STUDY ON THE SEQUENCE VOLATILITY OF FINANCIAL ASSETS BASED ON MARKOV CHAIN MONTE CARLO SIMULATION

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Abstract. In recent years, a new issue occurs in the financial academy and business circles, i.e., dynamics of the financial asset price and its volatility model. However, lots of problems in the financial asset price and its volatility model at present have made the motor behaviors of emergencies in the fitting financial market become difficult; whats more, the limitation of parameter estimation on the practical application of models can increase with the increase of model complexity. Estimation of financial temporal models is usually based on classical statistical methods, and the measuring standard of the volatility estimation model is calculated using the actual volatility of low-frequency data. Therefore, taking the price fluctuation of Shanghai Stock Exchange A-share index as an example, this study constructed a model and aimed to analyze the sequence volatility of financial assets based on Markov chain Monte Carlo simulation methods.

Keywords: Financial assets, Markov chain Monte Carlo method, volatility, GARCH Jump model.

1. Introduction

In recent years, under the influence of the increasingly innovating science and technology as well as the developing international financial market, the economic society is becoming more and more global. As a consequence, more and more financial products and the derived product financial market develop constantly, leading to the increasingly enhanced volatility of the financial market. This phenomenon can increase the complexity and risk of the financial market (Chaboud et al., 2010; Tadesse et al., 2016). At present, the most widely applied Bayesian statistical method belongs to the Markov chain Monte Carlo (MCMC) simulation method. On one hand, it considers the uncertainty of probability and predictive parameters, which can provide finite samples with precise inference.

In the meantime, it is efficient and flexible in dealing with complex non-standard parameter models, and it can have efficient statistical inference even under the condition that the parameters are restrained (Plotnikov and Shkarupa, 2015). On the other hand, as the Bayesian statistical method is becoming more and more mature, now there is a specific kind of Bayesian statistical software that is used for inference. At present, lots of researchers in China and abroad have carried out relevant studies as well.

For example, in 2012, Oliveira SC and Andrade MG (Oliveira and Andrade, 2012) introduced and explained how to use the MCMC technology to estimate the model parameters of item response theory (IRT). First of all, the basic principle of using MCMC method to estimate model parameters was introduced; secondly, general methods of using MCMC method to estimate model parameters, including Gibbs sampling, alternative sampling and Metropolis-Hastings algorithm, etc., were introduced; at last, the 2 parameters logistic (2PL) model of IRT was taken as an example to specifically introduce the algorithm process of using the M-H algorithm in Gibbs to estimate item parameters (1j2j). In 2012, Ketter W, Collins J and Gini M, et al. (Ketter et al., 2012) took the Standard Poor 500 share index, one of the most mature stock indexes worldwide, as the research object to astringently test the algorithm process of the construction of standard stochastic volatility model and the fat-tail stochastic volatility model based on Gibbs sampling using MCMC. In 2014, Agdas D, Davidson MT and Ellis RD (Agdas et al., 2014) used hybrid MCMC method to predict the permeability more reasonably according to multi-well test information and based on Bayesian method, thus to provide a new method for numerical simulation of fine reservoir. On the basis of MCMC simulation method, this study took the price fluctuation of Shanghai Stock Exchange A-share index as an example to explore the sequence volatility of financial assets.

2. MCMC method

MCMC method is a special kind of Monte Carlo method. In essence, Monte Carlo integrals are used in the Markoff process in Monte Carlo simulation (Nabok et al., 2011; Lynch, 2014). Besides, the traditional Monte Carlo simulation has the deficiency that only the static simulation is feasible; however, the introduction of the MCMC method can make up such deficiency and realize dynamic simulation. A Markov chain that can be used for enough time of simulation of a specific stable transition distribution $P(\theta|X)$ is constructed and the simulated current values are very close to the stable transition distribution $P(\theta|X)$; then based on the Markov chain, samples of $P(\theta|X)$ are obtained for statistical inference, which is the main idea of the MCMC method. Therefore, it is significantly important to construct a Markov chain that is stable to $P(\theta|X)$ using MCMC method. In following content, the MCMC sampling algorithm which is widely applied is mainly introduced (George et al., 2010; Tian et al., 2010; Ruggeri, 2015).

