A NOTE ON S-ACTS AND BOUNDED LINEAR OPERATORS

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Abstract. In this work, the properties of the certain operator have been studied by looking at the associated S-act of this operator, and conversely. Some operators, for example such operator, one to one, onto operators have been looked. On the other hand, basic mathematical interpretation understanding of S-acts, such as faithful, finitely generated, singular, separated, torsion free and noetherian acts. We have found out the properties may be associated with S-act which has any of these properties. Let V be a inner product space over a field F, T be a bounded operator on V, and let $S = \{e^{x+y}|x, y \text{ are independent variables in } R\}$ be the semigroup. Define $\theta : S \times V \longrightarrow V$ by $\theta(e^{x+y}, v) = e^{T+T^*}(v)$. This function makes V a left S-act, denote by V_{T+T^*} and we call it the associated S-act of $T + T^*$.

Keywords: Bounded linear operator, finite dimensional Banach space, S-act, fathful S-act, Notherian S-act.

1. Introduction

A non-empty set S with a binary operation $S \times S \to S$, $(s.s') \mapsto s.s'$, is called a groupoid. The operation of a groupoid is often called multiplication. Instead of s .s' we usually write ss'. The multiplication on a groupoid S is called associative if $a(bc) = (ab)c$ for all $a, b, c \in S$. A groupoid with associative multiplication or for short an associative groupoid is called a semi-group. A semigroup S with 1 is called monoid (see [1]). Let S be a monoid and A a non empty set. If we have a mapping $\mu : A \times S \longrightarrow A$,$(a, s) \mapsto as := \mu(a, s)$ Such that $a(st) = (as)t$,and $a.1 = a$, for $a \in A$,$s, t, \in S$. We call A a right S-act or a right act over S and write A_S, we can define a left S-act and write S_A. In [2], The module of an operator was study, let Vbe a vector space over afield F. Let T be a linear operator acting on the elements of V on the left. Let $R = F[x]$ be the ring of polynomials in x with coefficients in F. Define $\varphi : R \times V \longrightarrow V$ by $\varphi(p, v) = p.v = p(T)v$. That φ makes V a left R-module denoted V_T, and calls the associated R- module. Let H be a Hilbert space over a field K (K may be

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real or complex), let T be abounded linear operator on H. Define exponential operator e^T as follows: $e^T = \sum_{n=0}^{\infty} T^n/n!$. Note the definition of exponential operator is well defined, this means the sum in the definition of the exponential operator exists [3]. Let V be a Banach space over a field F,T be a bounded operator on V and $S = \{e^x : x \in R\}$ be the semi-group. Define $\mu : S \times V \longrightarrow V$ by $\mu(e^x, v) = e^T(v)$. This function makes V a left S-act, denoted by V_T. We call it the associated S-act of T [4]. In this paper the associated S-act of $T+T^*$ have been study, let $S = \{e^{x+y}|x,\ y\ are\ \ independent\ \ variables\ \ in\ R\}$ be the semigroup. Define $\theta : S \times V \to V$ by $\theta(e^{x+y}, v) = e^{T+T^*}(v)$. This function makes V a left S-act, denote by V_{T+T^*}, the form of every element in V_{T+T^*} is $e^{T+T^*}(v)$, and if two operators T and S are similar then $V_{(T+T^*)}$ is isomorphic to V_{S+S^*}. The relation between a bounded linear operator T and faithful S-act have been study. The relation between finite dimensional Banach space V and Noetherian S-act, discussed have been study by if V is finite dimension then V_{S+S^*} S-act is Noetherian.

2. Main results

Definition 2.1. Let $S = \{e^{x+y}|x, y\ are\ independent\ variables\ in\ R\}$ be the semigroup. Define $\theta : S \times V \longrightarrow V$ by $\theta(e^{x+y}, v) = e^{T+T^*}(v)$. This function makes V a left S-act, denote by $V_{(T+T^*)}$

We put $p_n(T+T^*) = \sum_{i=0}^{n}(T+T^*)^i/(i!) = I + (T+T^*) + (T+T^*)^2/2! + (T+T^*)^3/3! + \cdots + (T+T^*)^n/n!$.

