A VIEW ON QUASI λ-OPEN M-SETS IN M-TOPOLOGICAL SPACES

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Abstract. In this article the concepts of Λ-open M-sets, λ-open M-sets, λ-continuous M-set functions, quasi λ-open M-set functions and λ-irresolute M-set functions are introduced and studied. Also, their properties and characterizations are discussed.

Keywords: Λ-open M-sets, λ-open M-sets, λ-continuous M-sets functions, quasi λ-open M-set functions and λ-irresolute M-set functions.

1. Introduction

The concept of multiset have tremendous applications both in Mathematics and Computer Science. Girish and sunil Jacob John [8] introduced the concepts of M-topological spaces and open M-sets. In this article the authors studied many interesting topological properties via M-sets. As an application to M-topological spaces Chakrabarty et al. [6] studied n^k-bags via rough sets. Later on Amudhambigai B and Revathi G.K. et al. [1,2,3] contributed many articles in M-topological spaces. Maki [10] introduced the notion of Λ-sets in topological spaces. Arenas et al. [4] introduced and investigated the notion of λ-sets by involving Λ-sets and closed sets. Caldas et al. [5] introduced the notion of λ-closure of a set by utilizing the notion of λ-open sets defined in [4]. A new class of functions called slightly λ-continuous function has been defined and studied in topological spaces by Duraisamy and Vennila [7].

In this article the concepts of Λ-open M-sets, λ-open M-sets, λ-continuous M-set functions, quasi λ-open M-set functions and λ-irresolute M-set functions are introduced and studied. Also, their properties and characterizations are discussed.

2. Preliminaries

Definition 2.1 ([8])**.** Let $[X]^w$ be an M-set space and $\{M_1, M_2, \ldots\}$ be a collection of M-sets drawn from $[X]^w$. Then the following operations are possible under an arbitrary collection of M-sets.

a. The union $\prod_{i\in I} M_i = \{C_{M_i}(x)/x : C_{M_i}(x) = \max\{C_{M_i}(x) : x \in X\}\}$.

b. The intersection $I_{i\in I} = \{C_{\cap M_i}(x)/x : C_{\cap M_i}(x) = \min\{C_{M_i}(x) : x \in X\}\}$.

c. The M-set addition $\bigoplus_{i\in I} M_i = \{C_{\oplus M_i}(x)/x : C_{\oplus M_i}(x) = \sum_{i\in I} C_{M_i}(x), x \in X\}$.

d. M-set complement $M^c = Z \ominus M = \{C_{M^c}(x)/x : C_{M^c}(x) = C_z(x) - C_M(x), x \in X\}$.

Definition 2.2 ([8])**.** An M-set relation f is called an *M-set function* if for every element m/x in $Domf$, there is exactly one n/y in $Ranf$ such that $(m/x, n/y)$ is in f with the pair occurring as the product of $C_1(x, y)$ and $C_2(x, y)$.

Definition 2.3 ([8])**.** Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a *Multiset topology* of M if τ satisfies the following properties:

a. The M-set M and the empty M-set ϕ are in τ.

b. The M-set union of the elements of any sub collection of τ is in τ.

c. The M-set intersection of the elements of any finite sub collection of τ is in τ.

Definition 2.4 ([8])**.** A sub M-set N of an M-topological space M in $[X]^w$ is said to be closed if the M-set $M \ominus N$ is open. i.e., $N^c = M \ominus N$.

Definition 2.5 ([8])**.** Let M and N be two M-topological spaces. The M-set function $f : M \to N$ is said to be continuous if for each open sub M-set V of N, the M-set $f^{-1}(V)$ is an open sub M-set of M, where $f^{-1}(V)$ is the M-set of all points m/x in M for which $f(m/x) \in^n V$ for some n.

Definition 2.6 ([5])**.** Let B be a subset of a space (X, τ). B is a Λ-set (resp. $\vee$-set) if $B = B^\Lambda$ (resp. $B = B^\vee$), where $B^\Lambda = \cap\{U | U \supseteq B, U \in \tau\}$ and $B^\vee = \cup\{F | B \supseteq F,\ F^C \in \tau\}$.

Definition 2.7 ([7])**.** A subset A of a topological space (X,τ) is said to be λ-closed if $A = B \cap C$, where B is a Λ-set and C is a closed set of X. The complement of λ-closed set is called λ-open.

3. On quasi λ-open M-sets in M-topological spaces

Definition 3.1. Let (M,τ) be an M-topological space. Any sub M-set B of M is said to be a Λ-M-set if $B = B^{\Lambda}$ with $C_B(x) = C_{B^{\Lambda}}(x)$, for all $x \in X$, where $B^{\Lambda} = \cap\{U|U \supseteq B,\, U \in \tau\}$ with $C_{B^{\Lambda}}(x) = \min\{C_U(x) : U \supseteq B,\, U \in \tau\}$ and $B_{\vee} = \cup\{F|B \supseteq F, FC \in \tau\}$ with $C_B(x) = \max\{C_F(x) : B \supseteq F,\, F^C \in \tau\}$, for all $x \in X$.