2.1 Gibbs sampling

The Gibbs sampling method is a special kind of MCMC algorithm as well as a kind of iteration sampling method based on conditional distribution. Therefore, the conditional distribution is required, especially full conditional distribution. The required conditional distribution is something like $\pi(x_T|x_{-T})$, in which $X_T = \{X_i, i \in T\}$, $X_T = \{X_i, i \notin T\}$ and $T \subset N = \{1, \ldots n\} P(\theta|X)$. All variables are included in the above conditional distributions.

2.2 Distribution conditions of MCMC

The MCMC method is mainly constructed on the conditional distribution $\pi(x_T|x_T)$, in which $x_T = \{x_i, i \in T\}$, $x_T = \{x_i, i \notin T\}$, $T \subset N = \{1, 2, \ldots N\}$. The definition of full conditional distribution is that, all variables in distribution have been included in all conditions in distribution. Such kind of distribution is defined as the full conditional distribution (Charalambous et al., 2011; Moeini et al., 2011; Trutschnig and Snchez, 2014; Chakraborty et al., 2012). For any given $x \in \psi$ and $T \in N$, there is

$$\pi(x_T|x_T) = \frac{\pi(x)}{\int \pi(x) dx_T} \alpha \pi(x). \tag{2.1}$$

In the equation, α means there may be a scale factor that is not related to x. Thus in the product term of $\pi(x)$, only the items that are related to are remained. If $x, x' \in \psi$, and $x_T = x_T$, the following equation holds.

$$\frac{\pi(x'_T|x'_{-T})}{\pi(x_T|x_{-T})} = \frac{\pi(x')}{\pi(x)}. \tag{2.2}$$

Generally, if y represents the observed data, $x = (\theta, \phi, z)$; is parameter, φ is hyperparameter, z refers to missed data, $p(y, (z|\theta))$ refers to the density function of observed data, $\pi(\theta|\varphi)$ is prior distribution and $\pi(\varphi)$ refers to the distribution of hyper-parameter φ; then $\pi(x)$ can be expressed as $\pi(x|y)$ and $\pi(x|y) \alpha p(y, (z|\theta) \pi(\theta|\phi) \pi(\phi)$.

2.3 Calculation of value at risk using MCMC method

WinBUGS is a new software package developed by the Public Health Institute of University of Cambridge, which is exclusively used in the MCMC method and can have Bayesian statistical inference(King et al., 2011; Zhao et al., 2011; Marano et al., 2014). On the basis of such software, the Gibbs sampling can be easily achieved in common models like hierarchical model, cross-over design model and latent variable model, etc. (Bradlow et al., 2012; Sendjaya et al., 2011; Liaw et al., 2010). Moreover, models can be described more directly through digraphs and then Gibbs sampling dynamic graphs with parameters can be presented. Therefore, the software was selected in this study for the calculation of MCMC method.

3. Empirical study on the return rate of Shanghai Stock Exchange A-share index

3.1 Data source

Data samples of the Shanghai Stock Exchange A-share index were share index closing quotation of every R of the China Securities Index Co., Ltd, with totally 1813 trading days from January 4th, 2008 to June 21st, 2015.

3.2 The calculation method of return rates

The calculation equation of the return rate of Shanghai Stock Exchange Ashare index was:

$$R1 = 100(InP_t - InP_{t1}) \tag{3.1}$$

Pt is the share index closing quotation at (t) time point; P_{t1} is the former share index closing quotation at $(t-1)$ time.