Proposition 2.2. *If $K = \{V_j, j \in \Lambda\}$ is a basis for V, then each element of V_{T+T^*} can be written in the form*

$$\lim_{n\to\infty} \sum_{i=0}^{n} (T+T^*)^i/(i!) \sum_{j\in\Lambda} a_j v_j = \lim_{n\to\infty} p_n(T+T^*).V$$

The symbol $\sum_{j\in\Lambda}$ means the sum is taken over a finite subset of Λ.

Proof. We define $\mu : S \times V \longrightarrow V$, by $\mu(e^{x+y}.v) = e^{T+T*}(v) = \sum_{i=0}^{\infty}(T+T^*)^i/(i!)(v)$ Let $w \in V_{T+T^*}$ then $w = \sum_{i=0}^{\infty}(T+T^*)^i/(i!)(v) = (I + (T+T^*) + (T+T^*)^2/2! + ((T+T^*)^3/3! + \cdots)(v)$. Since $K = \{v_j, j \in \Lambda\}$ is a basis for V then $w = I + (T+T^*) + (T+T^*)^2/2! + ((T+T^*)^3/3! + \cdots (\sum_{j\in\Lambda}(a_j v_j)) = I(\sum_{j\in\Lambda} a_j v_j) + (T+T^*)(\sum_{j\in\Lambda}\ (a_j v_j) + (T+T^*)^2/2!(\sum_{j\in\Lambda}\ (a_j v_j) + ((T+T^*)^3)/3!(\sum_{j\in\Lambda}(a_j v_j) + \cdots$ But the series $\sum_{i=0}^{\infty} T +^i /(i!)$ converges in B(H), Where $T \in B(H)$. Therefore $\sum_{i=0}^{\infty}(T+T^*)^i/(i!)$ is converge in $B(H)$Then we get $w = lim_{n\to\infty} \sum_{i=0}^{\infty} T +^i /(i!) \sum_{j\in\Lambda}(a_j v_j) = \lim_{n\to\infty} p_n(T+T^*).V$ □

Examples 2.3. 1. Let$\{v_j | j \in \Lambda\}$ be a basis for a Banach space V. Let O be the zero operator on V, recall $O^o = I$. Let $w \in V_{0+0^*}$, then by proposition2.2 $w = e^{T+T^*}(v) = e^{0+0^*}(v)$ then $w = I(\sum_{j\in\Lambda} a_j v_j)$ $= \sum_{j\in\Lambda} a_j v_j$,since $e^0 = I$ [2], therefore, $e^{0+0^*} = I$

2. Let $I : V \longrightarrow V$ be the identity operator on V. $\{v_j | j \in \Lambda\}$ be a basis for V, and let $w \in V_{I+I^*}$, then by proposition 2.2 $w = e^{T+T^*}(v) = e^{I+I^*}(v) = e^{I+I^*}(\sum_{j\in\Lambda} a_j v_j)$. Since $I^* = I$ therefore $w = e^(I + I)(\sum_{j\in\Lambda} a_j v_j) = (I + I)\sum_{n=0}^{n} 1/n!(\sum_{j\in\Lambda} a_j v_j) = 2\sum_{n=0}^{n} 1/n!(\sum_{j\in\Lambda} a_j v_j) = \lim_{n\to\infty} 2\sum_{n=0}^{n} 1/n!(\sum_{j\in\Lambda} a_j v_j)$, put $a_n = \sum_{i=0}^{n} 1/i!$ then $w = 2\lim_{n\to\infty} a_n(\sum_{j\in\Lambda} a_j v_j)$.

3. Let $\{v_j : j \in \Lambda\}$ be a basis for a Banach space V, and T be a nilpotent operator on V (i.e. $T^n = 0$ and $T^{n-1} \neq 0$ for some positive integer n) let $w \in V_{T+T^*}$, then by proposition 2.2 $w = e^{T+T^*}(v) = (I + (T + T^*) + (T + T^*)^2/2! + ((T + T^*)^3)/3! + \cdots + ((T + T^*)^n/n!)(v) = [I + (T + T^*) + (T + T^*)^2/2! + ((T + T^*)^3/3! + \cdots + ((T + T^*)^{n-1})/(n - 1)!](\sum_{j\in\Lambda} a_j v_j) + (T+T^*)^n/n!(\sum_{j\in\Lambda} a_j v_j) = [I + (T+T^*) + (T+T^*)^2/2! + (T+T^*)^3/3! + \cdots + (T+T^*)^{n-1}/(n-1)!](\sum_{j\in\Lambda} a_j v_j) + [((n1)T^{n-1}T^* + (n : 2)T^{n-2}{T^*}^2 + \cdots + (n(n-1))T(T^*)^{n-1}/n!](\sum_{j\in\Lambda} a_j v_j) = \lim_{n\to\infty} p_{n-1}(T + T^*)(\sum_{j\in\Lambda} a_j v_j) + a_n(\sum_{j\in\Lambda} a_j v_j) = \lim_{n\to\infty}[p_{n-1}(T + T^*) + a_n].v$, where $a_n = ((n1)T^{n-1}T^* + (n2)T^{n-2}T^{*2} + \cdots + (n(n-1))T(T^*)^{n-1}/n!$.