Example 3.1. Let $X = \{a,b,c\}$, $w = 3$ and $M = \{2/a, 3/b, 1/c\}$. Let $\tau = \{M, \phi, \{2/a\}, \{2/a, 3/b\}\}$. Here, τ is an M-topology and (M,τ) is an M-topological space. Let $B = \{2/a\}$ be a sub M-set of M, then $B^{\Lambda} = \{2/a\}$ with $C_B(a) = 2$, $C_{B^{\Lambda}}(b) = 0$ and $C_{B^{\Lambda}}(c) = 0$. Also $C_B(a) = 2$, $C_B(b) = 0$ and $C_B(c) = 0$. Therefore, $C_B(x) = C_{B^{\Lambda}}(x)$, for all $x \in X$. Thus, $B = \{2/a\}$ is a Λ-M-set.

Definition 3.2. Let (M,τ) be an M-topological space. Any sub M-set A of an M-topological space (M,τ) is said to be a λ-closed M-set if $A = B \cap C$ with $C_A(x) = C_B \cap C_C(x)$, for all $x \in X$, where B is a Λ-M-set and C is a closed M-set of (M,τ).

Definition 3.3. Let (M,τ) be an M-topological space. Any sub$M$-set A of an M-topological space (M,τ) is called λ-open M-set, if $A^C = M \ominus A$ is λ-closed M-set and also $C_{A^c}(x) = W \ominus C_A(x)$, for all $x \in X$.

Example 3.2. From Example 3.1, clearly (M,τ) is an M-topological space. Here, the collection of all Λ-M-sets of (M,τ) is $\{M, \phi, \{2/a\}, \{2/a, 3/b\}\}$. Let $B = M$ be any Λ-M-set in (M,τ) and $C = \{3/b, 1/c\}$ be any closed M-set in (M,τ). Now, $A = B \cap C = \{3/b, 1/c\}$. Also, $C_A(x) = C_{B\cap C}(x)$, for all $x \in X$. Thus, $A = \{3/b, 1/c\}$ is a λ-closed M-set and $\{2/a\}$ is a λ-open M-sets of (M,τ).

Definition 3.4. Let (M,τ) be an M-topological space and A be any sub M-set of M. Then the λ-interior of A and λ-closure of A is respectively denoted and defined as follows.

1. $Int_{\lambda}(A) = \cup\{U|U \subseteq A, \text{each } U \subseteq M \text{ is a } \lambda\text{-open } M\text{-set}\}$ with $C_{int_{\lambda}(A)}(x) = \max\{C_U(x) : U \subseteq A, \text{ each } U \subseteq M \text{ is a } \lambda\text{-open } M\text{-set }\}$ for all $x \in X$.
2. $cl_{\lambda}(A) = \cap\{G|A \subseteq G, \text{each } G \subseteq M \text{ is a } \lambda\text{-closed } M\text{-set }\}$ with $C_{cl_{\lambda}(A)}(x) = \min\{C_G(x) : A \subseteq G, \text{ each } G \subseteq M \text{ is a } \lambda\text{-closed } M\text{-set}\}$, for all $x \in X$.

Example 3.3. From Example 3.1, clearly (M,τ) is an M-topological space. Let $A = \{2/a, 1/b\}$ be a sub M-set of M. Then, $Int(A) = \{2/a\}$ with $C_{Int_{\lambda}(A)}(x) =$

$\max\{C_U(x) : U \subseteq A$, each $U \subseteq M$ is a λ-open M-set$\}$, for all $x \in X$ and $Cl(A) = \{2/a, 3/b\}$ with $Cl_{cl_\lambda(A)}(x) = \min\{C_G(x) : B \subseteq G$, each $G \subseteq M$ is a λ-closed M-set$\}$, for all $x \in X$.

Definition 3.5. Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a λ-continuous M-set function if $f^{-1}(V)$ is a λ-open M-set in (M, τ) for every open M-set V of (N, σ).

Example 3.4. Let $X = \{a, b, c\}$, $w_1 = 3$ and $Y = \{x, y, z\}$, $w_2 = 3$. Let $M = \{2/a, 3/b, 1/c\}$ and $\tau = \{M, \phi, \{2/a\}, \{2/a, 3/b\}\}$. Here, τ is an M-topology and (M, τ) is an M-topological space. Let $N = \{3/x, 1/y, 2/z\}$ and $\sigma = \{N, \phi, \{1/y\}\}$. Then, σ is an M-topology and (N, σ) is an M-topological space. Let the function $f : (M, \tau) \to (N, \sigma)$ be defined by $f : \{(2/a, 1/y)/2, (3/b, 2/z)/6, (1/c, 3/x)/3\}$. Clearly $f^{-1}(V)$ is a λ-open M-set in (M, τ) for every open M-set V of (N, σ). Thus, f is λ-M-set continuous.

