{i+1}{2}}$ to $x_i$, when $i$=even, assign $D_{\frac{i}{2}}$ to $x_i$. Assign $C'_i$ to $y_{2i-1}, y_{2i}$ and assign $C_{\frac{n+1}{2}}$ to $y_n$.

- For $1 \leq i \leq \frac{n+1}{2}$, assign the colour $C_i$ to the vertex $y_{2i-1,2i}$.
- For $1 \leq i \leq \frac{n-1}{2}$, assign the colour $D_i$ to the vertex $y_{2i,2i+1}$. Assign the colour $C_1$ to the vertex $y_{1,n}$.
- When $i$ is odd, assign the colour $D_{\frac{i+1}{2}}$ to the vertex $xy_i$.
- When $i$ is even, assign the colour $C_{\frac{i}{2}}$ to the vertex $xy_i$, where $i \neq n$. Assign the colour $C_1$to the vertex $xy_n$.

If we introduce any new colour to any vertex, that will not be adjacent to all the other colours in the colour set. Hence by this colouring procedure, the above said colouring is $b$-chromatic and it is the maximal one.

**Example.**

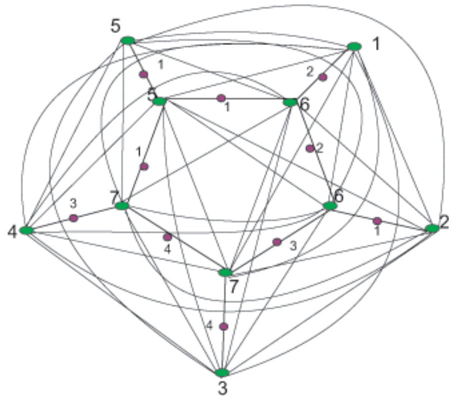

Figure 3: $\varphi[C(Sl_5)] = 7$

## 4. Observations

- For any Web graph $W_n$,$\psi[C(W_n)] = 3n, n \geq 3$.
- For any Web graph $W_n, \varphi[C(W_n)] = 2n, n \geq 3$.

## 5. Conclusion

In this paper, we have found the achromatic number of central graph of Crown graph and have noted that it is equal to the number of vertices in that graph. And we discussed the achromatic and $b$-chromatic number of central graph of Sunlet graph. Also we find the achromatic and $b$-chromatic number of central graph of web graph.

## References


[1] Effantin Brice, Kheddouchi Hamammache, *The b-chromatic number of some power graphs*, Discrete Mathematics and Theoretical Computer Science, 2003, 46-54.

[2] J. Gallian, *Dynamic Survey of Graph Labeling*, Elec. J. Combin. 14, No. DS6, Jan. 3, 2007.

[3] R.W. Irving, D.F. Manlove, *The b-Chromatic number of a graph*, Discrete Applied Mathematics, 91(1-3) 1999, 127-141.

[4] Jonathan Gross, Jay Yellan, Handbook of graph theory CRC Press, New York, 2004.

[5] Marko Jakovac, Sandi Klavzar, *The b-chromatic number of cubic Graphs*, Graphs and Combinatories, 26 (2010), 107-118.

[6] K. Thilagavathi, K.P. Thilagavathy, N. Roopesh, *The achromatic colouring of graphs*, Electronic Notes in Discrete Mathematics, 33 (2009), 153-156.

[7] K.P. Thilagavathy, A. Santha, *A Note on Achromatic and b-chromatic number of graphs*, International Journal of Applied Mathematics and Statistics, 53 (2016), 104-110.

[8] J. Vernold Vivin, K. Thilagavathi, *On Harmonious colouring of Central Graphs*, Far East. Math. Sci (FJMS), 2 (2006), 189-197.

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