4. Let $\{v_j : j \in \Lambda\}$ be a basis for a Banach space V, and T be a Self -adjoint operator on H (a bounded linear operator $T : H \longrightarrow H$ on a Hilbert space H is called self -adjoint operator if $T = T^*$ and let $w \in V_{T+T^*}$, then by propositon 2.2 $w = e^(T + T)(\sum_{j\in\Lambda} a_j v_j) = e^{2T}(\sum_{j\in\Lambda} a_j v_j) = (e^T)^2(\sum_{j\in\Lambda} a_j v_j) = (I + T + T^2/2! + T^3/3! + \cdots)2(\sum_{j\in\Lambda} a_j v_j)$.

Proposition 2.4. *Let T and S be two bounded operators on V. If S and T are similar. Then V_{S+S^*} and V_{T+T^*} are isomorphic.*

Proof. Assume that T and S are similar, and i.e. there exist an invertible operator h on V, such that $hTh^(-1) = S$ [6], then $(hSh^(-1) = T).h$, this gives $h(S+S^*) = (T+T^*)h$. Since $hS = Th$, then $h(S+S^*)n = h(S+S^*)(S+S^*)n-1 = (T+T^*)h(S+S*)(S+S*)n-2 = (T+T*)(T+T*)h(S+S*)(S+S*)n-3 = .. = (T + T*)^n h$, then

$$he^{S+S^*} = e^{T+T^*}h. \tag{1}$$

Define $h' : V_{S+S^*} \longrightarrow V_{T+T^*}$ by

$$(e^{S+S^*}(v))h' = e^{T+T^*}(h(v)). \tag{2}$$

To prove h' is isomorphism we must prove:

1. h' is well defined

Let $e^{S+S^*}(v_1) = e^{S+S^*}(v_2)$ then $h(e^(S + S^*(v_1)) = h(e^{S+S^*}(v_2))($ Since h is well defined). Then by equation 1 we get $e^{T+T^*}(h(v_1)) = h(e^{S+S^*}(v_1)) = h(e^{S+S^*}(v_2)) = e^{T+T^*}(h(v_2))$, this give

$$e^(T + T^*(h(v_1)) = e^{T+T^*}(h(v_2)). \tag{3}$$

Then, by equations 2, 3 we get $(e^{S+S^*}(v_1))h' = (e^{S+S^*}(v_2))h'$. Thus h' is well defined.

2. h' is one to one.

Let $(e^{S+S^*}(v_1))h' = (e^{S+S^*}(v_2))h'$ Then by equation 2, we get $e^{T+T^*}(h(v_1)) = e^{T+T^*}(h(v_2))$ Then by equation 1 we get $h(e^{S+S^*}(v_1)) = h(e^{S+S^*}(v_2))$ But h is invertible then $h^(-1)h(e^{S+S^*}(v_1)) = h^(-1)h(e^{S+S^*}(v_2))$. This give $(e^{S+S^*}(v_1)) = (e^{S+S^*}(v_2))$, therefore h' is one to one.

3. h' is onto.

Let $e^T(v) \in V_T$ since $v \in V$ then $h^{-1}(v) \in V$ and $e^{S+S^*}(h^{-1}(v)) \in e^{S+S^*}$, then by equation 2 we get $(e^{S+S^*}(h^(-1(v))h' = e^{T+T^*}(h(h^{-1}(v)) = e^{T+T^*}(v)$, then h' is onto. Note that $(e^{S+S^*}(v))h' = e^{T+T^*}(h(v)) = h(e^{S+S^*}(v))$, thus $(e^{S+S^*}(v))h'=h(e^{S+S^*}(v))$ But h it is an operator (linear) on V, thus h is S-homomorphism,this give h' is S-homomorphism. Then h' is an S-isomorphism. □

Recall that the left S-act A_s is faithful if for $s, t \in S$ the equality $sa = ta$ for all $a \in A_s$, implies $s = t$. The relation between faithful S-act and bounded linear operator T have been explained in the following proposition.