Notation 3.1.

a. The collection of all open sub M-sets of (M, τ) is denoted by $O(M)$ and $O(M, x) = \{V \in O(M) | x \in mV\}$, for $x \in^m M$.

b. The collection of all λ-open sub M-sets of (M, τ) is denoted by $O(M)$ and $O(M, x) = \{V \in \lambda O(M) | x \in^m V\}$, for $x \in^m M$.

Proposition 3.1. *Let (M, τ) and (N, σ) be any two M-topological spaces. For an M-set function $f : (M, \tau) \to (N, \sigma)$, the following statements are equivalent:*

a. f is a λ-continuous M-set function;

b. For every open sub M-set V of (N, σ), $f^{-1}(V) \in \lambda O(M)$;

c. For each $x \in^m M$ and each $V \in O(N, f(x))$, there exists a $U \in \lambda O(M, x)$ such that $f(U) \subseteq V$ with $C_{f(U)}(x) \leq C_V(x)$, for all $x \in X$.

Definition 3.6. Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a quasi λ-open (resp., λ- closed) M-set function if for each λ-open (resp., λ- closed) sub M-set V of M, its image $f(V)$ is an open (resp., closed) sub M-set of (N, σ). That is, $f(V)$ is the M-set of all points n/x in N for $x \in^m V$ for some m.

Example 3.5. In Example 3.4, clearly, the image of every λ-open M-set is an open M-set in (N, τ). Therefore, f is a quasi λ-open M-set function. Consequently, the image of every λ-closed M-set is a closed M-set in (N, σ). Hence f is a quasi λ-closed M-set function.

Proposition 3.2. *Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a quasi λ-open M-set function if and only if for every sub M-set U of M, $f(Int_\lambda(U)) \subseteq Int((U))$ with $C_{f(Int_\lambda(U))}(x) \leq C_{Int(f(U))}(x)$, for all $x \in X$.*

Proposition 3.3. *Let (M, τ) and (N, σ) be any two M-topological spaces.*

If $f : (M, \tau) \to (N, \sigma)$ is a quasi λ-open M-set function, then $Int_\lambda((G)) \subseteq (Int(G))$ with, for every sub M-set G of N and $x \in X$.

Proposition 3.4. *Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a quasi λ-open M-set function if and only if for any sub M-set B of N and for any λclosed M-set F of M containing $f^{-1}(B)$ there exists a closed M-set G of N containing B such that $f^{-1}(G) \subseteq F$ with $C_{f^{-1}(G)}(x) \leq C_F(x)$, for all $x \in X$.*

Proposition 3.5. *Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a quasi λ-open M-set function if and only if $f^{-1}(Cl(B)) \subseteq Cl_\lambda((B))$ with $C_{f^{-1}(cl(B))}(x) \leq C_{cl_\lambda(B)}(x)$, for every sub M-set B of N and for all $x \in X$.*

Proposition 3.6. *Let (M, τ), (N, σ) and (P, η) be any three M-topological spaces and $f : (M, \tau) \to (N, \sigma)$ and $g : (N, \sigma) \to (P, \eta)$ be two M-set functions and $g \circ f : (M, \tau) \to (P, \eta)$ is a quasi λ-open M-set function. If g is M-set continuous injective, then f is quasi λ-open M-set function.*

Definition 3.7. Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be λ-closed M-set function if for each λ-closed M-set U in (M, τ), $f(U)$ is a λ-closed M-set in (N, σ).

Definition 3.8. Let (M, τ) and (N, σ) be any two M-topological spaces. An M-set function $f : (M, \tau) \to (N, \sigma)$ is said to be a λ-irresolute M-set function if $f^{-1}(V)$ is a λ-closed M-set (resp. λ-open M-set) in (M, τ) for every λ-closed M-set V (resp. λ-open M-set) of (N, σ).

Proposition 3.7. *Let (M, τ), (N, σ) and (P, η) be any three M-topological spaces and $f : (M, \tau) \to (N, \sigma)$ and $g : (N, \sigma) \to (P, \eta)$ be any two M-set functions. Then:*

a. *If f is a λ-closed M-set function and g is a quasi λ-closed M-set function, then $g \circ f$ is a quasi λ-closed M-set function;*

b. *If f is a λ-irresolute M-set function and g is a quasi λ-closed M-set function, then $g \circ f$ is a λ-continuous M-set function;*

c. *If f is a λ-closed M-set function and g is a λ-continuous M-set function, then $g \circ f$ is a quasi λ-closed M-set function.*

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