OPTIMAL MATHEMATICAL MODEL OF DELIVERY ROUTING AND PROCESSING TIME OF LOGISTICS

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Abstract. As a newly-developing service industry that has a broad prospect and enormous market potential under the environment of electronic commerce, logistics is characterized by intellectualization, exibility, informatization, networking and automation, etc. However, with the rapid development of economy, clients requirements on logistics services are increasing gradually. Therefore, how to reasonably control delivery costs while still maintaining the high-standard quality and timeliness has become a new challenge for logistics. On the basis of the earliest and latest service time allowed by clients, the problem of time window constraints should be especially considered for the distribution problems of multiple batches, multiple species and small batch quantity. Due to the practical significance of studying the routing problem of vehicles with time windows, this study explored the key optimization problem of logistics as well as the problem of distribution routing selection and processing time according to characteristics of logistics under the environment of electronic commerce, using ant colony algorithm (ACA) and particle swarm optimization (PSO).

Keywords: Logistics, route optimization, mathematical model, ant colony algorithm, particle swarm optimization.

1. Introduction

With the popularization of Internet in peoples daily life as well as the increasing development of scientific technology, electronic commerce has become a more and more important part in our life. As the key of electronic commerce, commodity delivery is also developing vigorously [1-2]. The key of upgrading the level of the whole logistics industry is to improve the delivery efficiency and thus reduce delivery costs using scientific and reasonable delivery schemes. Functions of logistics include various contents, but the core is article delivery. Delivery refers to products delivery and collection according to requirements of target clients. Delivery flow includes products collection, subpackage, transportation and distribution. In order to provide the best services for clients as well as reduce delivery costs as much as possible at the same time, scheduling and optimization of delivery vehicles has been selected as the research emphasis [3-5].

Ant colony algorithm (ACA) was initially adopted to solve the famous Chinese traveling salesman problem (TSP) [6]. However, with the development of researches carried out by scientific researchers all over the world, ACA was gradually applied to other industries [7-8]; moreover, ACA has transformed from dealing with one-dimensional static optimization problems to multi-dimensional dynamic combination optimization problems, as well as from exploring in discrete type range to exploring in successive type range. Due to great changes of ACA in application spaces, the latest evolutionary optimization algorithm of ACA showed vigorous growth potential [9-10]. At present, the algorithm has been extensively applied to scheduling problems, network route problems, vehicle routing, robot field, image processing, electrical power system, data mining, fault diagnosis and controlling parameter optimization, etc. [11]. After the continuous improvement of worldwide academic researches in several years, particle swarm optimization (PSO) has achieved satisfactory results in dealing with optimization solutions of most continuous problems. During years of improvement, PSO has been successively introduced to a wide range of fields, such as biological medicine, combination optimization, communication network, control engineering, electron and electromagnetic field, finance, image processing, task scheduling and other engineering application problems [12-15].

On the basis of ACA, PSO and ACA combined PSO mixed algorithm, this study analyzed and solved the vehicle scheduling model with time windows, and results showed that the mixed algorithm was superior to single algorithm.

2. Construction of vehicle routing problem with time windows model

Fundamental assumptions

In order to abstract vehicle routing problem with time window (VRPTW) constraints [16] in reality as a mathematical model, following fundamental assumptions were established in this study: products delivery was one-way, i.e., pure products delivery; both the starting point and terminal point of every delivery routine should be the distribution center and there was only one distribution center; position coordinates of distribution center and clients points were known; vehicles types were the same and the maximum capacity of the vehicles was known; product weight should not exceed the rated loading capacity of the vehicle; demanded quantity of each client was known; each client was visited once and only once; each client had a specific service time window, thus the arrival time of vehicles should be within this time range; one vehicle for one routine; all routines were unhampered.

Penalty function

Former researches on delivery problems of vehicles with time window constraints only focused on the delivery costs. But in fact, violation of time window constraints of clients can result in certain economic losses which can be regarded as the time cost effect caused by violating time window constraints of clients. Therefore, while pursuing the minimization of overall costs, the time-effect costs should also be considered [24].