3.3 Descriptive statistics of the daily return rate of Shanghai Stock Exchange A-share index

Figure 1 shows that, the average value of daily return rates R1 of the Shanghai Stock Exchange A-share index was 0.052163, indicating that the daily return rates of the Shanghai Stock Exchange A-share index from January 4th, 2008 to June 21st, 2015 were generally JH. The skewness was -0.373126 which was smaller than zero and had left skewness. Kurtosis was 5.556129 which was bigger than 3, indicating that the daily return rates R1 of the Shanghai Stock Exchange A-share index was characterized by peak and fat tail. The value of Jarque-Bera statistics was 535.52196 and the significance probability P was smaller than 0.01, indicating that the daily return rates R1 of the Shanghai Stock Exchange A-share index was not normal distribution (Zellner, 2012; Garca et al., 2010; Vazquezleal et al., 2011).

4. Markov switching autoregressive model of the return rates of Shanghai Stock Exchange A-share index

In order to explore the structural changes of the daily return series of Shanghai Stock Exchange A-share index, the Chow Test was adopted to test whether the linear model of series had structural changes or not. The results of Chow test were below the 5% significance level, thus the linear model of series did have structural changes.

Due to the structural changes of the daily return rate R1, the daily return rate of Shanghai Stock Exchange A-share index was considered from the aspect of Markov switching. It could be confirmed from the observation of R1 curve that, the two states of Markov switching autoregressive model were high return rate and low return rate. The values of state variable were 1 and 2, thus the

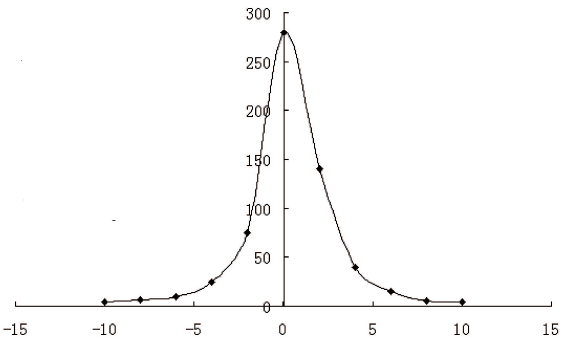

Figure 1: Descriptive statistics of the daily return rates R1 of the Shanghai Stock Exchange A-share index

Parameter	μ	ω	α	β	ν_1	τ	θ	δ^2
Rhat	1.0023	1.0213	1.0012	1.0053	1.0090	1.0090	1.0129	1.0011

Table 1: BGR deviation ratios of each parameter of the model

state variable St transformed between 1 and 2. The maximum likelihood estimation method was adopted in this study to estimate the parameters of Markov switching autoregressive model (Komprej et al., 2013; Zeebari et al., 2012; Murariu et al., 2011). Residual terms were analyzed respectively from the normal distribution and t-distribution aspects. According to the Akaike information criterion (AIC), it was confirmed that the number of lagged ranks was 4.

5. Analysis of GARCH-Jump model using MCMC method

The volatility of Shanghai Stock Exchange A-share index was verified based on the MCMC method. Prior distributions of the parameters of GARCH-Jump model were designed as: $\mu \sim N(0, 1.0E-6)$, $\omega \sim U(0, 10)$, $\alpha \sim U(0, 0.3)$, $\beta \sim U(0.8, 1)$, $\nu_1 \sim U(0, 1)$, $\tau \sim U(0, 0.5)$, $\theta \sim N(0, 1.0E-3)$ and $\delta^2 \sim IGamma(4, 0.06)$. Combing with the OpenBUGs software, two-chain setting was carried out to set different initial values to each chain; iterations were increased once and the convergence of model was judged mainly according to the autocorrelation coefficient map and BGR deviation ratio. In this verification, we discovered that it was a convergence model if 12000 times of Gibbs sampling were carried out; and in the former 5000 times of sampling, we adopted the random walk algorithm and took it as the combustion period (Kharol et al., 2012; Sharma et al., 2012); in the latter 7000 times, the Metropolis Hasting (MH) algorithm was used for sampling. Table 1 shows the BGR deviation ratios of each parameter of the model.

After certain times of iterations of the two Markov chains of each parameter, the autocorrelation coefficient approached to zero rapidly. In addition, according to table 1, we could easily find that the deviation ratio of each parameter was very close to 1. Therefore, we could regard that the model had achieved convergence. The posterior distribution images of each parameter were shown in figure 2.

