PSEUDO-TOPOLOGICAL HYPERVECTOR SPACES AND THEIR PROPERTIES

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Abstract. In this paper we introduce and study the concepts of a pesudo-topological hypervector space and strongly pseudo-topological hypervector space. Further, we define a regular equivalence relation on a hypervector space and construct a new hypervector space with respect to this regular equivalence relation. Finally, by using a topology on the power set of a hypervector space, we introduce a topological hypervector space and investigate some of its properties.

Keywords: Hypervector space, pseudo-continuous, strongly pseudo-continuous, psudo-topological hypervector space, strongly pesudo-topological hypervector space, regular equivalence relation.

1. Introduction and preliminaries

Hyperstructures theory was initially introduced in 1934 by Marty [10]. He defined hypergroups that are a generalization of groups and studied some properties and applications of them in non-commutative groups, rational functions and algebraic functions. In the recent years, various aspects of hyperstructures are studied [3, 4]. Especially, many connections between hyperstructures and topological concepts has been established and investigated. For example, the notions of topological hypergroups, topological hypergroupoids, fuzzy pseudo-topological hypergroupoids and intuitionistic fuzzy topological polygroups were introduced and studied in [1, 2, 5, 6, 7, 8, 9].

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In 1998, the notion of hypervector spaces was initially introduced and studied by Tallini [13, 15]. Also some properties of hypervector spaces were studied by Raja and Taghavi in [11, 12].

In this paper we study some new structures. The organization of this paper is as follows:

In section 2; we introduce the concepts of a pesudo-topological hypervector space and strongly pseudo-topological hypervector space and study some of their properties. Further, we introduce a regular equivalence relation on a hypervector space and construct a new hypervector space with respect to this regular equivalence relation.

In section 3, by using a topology on the power set of a hypervector space, we present a new topological hypervector space and investigate some of its properties.

We begin with the following definitions which are used in our paper.

Definition 1.1. A hypervector space over a field K is a quadruplet $(V, +, \circ, K)$, that $(V, +)$ is an abelian group and

$$\circ : K \times V \to \mathcal{P}^*(V)$$

is a mapping of $K \times V$ into the power set of V (deprived of the empty set), such that the following properties holds:

$$(a+b)\circ x \subseteq (a\circ x)+(b\circ x), \quad \forall a,b \in K,\ \forall x \in V, \tag{1.1}$$

$$a\circ(x+y) \subseteq (a\circ x)+(a\circ y), \quad \forall a \in K,\ \forall x,y \in V, \tag{1.2}$$

$$a\circ(b\circ x) = (ab)\circ x, \quad \forall a,b \in K,\ \forall\ x \in V, \tag{1.3}$$

$$x \in 1\circ x, \quad \forall x \in V. \tag{1.4}$$

$$a\circ(-x) = -a\circ x, \quad \forall a \in K,\ \forall x \in V. \tag{1.5}$$

If in (1.1) the equality holds, the hypervector space is called strongly left distributive. If in (1.2) the equality holds, the hypervector space is called strongly right distributive. The distributive hypervector spaces were studied in [13].

Let V be a hypervector space over the field K; $H \subseteq V$ is a subspace of V, if

$$H \neq \phi,\ H - H \subseteq H,\ a\circ H \subseteq H,\ \ \forall a \in K.$$

Definition 1.2. Let $(V, +, \circ, K)$ and $(V', +', \circ', K)$ be two hypervector spaces over the same field K. A mapping

$$f : V \to V'$$

is called a homomorphism between V and V', if $\forall a \in K,\ \ \forall x, y \in V$:

$$f(x+y) = f(x) +' f(y), \tag{1.6}$$

$$f(a\circ x) \subseteq a\circ' f(x). \tag{1.7}$$

If in (1.7) the equality holds, then f is called strong homomorphisms.

Theorem 1.3 ([14]). *Let V and W be hypervector spaces. If H is a subspace of V, then $f(H)$ is a subspace of W. In particular Imf is a subspace of W, conversely if H' is a subspace of W, then $f^{-1}(H')$ is a subspace of V.*

Throught out of the paper, the ground field of a hypervector space V is presented with K, This field is usually considered by $\mathbb{R}$ or $\mathbb{C}$.