Costs caused by violation of time window should be expressed in an appropriate way to avoid neglecting intangible costs. Therefore, it can be regarded as the penalty cost function of the distribution center violating time window of clients, i.e., using functions to simulate time windows of clients. Reasonable penalty costs can lower costs of the distribution center as well as guarantee the service quality at the same time; usually the limit of penalty costs is determined by the balance between costs and clients degree of satisfactory. In another word, the more the arrival time window of vehicle deviating the time window constraint, the higher the penalty costs. To simplify the problem, we assume that penalty costs increase linearly.

Therefore, before the determination of penalty cost function, the presumption and hypothesis of penalty cost were as follows: penalty cost is expressed in function; the distribution center does not need to pay any penalty cost if the vehicle serves within the time window; the smaller the width of time window, the higher the marginal effect of penalty costs; the penalty cost increases linearly with the increase of penalty degree whether the vehicle arrives early or late. Function expression of penalty cost is as follow:

$$p_i(s_i) = \begin{cases} p(e_i - s_i), & s_i < e_i \\ 0, & e_i \le s_i \le l_i \\ q(s_i - l_i), & s_i > l_i. \end{cases} \tag{2.1}$$

The above function can also be unified as:

$$p_i(s_i) = p \max(e_i - s_i, 0) + q \max(s_i - l_i, 0). \tag{2.2}$$

In above equations, e_i refers to the starting point of service time window allowed by client i; l_i refers to the end point of client allowed service time window; s_i is the arrival time of the vehicle at client i; p refers to the waiting opportunity cost of unit time of the vehicle arriving at the location of client in advance; q means the penalty cost of unit time of vehicle arriving late than time window of client. If vehicle k arrives at the client node i before the time window e_i and the vehicle waits there, then the loss of opportunity cost is $p(e_i - s_i)$; if the vehicle arrives late than the time window and the service is delayed, then the delay cost is $q(s_i - l_i)$; if the vehicle arrives within the time window $[e_i; l_i]$, then the time effect cost is 0. Values of p and q are determined by practical situations. For some important clients or clients who have high requirements on time, then values of p and q can be big. Hard time window can be regarded as the soft time window, i.e., when the value of p and q is the infinitely-great positive number M, the soft time window penalty function can be corrected as hard time window and its penalty function expression is:

$$p_i(s_i) = M \max(e_i - s_i, 0) + M \max(s_i - l_i, 0). \tag{2.3}$$

Mathematical model

Suppose a distribution center provides delivery services for n clients and the client set was $C = \{1, 2, \dots, n\}$; the demanded quantity of each client was $q_i (i \in C)$; the distribution center distributed m same type vehicles and the vehicle set is $V = \{1, 2, \dots, n\}$; loading capacity is Q; d_{ij} is the distance between client i and j; P refers to the unit delivery cost on the distribution route; t_i is the time of vehicle arriving at client i; the number of clients served by the kth vehicle is N_k; the least number of clients served by each vehicle is p, and r_k^q refers to the serial number of the qth client in the client set served by the kth vehicle (r_0^k refers to the serial number of the distribution center); L_i refers to the delayed time of vehicle served for client p and P_1 is the late penalty factor; E_i is the advanced time of vehicle serving client p and P_i is the penalty factor of early wait; thus the mathematical model of VRPTW is:

$$
\begin{aligned}
F = \min \sum_{k=1}^{m} [P \cdot d_{r_k^0 r_k^1} + P_1 \cdot L_{r_k^1} + P_2 \cdot E_{R_k^1} \\
+ \sum_{i=1}^{N_k} \sum_{j=1}^{N_k} x_{ijk} (P \cdot d_{r_k^i r_k^j} + P_1 L_{r_k^i} + P \cdot d_{r_k^{N_k} r_k^0} + P_1 \cdot L_{r_k^0})].
\end{aligned} \tag{2.4}
$$

Constraint conditions are:

$$\sum_{i=1}^{N_k} q_{r_k^i} \leq Q \tag{2.5}$$

$$\sum_{k=1}^{m} \sum_{i=0}^{N_k} x_{ijk} = 1 \tag{2.6}$$

$$\sum_{k=1}^{m} \sum_{j=0}^{N_k} x_{ijk} = 1 \tag{2.7}$$

$$p \leq N_k \leq n \tag{2.8}$$

$$x_{ijk} = \begin{cases} 1, & \text{Vehicle } k \text{ from } r_k^i \text{ to } r_k^i \\ 0, & \text{others} \end{cases} \tag{2.9}$$