Proposition 2.5. *For any bounded linear operator T then V_{T+T^*} is a faithful S-act.*

Proof. We want to show that V_{T+T^*} is a faithful S-act, for any bounded operator T. Let $e^{x_1+y_1}.e^{T+T^*}(v) = e^{x_2+y_2}.e^{T+T^*}(v)$. Since e^T is operator then e^T is linear transformation, this give $e^{x_1+y_1}.e^{T+T^*}(v) = e^{x_2+y_2}e^{T+T^*}(v) = e^{T+T^*}(e^{x_1+y_1}.v) = e^{T+T^*}(e^{x_2+y_2}.v)$. Since e^T is one to one, therefore e^{T+T^*} is one to one. Hence $e^{x_1+y_1}.v = e^{x_2+y_2}.v$, then $e^{x_1+y_1}.v - e^{x_2+y_2}.v = 0$. Then $(e^{x_1+y_1} - e^{x_2+y_2})v = 0$, thus $e^{x_1+y_1} = e^{x_2+y_2}$. Therefore $V_{T+T^*}$is faithful S-act. □

Remark 2.6. If V is a finite dimensional Banach space, then V_{T+T^*} is finitely generated S-act.

In [7], show that a subspace W of V is said to be an invariant subspace of V under T if $Tw \in W$ for all $w \in W$.

The following proposition shows under what condition the vector space V is finite dimension.

Proposition 2.7. *If T is one to one and onto and $V_T + T^*$ is finitely generated, then V is finite dimensional.*

Proof. Assume that V is not finite dimensional. Let $K(T) = \{w \in V | (T + T^*)w = 0\}$. It is clear that K is as invariant subspace of V(since $K \subseteq V$ and $\forall w \in K, (T + T^*)(w) = 0 but 0 \in K then (T + T^*)(K) \subseteq K$) and by the first isomorphic theorem of S-act,then $(T + T^*)V \cong V/K$ [1], since T is one to one then T^* is onto and T is onto then $(T + T^*)V = V$, therefore $V \cong V/K$. By assuming that V is not finite dimensional then either K is infinite dimensional or K is finite dimensional. K is an invariant subspace of V then we can consider K_{T+T^*}. If K is finite dimensional then K_{T+T^*} is finite generated, remark2.6,the subact K_{T+T^*} is generated by the set $\{e^{T+T^*}(w_j)|j \in \Lambda\}$ where $\{w_j|j \in \Lambda\}$ is a basis for K. But $w_j \in K$ given that $Tw_j = 0$, since the restriction of T on K is the zero operator O. Thus $K_{T+T^*} = K_{O+O^*}$, therefore K_{T+T^*} can not be finitely generated (see example 1 from 2.3, this contradiction. Hence K must be infinite dimensional. The subact K_{T+T^*} is generated by the set $\{e^{T+T^*}(w_j)|j \in \Lambda\}$ where $\{w_j|j \in \Lambda\}$ is a basis for K. But $w_j \in K$ given that $Tw_j = 0$, since the restriction of $T + T^*$ on K is the zero operator O. Thus $K_{T+T^*} = K_{O+O^*}$, therefore K_{T+T^*} can not be finitely generated (see example 1 from 2.3, but K_{T+T^*} is a subact of V_{T+T^*} and V_{T+T^*} is finitely generated. This mean infinitely generated contains in finitely generated. This contradiction shows that V is finite dimensional. □

Recall that an S-act A separated if for each $a \neq b$ in A there exists $s \neq e$ such that $sa \neq sb$ [1]. In [8], let M_s be an S-system and H a subset of S, then H is called reductive on M_s if and only if for each $a, b \in M_s$,$ah = bh$ for all $h \in H$ implies $a = b$,an singular relation ψ_M on M_s by the set $\{(a, b) \in M \times M|ah = bh$ for some $h \in H$ for some reductive subset H of $S\}$.