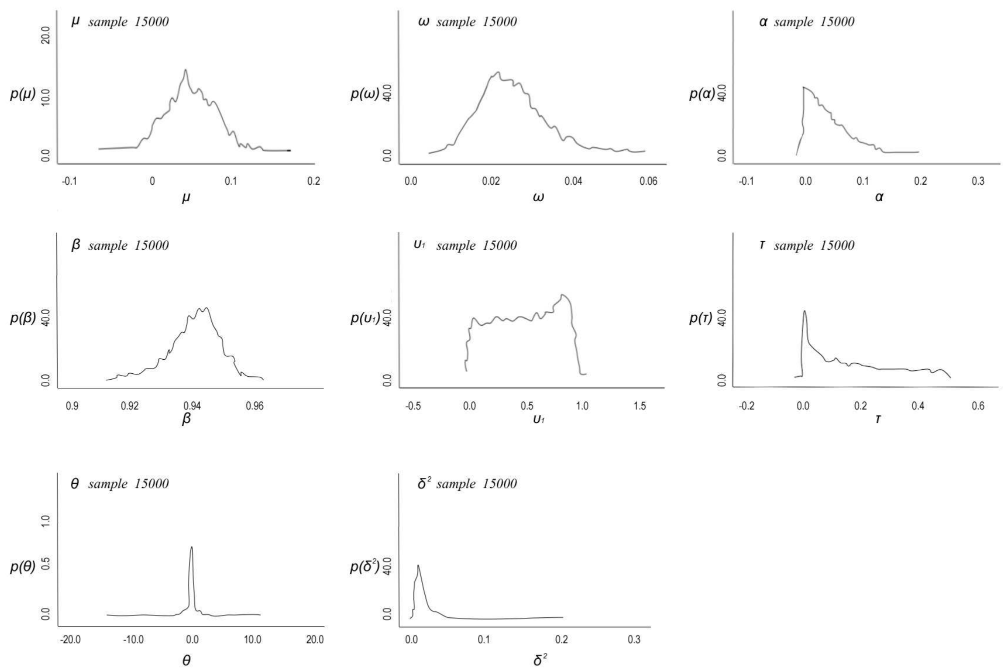

Figure 2: Posterior distribution images of each parameter

The experimental verification showed that, the four kinds of loss function values calculated based on ML algorithm were bigger than function values calculated using the MCMC method. Thus we could infer that, the models constructed using the MCMC method had better results. In order to further verify whether the loss function indexes had significant differences in these two kinds of methods, we used Durbin-Watson (DW) statistics to further verify these loss function indexes. In the meantime, we adopted relevant software to calculate DM values and other relevant values, as shown in table 2. This study showed that, the Shanghai Stock Exchange A-share index was very sensitive to the volatility of return rates, which showed significant jumping characteristic. Because the total volatility was the sum of smooth volatility h_1 and jumping volatility $\sigma_{t,2}^2$, according to estimation results of parameters of the GARCH-Jump model, it was calculated that the jump volatility accounted for 58% of the total volatility. Meanwhile, because $E(\lambda_t) = \tau * \omega/(1-\alpha-\beta)/(1-v_1)$, it

Loss function indexes	DW statistics	P value
MSE	-3.1621	0.0015
QLIKE	-27.0921	$< 2.2e-16$
MAE	-13.9416	$< 2.2e-16$
R2LN	-34.1524	$< 2.2e-16$

Table 2: DW values and some relevant values

was calculated that the average unconditional jump intensity was 0.4259, which verified that we could not ignore the jumping characteristic and the jumping volatility of stocks.