$$L_i = \begin{cases} t_i - b_i, & If t_i > b_i \\ 0, & \text{or} \end{cases} \tag{2.10}$$

$$E_i = \begin{cases} a_i - t_i, & \text{if } f a_i > t_i \\ 0, & \text{or} \end{cases} \tag{2.11}$$

$$k = 1, 2, \dots, m. \tag{2.12}$$

Vehicle k from r_i^k to r_j^k Equation (4) is the objective function of minimum delivery cost; equation (5) presents loading capacity constraint of the vehicle; equation (6) and (7) indicate that, every vehicle starts from the distribution center and each client is served by one vehicle and only for once; all vehicles returns to the distribution center after delivery; equation (8) shows the minimum number of clients served by each vehicle, which should not exceed the overall number of clients; equation (9), (10) and (11) are all decision variables.

3. Swarm intelligence algorithm

In VRPTW mathematical model which takes the minimum total distribution cost as the objective function, the penalty function with regard to the violation of client time window give consideration to the waiting opportunity cost generated because of the early arrival of vehicles and the delay penalty cost generated because of delayed arrival. In VRPTW model, besides the limitation of VRP, time window limits for demand points also need to be satisfied. With the increase of the number of clients and the complexification of constraint conditions, solution becomes increasingly complicated. The traditional accurate algorithm is not quite suitable to the solution of VRPTW as the calculated quantity will be in exponential growth with the increase of issue scale. Swarm intelligence algorithm suggests favorable search effect in discrete and continuous solution; routine selection mechanisms established through swarm intelligence behaviors can make the algorithmic search approximate to the optimal solution in the highest speed; it has become the research hotspot in the solution to VRPTW for its advantages of absence of centralized control, multi-agent mechanism, simple algorithm structure, implied parallelism, easy understanding and easy implementation. Ant colony algorithm and particle swarm optimization among swarm intelligence algorithm have high application values and development potentials.

Vehicle routing problem (VRP) is an important content of logistics system research, which can be transformed to TSP problem according to vehicle routing model. ACA shows significant achievements in dealing with famous problems like TSP; however, when it comes to large-scale problems, disadvantages like slow rate of convergence and long consumed time may occur.

PSO is a global optimizing algorithm which has following advantages [17-18]: ability of global research in a wide range; ability of researching from the colony, which has good stability; evaluation function values are used for inspiration while searching; fast rate of convergence and simple parameter adjustment; it has good extendibility which can be combined with other algorithms. Disadvantages of PSO include insufficient usage of feedback information and poor local search ability at later stage of algorithm.

This study analyzed and solved the constructed vehicle scheduling model with time window using ACA, PSO and ACA combined PSO algorithm.

Ant colony algorithm ACA is a kind of heuristic algorithm of swarm intelligence, which was put forward by Dorigo in his doctoral thesis in 1991 and applied to planning of delivery vehicle routing. ACA is a kind of bionics algorithm and its basic principle is to simulate the foraging of a colony of ants:

during the process of foraging, ants release certain amount of pheromone from their nest to the location where food is discovered as a kind of mark, which can be used by following ants to bypass obstacles to find better foraging routes; the disadvantage is that the algorithm may be caught in local optimum [19-20].

Basic steps of ACA are as follows:

The first step is the initialization of parameters. Initial pheromones are set between client routes.

The second step is to distribute ants randomly to different nodes and the current positions of ants are added to the search tabu table $tabu_k$.

The third step is to calculate the next client according to the equation and thus to determine the next client j.

Step four is to judge whether the loading capacity of the vehicle overpasses the maximum loading capacity; if so, step five is carried out; if not the third step is repeated.

Step five is to judge whether the service arrival time meets the requirements of clients; if so, j is added to $tabu_k$ and costs from position i to position j is calculated; if not, then j is added to $tabu_k$ and costs from position i to position j is calculated, including time penalty costs.

In step six, after all ants have moved once, new pheromones on each route are adjusted according to local pheromone adjusting formula.

Step seven is to check whether the tabu table is completely filled, i.e., whether all points are covered; if so, step 9 is carried out; if not, step three is repeated. Step seven is to check whether the tabu table tabuk is completely filled, i.e., whether all points are covered; if so, step 9 is carried out; if not, step three is repeated.

In step eight, the optimal path is updated.