2. Pseudo-topological hypervector spaces

In this section, the concepts of a pesudo-topological hypervector space and strongly pseudo-topological hypervector space are introduced and some of their properties are studied.

Definition 2.1. Let $(V, +, \circ, K)$ be a hypervector space and (V, τ) be a topological space. The mapping $\circ$ is called:

i) pseudo-continuous, or for short p-contiuous, if for every open set U (member of τ), the set

$$U^* = \{(k, x) \in K \times V \mid k \circ x \subseteq U\}$$

is open in $K \times V$, with respect to the product topology on $K \times V$.

ii) Strongly pseudo-continuous, or for short strongly p-contiuous, if for every open set U (member of τ), the set

$$U_* = \{(k, x) \in K \times V \mid k \circ x \cap U \neq \phi\}$$

is open in $K \times V$, with respect to the product topology on $K \times V$.

Definition 2.2. Let $(V, +, \circ)$ be a hypervector space over the field K and also (V, τ) be a topological space endowed with a topology τ. V is called:

i) Pseudo topological hypervector space (or for short p-topological hypervector space), if the mapping $+ : V \times V \to V$ is continuous and the mapping $\circ : K \times V \to \mathcal{P}^*(V)$ is p-contiuous.

ii) Strongly pseudo topological hypervector space (or for short strongly p-topological hypervector space), if the mapping $+ : V \times V \to V$ is continuous and the mapping $\circ : K \times V \to \mathcal{P}^*(V)$ is strongly p-contiuous.

Example 2.3. Let $V = \mathbb{R}$. Define

$$\circ : \mathbb{R} \times \mathbb{R} \to \mathcal{P}^*(\mathbb{R})$$

$$a \circ x = \{ax\},$$

for all $a, x \in \mathbb{R}$. If $\mathbb{R}$ is endowed with the discrete topology, then for all $U \in \mathbb{R}$,

$$U^* = \{(a, x) \ : \ a \circ x \subseteq U\},$$

is an open set in $\mathbb{R} \times \mathbb{R}$ with respect to the product topology. So $(\mathbb{R}, +, \circ, \mathbb{R})$ is a p-topological hypervector space.

Example 2.4. Let $V = \mathbb{R}^2$. Define

$$\circ : \mathbb{R} \times \mathbb{R}^2 \to \mathcal{P}^*(\mathbb{R}^2)$$

Such that for all $a \in \mathbb{R}$ and $(x, y) \in \mathbb{R}^2$,

$$a \circ (x, y) = \begin{cases} \{(ax, ay)\} & a \neq 0 \\ \{(0, 0)\} & a = 0 \end{cases},$$

Let $\mathbb{R}$ be endowed with the discrete topology, $\mathbb{R}^2$ be endowed with the standard topology and $U = B_r(x', y')$ ($B_r(x', y')$ is an open ball in $\mathbb{R}^2$ centered at a point (x', y'), with radius $r > 0$). So we have

$$U^* = \{(a, (x, y)) \ : \ a \circ (x, y) \subseteq U\}.$$

If $(0, 0) \in U$, then

$$U^* = \bigcup_{a \neq 0} \{a\} \times B_{\frac{r}{|a|}}(\frac{x'}{a}, \frac{y'}{a}) \cup \{0\} \times \mathbb{R}^2,$$

that is open set in $\mathbb{R} \times \mathbb{R}^2$ with respect to the product topology, if $(0, 0) \notin U$, then

$$U^* = \bigcup_{a \neq 0} \{a\} \times B_{\frac{r}{|a|}}(\frac{x'}{a}, \frac{y'}{a}),$$

that U^* is an open set in $\mathbb{R} \times \mathbb{R}^2$. So $(\mathbb{R}^2, +, \circ, \mathbb{R})$ is a p-topological hypervector space.

Remark 2.5. Let H be a subspace of hypervector space $(V, +, \circ)$. Then $(H, +, \circ|_H)$ is obviously a hypervector space.