Proposition 2.8. *For any bounded linear operator T, if V_{T+T^*} is singular S-act then V is generated by one element.*

Proof. Since V_{T+T^*} is singular S-act, then

$$\psi_V=\{(e^{T+T^*}(v_1), e^{T+T^*}(v_2))\in V_{T+T^*}\times V_{T+T^*}|e^{x+y}e^{T+T^*}(v_1) = e^{x+y}e^{T+T^*}(v_2) \text{ for some } e^x \in H \text{ for some reductive subset } H \text{ of } S\}$$

then $e^{x+y}e^{T+T^*}(v_1) = e^{x+y}e^{T+T^*}(v_2)$ ..... (2-1). This gives $e^{T+T^*}(e^{x+y}v_1) = e^{T+T^*}(e^{x+y}v_2)$ since e^{T+T^*} is one to one, therefore $e^{x+y}v_1 = e^{x+y}v_2$ then $e^(x + y)v_1 + (-1)e^{x+y}v_2 = 0$ .....(2-2) but H is reductive subset of S, then by (2-1) find $e^{T+T^*}(v_1) = e^{T+T^*}(v_2)$ thus $v_1 = v_2$, we replies $v_1 = v_2$ on (2-2) then $e^{x+y}v_1 + (-1)e^{x+y}v_1 = 0$, therefore V is generated by one element this gives V is a finite dimensional. □

Recall that an S-act is separated if for each $a \neq b$ in A there exists $s \neq e \in S$, where e is the identity element such that $sa \neq sb$ [1]. In the following proposition, we explain the relationship between a bounded linear operator T and separated S-act.

Proposition 2.9. *For any bounded linear operator T then V_{T+T^*} is separated S-act.*

Proof. Let $a \neq b$ in V_{T+T^*} to prove V_{T+T^*} is separated we have to show that there exist s, e in S, $s \neq e$ where e is the identity element such that $sa \neq sb$. Assume $sa = sb$, $e \neq s \in S$, such that $s = e^{x+y}$, e is the identity element, $a, b \in V_{T+T^*}$,this gives $e^{x+y}.e^{T+T^*}(v_1) = e^{x+y}.e^{T+T^*}(v_2)$, $v_1, v_2 \in$

$V, x, y \in R$, since e^{T+T^*} is operator then e^{T+T^*} is linear transformation, this gives $e^{x+y}.e^{T+T^*}(v_1) = e^{x+y}.e^{T+T^*}(v_2)$, thus $e^{T+T^*}(e^{x+y}.v_1) = e^{T+T^*}(e^{x+y}.v_2)$, but e^{T+T^*} is one to one, then $e^{x+y}.v_1 = e^{x+y}.v_2$, hence $(v_1 - v_2)e^{x+y} = 0$, since $e^{x+y} \neq 0$. Then $v_1 = v_2$, this gives either $e^{T+T^*}(v_1) \neq e^{T+T^*}(v_2)$ or $e^{T+T^*}(v_1) = e^{T+T^*}(v_2)$, but if $e^{T+T^*}(v_1) \neq e^{T+T^*}(v_2)$, this give $v_1 \neq v_(2)$ this contradiction with $v_1 = v_2$, then $e^{T+T^*}(v_1) = e^{T+T^*}(v_2)$, means $a = b$ which a contradiction, then V_{T+T^*} is separated S-act. □

The converse of the proposition 2.9 have been study in the following proposition.