6. Conclusion

We discover from the experimental verification that, the GARCH-Jump model estimated based on MCMC method has better results. It can better simulate the volatility characteristics of Shanghai Stock Exchange A-share index. Abnormal fluctuation that occurs to stock market under the effects of events will last for a period and would not disappear in a short term. Stronger persistence of the fluctuation indicates the stronger capability of the predicted GARCH-Jump model in capturing the sequence information of financial events based on MCMC method and higher accuracy in describing the fat tail of the sequence of return on assets. The fluctuation of yield rate of single asset was fitted by a model, and the structural relationship between assets was described by Copula function. The risk value of asset portfolio was obtained based on the above results. Then VaR value was calculated using MCMC method. The risks were estimated, and the optimal decision was made to guide investors in the theoretical level. However, due to the lack of resources and conditions during experimental verification and data acquisition, this study still has some deficiencies to be improved in the future.

References

[1] A.P. Chaboud, B. Chiquoine, E. Hjalmarsson et al., *Frequency of observation and the estimation of integrated volatility in deep and liquid financial markets*, Journal of Empirical Finance, 17 (2010), 212-240.

[2] G. Tadesse, B. Algieri, M. Kalkuhl et al., *Drivers and triggers of international food price spikes and volatility,* Food Price Volatility and Its Implications for Food Security and Policy.Springer International Publishing, (2016),117-128.

[3] M.Y. Plotnikov, E.V. Shkarupa, *A combined approach to the estimation of statistical error of the direct simulation Monte Carlo method*, Computational Mathematics and Mathematical Physics, 55 (2015), 1913-1925.

[4] S.C. Oliveira, M.G. Andrade, *Comparison between the complete Bayesian method and empirical Bayesian method for ARCH models using Brazilian financial time series*, Pesquisa Operacional, 32 (2012), 293-313.

[5] W. Ketter, J. Collins, M. Gini, A. Gupta, P. Schrater, *Real-Time Tactical and Strategic Sales Management for Intelligent Agents Guided by Economic Regimes*, Information Systems Research, 23 (2012), 1263-1283.

[6] D. Agdas, M.T. Davidson, R.D. Ellis, *Efficiency Comparison of Markov Chain Monte Carlo Simulation with Subset Simulation (MCMC/ss) to Standard Monte Carlo Simulation (sMC) for Extreme Event Scenarios*, International Symposium on Uncertainty Modeling and Analysis and Management; and Fifth International Symposium on Uncertainty Modeling and Analysis, (2014), 86-95.

[7] D. Nabok, P. Puschnig, C. Ambrosch-Draxl, *An efficient implementation of van der Waals density functionals based on a Monte-Carlo integration technique*, Computer Physics Communications, 182 (2011), 1657-1662.

[8] T.B. Lynch, *Effects of measurement error on Monte Carlo integration estimators of tree volume: critical height sampling and vertical Monte Carlo methods,* Canadian Journal of Forest Research, 45 (2014), 463-470.

[9] K. George, K. Dmitri, R. Benjamin, T. Mark, *A limited-memory acceleration strategy for mcmc sampling in hierarchical bayesian calibration of hydrological models*, Water Resources Research, 46 (2010), 227-235.

[10] J. Tian, Y. Liang, J. Qian, *A New MCMC Particle Filter: Re-sampling Form the Layered Transacting MCMC Algorithm*, International Conference on Genetic and Evolutionary Computing. IEEE Computer Society, 2010, 75-78.

[11] P. Ruggeri, *Systematic evaluation of sequential geostatistical resampling within MCMC for posterior sampling of near-surface geophysical inverse problem*, Geophysical Journal International, 2015, 85-92.

[12] C.D. Charalambous, I. Tzortzis, F. Rezaei, *Stochastic Optimal Control of Discrete-Time Systems Subject to Conditional Distribution Uncertainty*, Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on. IEEE, (2011), 6407-6412.

[13] A. Moeini, B. Abbasi, H. Mahlooji, *Conditional Distribution Inverse Method in Generating Uniform Random Vectors Over a Simplex*, Communication in Statistics- Simulation and Computation, 40 (2011), 685-693.

[14] W. Trutschnig, J.F. Snchez, *Copulas with continuous, strictly increasing singular conditional distribution functions*, Journal of Mathematical Analysis and Applications, 410 (2014), 1014-1027.