Theorem 2.6. *Let V be a p-topological hypervector space and H be an open subspace of V. Then H is a p-topological hypervector space with respect to subspace topology.*

Proof. According to Remark 2.5, H is a hypervector space. Also $+ : H \times H \to H$ is obviously continuous. Now we show that $\circ : K \times H \to \mathcal{P}^*(H)$ is p-continuous. Let U_H be an open set in H. So $U_H = U \cap H$ where U is an open set in V. Since V is a p-topological hypervector space, thus

$$U^* = \{(k, x) \in K \times V \mid k \circ x \subset U\}$$

is an open set in $K \times V$. It is enough to show that $U_H^* = U \cap (K \times H)$. Suppose that $(k, h) \in U_H^* \iff$

$$k \circ h \subset U_H = U \cap H \Leftrightarrow (k, h) \in U^* \Leftrightarrow (k, h) \in U^* \cap (K \times H). \qquad \square$$

Theorem 2.7. *Let $(V, +, \circ)$ be a p-topological hypervector space and $(V', +', \circ')$ be a hypervector space endowed with topology τ, $f : V \to V'$ be a strong homomorphism that is open and continuous and H be an open subspace of V. Then the following statements are valid:*

i) *$f(H)$ is a p-topological hypervector space.*

ii) *$Im(f)$ is a p-topological hypervector space.*

iii) *If f is onto, then V' is a p-topological hypervector space.*

Proof. i) According to Theorem 1.3, $f(H)$ is a subspace of V'. Since f is a continuous strong homomorphism. It can be easily seen that the map $+ : f(H) \times f(H) \to f(H)$ is continuous.

Suppose that U is an open set in $f(H)$. It is enough to show that

$$U_* = (id_K \times f)((f^{-1}(U))_*).$$

Let $z \in U_*$. Then $z = (k, f(x)) \in K \times f(H)$ and also $k \circ' f(x) \subset U$. Since f is a strong homomorphism, so we have

$$k \circ' f(x) = f(k \circ x) \subset U.$$

It implies that $k \circ x \subset f^{-1}(U)$. So $z = (k, f(x)) \in (id_K \times f)((f^{-1}(U))_*)$. Simply we see that

$$(id_K \times f)((f^{-1}(U))_*) \subset U_*.$$

The proof of (ii) and (iii) is immediate from the part (i). □

Theorem 2.8. *Let $(V, +, \circ)$ be a hyper topological space that endowed with topology τ, $(V', +', \circ')$ is a p-topological hypervector space and $f : V \to V'$ be a strong homomorphism that is open and continuous. Then the following assertions hold:*

1) *$f^{-1}(H)$ is a p-topological hypervector space.*

2) *In particular, V is p-topological hypervector space.*

Proof. If U is an open set in $f^{-1}(H)$, then $U_* = (id_k \circ f^{-1})(f(U)_*)$. □

Proposition 2.9. *Let $(V, +, \circ)$ and $(V', +', \circ')$ be two hypervector spaces. If $f : V \to V'$ is an one-to-one open continuous strong homomorphism, then $f^{-1} : V' \to V$ is an open strong homomorphism.*

Proof. Let $y, z \in V'$. Then there exist $x_1, x_2 \in V$ such that $y = f(x_1), z = f(x_2)$. Now we have

$$\begin{aligned} f^{-1}(y + z) &= f^{-1}(f(x_1) + f(x_2)) \\ &= f^{-1}(f(x_1 + x_2)) \\ &= x_1 + x_2 \\ &= f^{-1}(y) + f^{-1}(z). \end{aligned}$$

and

$$f^{-1}(a \circ' y) = f^{-1}(a \circ' f(x_1)) = f^{-1}(F(a \circ x_1)) = a \circ x_1 = a \circ f^{-1}(y).$$

Since f is continuous, so f^{-1} is an open strong homomorphism. □

Definition 2.10. Let $(V, +, \circ)$ be a hypervector space and ρ be an equivalence relation on V. If A, B are non-empty subset of V, then $A\overline{\rho}B$ means that $\forall\, a \in A,\ \exists b \in B$ such that $a\rho b$ and $\forall\, b' \in B,\ \exists a' \in A$ such that $a'\rho b'$.

Definition 2.11. Let $(V, +, \circ)$ be a hypervector space and ρ be an equivalence relation on V. The equivalence relation ρ is called regular if for all $r \in K$, from $a\rho b$, it follows that $(r \circ a)\overline{\rho}(r \circ b)$.