In step nine, conditions of circulation termination are judged. If the number of circulation times is equal to or greater than the maximum iterations, the circulation is terminated and the optimal route and costs are output; if not, the tabu table tabuk is reset and the whole process is repeated from step two.

Particle swarm optimization

Inspired by foraging behaviors of birds, Kennedy and Eberhart in America put forward the PSO theory in 1995. The basic model of PSO is similar to the genetic algorithm and PSO is a kind of optimizing tool based on iteration. Compared with the genetic algorithm, particles can be updated by following two extreme values of particles in PSO and the optimal particle is obtained in iterative process to find the global optimum; however, in genetic algorithm, the optimal values can only be obtained through multi-step operation and multiple iterations such as crossover and variation. Therefore, the advantages of using PSO to solve VRP are that fewer parameters are considered and the algorithm is much simpler, etc. [21-23].

Steps based on PSO algorithm are as follows: In step one, particle position x_i^0 and v_0^i in n-dimensional space are initialized and the number of iterations is specified.

Every particle is evaluated in step two. Current adaptive values pi of all particles are obtained according to the fitness function.

In step 3, the obtained adaptive value p_i in step two was compared with the past optimal adaptive value pBest; if $p < pBest$, then the history optimal solution value of particle was pi; thus the former particle is replaced by the new particle, i.e., $p = pBest$.

In step 4, the history optimal adaptive values pBest of every particle are compared with the optimal value gBest of the whole particle swarm. If $gBest > pBest$, then the former global optimum adaptive value should be replaced by the optimal adaptive value of the new particle; meanwhile, the up-to-date state of the particle is recorded, i.e., $gBest = pBest$.

After the latest speed and position of the particle is calculated according to the adjusting formula, the particle is moved to a new position, thus a new particle comes into being. If the circulation times exceed the maximum iterations in step one or are lower than the expected convergence precision, then the algorithm running is ended and the final solution is output; or the whole process is repeated from step two.

ACA combined PSO

The initial pheromone information of ACA is random, thus there is no reasonable set; moreover, ACA has a slow rate of convergence. In PSO algorithm, when a local optimal solution is obtained after multiple times of circulation calculation of particles, speed change of particles mainly depends on the current flying speed of particles. In order to improve disadvantages of ACA and PSO, ants accomplish traversal once according to ACA, then appropriate adjustment is accomplished according to locally optimal solution and globally optimal solution [25-27]. Specific steps of mixed algorithm are as follows:

Step one: $nc \leftarrow 0$; refers to the number of iterations; after initialization, a large number of routes are generated and some good ones are chosen out of them; pheromones of those routes are left and m ants are placed at n peaks.

Step two: according to the current position, adaptive value Itsp 0 is calculated; suppose the current adaptive value is individual extremum Ptbest and the current position is individual extremum position pcbest; global extremum bgtest and global extremum position gcbest are discovered according to individual extremum Ptbest of each particle.

Step three: the initial starting point of each ant $k(k = 1, 2, \ldots, m)$ is placed in the current solution set; each ant can move to the next peak j according to the probability p_{ij}^k and the peak j is placed in the current solution set.

Step four: following operations are carried out for each ant. The j-th ant route $Co(j)$ intersects with gcbest to obtain $C_1'(j); C_1'(j)$ interacts with pbcest to obtain $C_1''(j)$ and $C_1''(j)$ mutates into $C_1(j)$; route length Itsp 1 is calculated according to current position; if the new target function is good, then the new value is accepted; otherwise, the route $C_1(j)$ of the j-th particle route changes to $Co(j)$. Individual extremum ptbest and extremum position of each ant are discovered again to find the global extremum bgtest and global extremum position gcbest.

Step five: route length $Lk(k = 1, 2, \ldots, m)$ of each ant is calculated and the optimal solution is recorded.

Step six: route length that is smaller than the set value is corrected according to the equation $\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t)$.

Step seven: $nc \leftarrow nc + 1$.

Step eight: if $nc <$ predetermined iterations and has no degradation behavior, i.e., the discovered solutions are all the same, then the procedure returns back to step 2.

Step nine: the current optimal solution is output.