Proposition 2.10. *If V_{T+T^*} is separated S- act then T is one to one.*

Proof. Assume that V_{T+T^*} is separated, we want to prove T is one to one. let $v_1 \neq v_2$, we must prove $T(v_1) \neq T(v_2)$. since $v_1 \neq v_2$ then either $e^{T+T^*}(v_1) \neq e^{T+T^*}(v_2)$ or $e^{T+T^*}(v_1) = e^{T+T^*}(v_2)$. If $e^{T+T^*}(v_1) = e^{T+T^*}(v_2)$, this contradiction with $v_1 \neq v_2$, hence $e^{T+T^*}(v_1) \neq e^{T+T^*}(v_2)$, but V_{T+T^*} is separated S-act then $\exists e \neq s$, $s = e^{x+y} \in S$ such that $e^{x+y}.e^{T+T^*}(v_1) \neq e^{x+y}.e^{T+T^*}(v_2)$, since e^{T+T^*} is an operator, then e^{T+T^*} is linear transformation, and hence $e^{T+T^*}(e^{x+y}.v_1) \neq e^{T+T^*}(e^{x+y}.v_2)$, this means $(I + (T + T^*) + (T + T^{*2}/2! + (T + T^*)^3/3! + \cdots)(e^{x+y}.v_1) \neq ((I + (T + T^*) + (T + T^*)^2/2! + ((T + T^*)^3)/3! + \cdots)(e^{x+y}.v_2)$ we get, $e^{x+y}.v_1 + (T+T^*)(e^{x+y}.v_1) + (T+T^*)^2/2!(e^{x+y}.v_1) + ((T+T^*)^3)/3!(e^(x+y.v_1) + \cdots \neq e^{x+y}.v_2 + (T+T^*)(e^{x+y}.v_2) + (T+T^*)^2/2!(e^{x+y}.v_2 + ((T+T^*)^3/3!(e^{x+y}.v_2) + \cdots$ $e^{x+y}.v_1 \neq e^{x+y}.v_2$, this gives $v_1 \neq v_2$ and $(T+T^*)(e^{x+y}.v_1) \neq (T+T^*)(e^{x+y}.v_2)$, but $(T+T^*)$ is an operator, this gives $e^{x+y}.(T+T^*)(v_1) \neq e^{x+y}.(T+T^*)(v_2)$,since $e^{x+y} \neq 0$, then $(T+T^*)(v_1) \neq (T+T^*)(v_2)$, and since $(T+T^*)^2/2!(e^{x+y}.v_1) \neq (T+T^*)^2/2!(e^{x+y}.v_2)$, therefore $(T+T^*)/2!(T+T^*)(e^{x+y}.v_1) \neq (T+T^*)/2!(T+T^*)(e^{x+y}.v_2)$ this give $(T+T^*)(e^{x+y)}.v_1) \neq (T+T^*)(e^(x+y).v_2)$by using the same way, we get $(T+T^*)(v_1) \neq (T+T^*)(v_2)$. Then we proof $T+T^*$ is one to one, thus T is one to one. □

Recall that An act A_S is torsion free if for any $x, y \in A_S$, and for any right cancellable element $c \in S$, the equality $xc = yc$ this implies $x = y$ (see [1]). In the following proposition the relation between a bounded linear operator T and torsion free S-act have been explain.

Proposition 2.11. *For any bounded linear operator T then V_{T+T^*} is torsion free S-act.*

Proof. Assume $e^{x+y}e^{T+T^*}(v_1) = e^{x+y}e^{T+T^*}(v_2)$,$\forall e^{x+y}$ is cancellable element in S, this give $e^{T+T^*}(e^{x+y}.v_1) = e^{T+T^*}(e^{x+y}.v_2)$, since e^{T+T^*} is one to one, therefore $(e^{x+y}.v_1) = (e^{x+y}.v_2)$, then $v_1 = v_2$, thus either $e^{T+T^*}(v_1) = e^{T+T^*)}(v_2)$ or $e^{T+T^*}(v_1) \neq e^{T+T^*}(v_2)$, if $e^(T+T^*(v_1) \neq e^{T+T^*}(v_2)$, we get contradiction with $v_1 = v_2$, then $e^{T+T^*}(v_1) = e^{T+T^*}(v_2)$, this give V_{T+T^*} is torsion free. □

Recall that a monid S is right Noetherian if and only if it satisfies the ascending chain condition for right ideals, this mean for every ascending chain $K_1 \subseteq K_2 \subseteq K_3 \subseteq \cdots \subseteq K_n \subseteq K_{n+1} \subseteq \cdots$, of its right subacts, there exists $n \in N$ such that $K_n = K_{n+1} = \cdots$

Theorem 2.12 ([8])**.** *If S is Noetherian and A is finitely generated S-act then A is Noetherain S-act.*

Proposition 2.13. *Let V be a finite dimensional normed space and T is similar to any operator J from R to R then is Noetherian S-act if and only if S is Noetherian.*

Proof. Since V is finite dimensional then it is finitely generated S-act by remark 2.3, therefore is Noetherian S-act, by theorem 2.12. Let$K_1 \subseteq K_2 \subseteq K_3 \subseteq \cdots \subseteq K_n \subseteq K_{n+1}$ be any ascending sequence ideals of S, then it is a sequence of subacts of S_S denoted by S_J , where J any operator from R to R, since T is similar to J, then by proposition 2.5, V_{T+T^*} is isomorphic S_{J+J^*}, thus S_{J+J^*} is Noetherian S- act, therefore this sequence is finite, then S is Noetherian. □

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