[15] S. Chakraborty, E. Fischer, Y. Goldhirsh, A. Matsliah, *On the power of conditional samples in distribution testing*, Conference on Innovations in Theoretical Computer Science. ACM, 2012, 561-580.

[16] M.D. King, F. Calamente, A. Clark, *Markov Chain Monte Carlo Random Effects Modeling in Magnetic Resonance Image Processing Using the BRugs Interface to WinBUGS*, Gadian, D.G., Journal of Statistical Software, 44 (2011), 1-23.

[17] L. Hao, M.A. Morgan, L.A. Parsels, J. Maybaum, T.S. Lawrence, D. Normolle, *Bayesian hierarchical changepoint methods in modeling the tumor growth profiles in xenograft experiments*, Clinical Cancer Research An Official Journal of the American Association for Cancer Research, 17 (2011), 4613-4613.

[18] G. Marano, P. Boracchi, E.M. Biganzoli, *Estimation of a Piecewise Exponential Model by Bayesian P-splines Techniques for Prognostic Assessment and Prediction*, Computational Intelligence Methods for Bioinformatics and Biostatistics. Springer International Publishing, 2014, 183-198.

[19] E.T. Bradlow, A.M. Zaslavsky, *A Hierarchical Latent Variable Model for Ordinal Data from a Customer Satisfaction Survey with "No Answer" Responses*, Journal of the American Statistical Association, 94 (2012), 43-52.

[20] A. Moeini, B. Abbasi, H. Mahlooji, *Servant Leadership Behaviour Scale: A hierarchical model and test of construct validity*, European Journal of Work and Organizational Psychology, 20 (2011), 416-436.

[21] Y.J. Liaw, N.W. Chi, A. Chuang, *Examining the Mechanisms Linking Transformational Leadership, Employee Customer Orientation, and Service Performance: The Mediating Roles of Perceived Supervisor and Coworker Support,* Journal of Business and Psychology, 25 (2010), 477-492.

[22] A. Zellner, *Bayesian and Non-Bayesian Analysis of the Log-Normal Distribution and Log-Normal Regression*, Journal of the American Statistical Association, 66 (2012), 327-330.

[23] V.J. Garca, E. Gmez-Dniz, F.J. Vzquez-Polo, *A new skew generalization of the normal distribution: Properties and applications*, Computational Statistics and Data Analysis, 54 (2010), 2021-2034.

[24] H. Vazquezleal, R. Castanedasheissa, U. Filobellonino, A. Sarmientoreyes, J.S. Orea, *High Accurate Simple Approximation of Normal Distribution Integral*, Mathematical Problems in Engineering, 2012 (2011), 244-247.

[25] A. Komprej, S. Malovrh, G. Gorjanc et al., *Genetic and environmental parameters estimation for milk traits in Slovenian dairy sheep using random regression model*, Czech Journal of Animal Science, 58 (2013), 125-135.

[26] Z. Zeebari, G. Shukur, B.M.G. Kibria, *Modified Ridge Parameters for Seemingly Unrelated Regression Model*, Communication in Statistics-Theory and Methods, 41 (2012), 1675-1691.

[27] G. Murariu, A. Caldararu, L. Georgescu, M. Voiculescu, G. Puscasu, A. Basset, *Investigation of Water Parameters in a River System with a Two-Dimensional Regression Analysis Model*, Asia Pacific Media Educator, 1387 (2011), 619-643.

[28] S.K. Kharol, K.V.S. Badarinath, A.R. Sharma, D.V. Mahalakshmi, D. Singh, K. Vadrevu, *Black carbon aerosol variations over Patiala city*, Punjab, IndiaA study during agriculture crop residue burning period using ground measurements and satellite data, Journal of Atmospheric and Solar-Terrestrial Physics, 84-85 (2012), 45-51.

[29] D. Sharma, D. Singh, S. Kumar, *Case studies of black carbon aerosol characteristics during agriculture crop residue burning period over Patiala*, Punjab, India using the synergy of ground based and satellite observations, 2 (2012), 2277-2081315.

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