Comparison of simulation results

Taking ei151, st70 and wi176 as examplse, matrix laboratory (MATLAB) is used to verify the effectiveness of algorithms. First, the second-best solution is obtained after 20 iterations using PSO; then based on the route length of the secondbest solution, distribution of initial pheromone in ACA is obtained according to $\tau_S = \tau_\varepsilon + \tau_\rho$; in ACA, $\rho = 0.02$, the number of ants is equal to the number of cities and $\alpha = 1.0$, $\beta = 5.0$ and $T_{min} = 0.0001$. Table 1 shows the comparison of solving capacity and time efficiency of PSO combined ant swarm (PSOAS) algorithm and basic ACA.

	The known optimal solution	ACA		PSO		PSOAS	
		The shortest route	Evolution algebra	The shortest route	Evolution algebra	The shortest route	Evolution algebra
eil51	426	429	823	430	152	429	152
st70	675	678	1034	679	235	678	235
ei176	538	545	1322	546	281	545	281

Figure 1: *
Table 1. Comparison of simulation experiment results.

4. Conclusion

To solve the optimal distribution route issue in logistic process, this study established VRPTW model and designed the PSOAS algorithm based on ACA and PSO and performed analysis and solution. The algorithm combined the advantages of both ACA and PSO. The simulation experiment suggested that, PSOAS algorithm was advantageous in the solution to VRPTW; it was superior to ant colony algorithm in the aspect of time efficiency and to particle swarm algorithm in the aspect of optimizing capability. But it also has deficiencies. For example, its universality is weak. More theoretical analyses are required to improve the performance and applicability of the algorithm and further investigate the optimization of distribution route involving multiple distribution centers and vehicle types. In conclusion, PSOAS algorithm is a brand-new heuristic algorithm which is of great significance to solve the route selection and distribution time issues in the field of logistics.

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References

[1] R. He, C. Ma, C. Ma et al, *Optimisation algorithm for logistics distribution route based on Prufer codes*, Int. J. Wireless Mobile Comput, 9 (2015), 205-210.

[2] Q. Fan, X.X. Nie, K. Yu et al., *Optimization of Logistics Distribution Route Based on the Save Mileage Method and the Ant Colony Algorithm*, Appl. Mech. Mater, (2013), 448-453:3683-3687.

[3] G.Q. Jiang, Y. Pan, F.Y. Hu et al., *Research on logistics distribution route based on genetic algorithm and ant colony optimization algorithm,* Compu. Eng. Appl, 2015, 758-762.

[4] J. Li, *Logistics Distribution Route Optimization Based on Chaotic Cuckoo Algorithm*, Adv. Mater. Res., 2014, 1049-1050:1681-1684.

[5] K.J. Xin, Z.Y. Qin, *Study on Logistics Distribution Route Optimization Based on Clustering Algorithm and Ant Colony Algorithm*, Open Cybernet. Syst. J., 9 (2015), 1245-1250.

[6] S.A.S. Alhamdy, A.N. Noudehi, M. Majdara, *Solving traveling salesman problem (TSP) using ants colony (ACO) algorithm and comparing with tabu searchsimulated annealing and genetic algorithm*, J. Appl. Sci. Res., (1) 2012, 434-440.

[7] S.R. Balseiro, I. Loiseau, J. Ramonet, *An Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows*, Comput. Operat. Res., 38 (2011), 954-966.

[8] S. Ghafurian, N. Javadian, *An ant colony algorithm for solving fixed destination multi-depot multiple traveling salesmen problems*, Appl. Soft Comput., 11 (2012), 1256-1262.

[9] S.K. Mustafa, Eren, G. Mesut et al., *A novel hybrid approach based on Particle Swarm Optimization and Ant Colony Algorithm to forecast energy demand of Turkey,* Energ. Convers. Manage., 53 (2012), 75-83.

[10] J.J.S. Chvez, J.W. Escobar, M.G. Echeverri, *A multi-objective Pareto ant colony algorithm for the Multi-Depot Vehicle Routing problem with Backhauls*, Int. J. Ind. Eng. Comput., 2016, 35-48.

[11] B. Xu, H.Q. Min, *Solving minimum constraint removal (MCR) problem using a social-force-model-based ant colony algorithm,* Appl. Soft Comput., 43 (2016), 553-560.

[12] S.H. Xu, X.D. Mu, D. Chai et al., *Multi-objective quantum-behaved particle swarm optimization algorithm with double-potential well and share-learning*, OPTIK, 127 (2016), 4921-4927.

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