Proposition 2.12. *Let $(V, +, \circ)$ be a hypervector space, ρ be an equivalence relation on V and $V/\rho = \{[v] \ :\ v \in V\}$. Then V/ρ is a hypervector space with respect to actions*

$$\oplus : V/\rho \times V/\rho \to V/\rho,$$

where $([x], [y]) \mapsto [x] \oplus [y] = [x + y], \quad \forall x, y \in V$. *and*

$$\odot : K \times V/\rho \to V/\rho$$

$$k \odot [x] = \{[z] \ :\ z \in k \circ x\},$$

if and only if ρ is regular.

Proof. First we check that the mappings $\oplus$ and $\odot$ are well-defined on V/ρ. Simply, we see that $\oplus$ is well-defined. It is enough to show that $\odot$ is well-defined. Let $(r, [x]) = (r', [x']$. So $r = r'$ and $[x] = [x']$. Now we check that $r \odot [x] = r \odot [x']$. We have $x\rho x'$. Since ρ is regular it follows that $(rox)\overline{\rho}(rox')$. Hence, for all $z \in r \circ x$, there exists $z_1 \in r \circ x'$ such that $z\rho z_1$, which means that $[z] = [z_1]$. It follows that $r \odot [x] \subseteq r \odot [x']$ and similary we obtain the converse inclusion.

Now we show that $(V/\rho, \oplus, \odot)$ is a hypervector space. Simply we see that $(V/\rho, \oplus)$ is an abelian group. It is enough to show that $\odot$ has the properties of Definition 1.1. For all $a, b \in K$ we have

$$\begin{aligned} (a + b) \odot [x] &= \{[z] \ :\ z \in (a + b) \circ x\} \\ &\subseteq \{[z] \ :\ z \in a \circ x + b \circ x\} \\ &= \{[c] \oplus [d] \ \mid\ c \in a \circ x, d \in b \circ x\} \\ &= a \odot [x] \oplus b \odot [x]. \end{aligned}$$

Also the properties (ii), (iii), (iv) in Definition 1.1 are easily obtained. □

Theorem 2.13. *Let $(V, +, \circ)$ be a hypervector space and ρ be a regular equivalence relation on V. Then the canonical projection $\pi : V \to V/\rho$, such that $\pi(x) = [x]$, is a strong epimorphism.*

Proof. Suppose that $x, y \in X$. Simply, we see that $\pi(x+y) = \pi(x) \oplus \pi(y)$. Now we show that, for all $r \in K$ and $x \in V$, $\pi(r \circ x) = r \odot \pi(x)$. Let $[z] \in \pi(r \circ x)$, there exists $z' \in r \circ x$ such that $[z] = [z']$. We have $[z] = [z'] \in r \odot [x] = r \odot \pi(x)$. Conversely, if $[z] \in r \odot \pi(x) = r \odot [x]$, then there exists $z_1 \in r \circ x$ such that $[z] = [z_1] \in \pi(r \circ x)$. □

Remark 2.14. Let V be a hypervector space that endowed with the topology τ and ρ be a regular equivalence relation on V. Then V/ρ is a topological space that endowed with the quotient topology. So the canonical projection $\pi : V \to V/\rho$ is a continuous and open map.

Theorem 2.15. *Let V be a p-topological hypervector space and ρ be a regular equivalence relation on V. Then V/ρ is a p-topological hypervector space.*

Proof. Since the canonical map $\pi : V \to V/\rho$ is onto, open strong homomorphism and V is a p-topological hypervector space. By Theorem 2.7, V/ρ is a p-topological hypervector space. □

Theorem 2.16. *If $(V, +, \circ)$ and $(V', +', \circ')$ are hypervector spaces over field K and $f : V \to V'$ is a strong homomorphism, then the equivalence relation R^f associated with the map f, that $xR^f y \quad \Leftrightarrow \quad f(x) = f(y)$, is regular and V/R^f is a hypervector space. And also $\phi : f(V) \to V/R^f$, defined by $\phi(f(x)) = [x]$, is an isomorphism map.*

Proof. Let $aR^f b$ and r be arbitrary element of K. If

$$u \in r \circ a \Longrightarrow f(u) \in f(r \circ a) = r \circ' f(a) = r \circ' f(b) = f(r \circ b),$$

then, there exist $v \in r \circ b$ such that $f(u) = f(v)$, which means that $uR^f v$. Hence, R^f is regular. So according to Proposition 2.12, $(V/R^f, \oplus, \odot)$ is a hypervector space. Now we check that ϕ is an isomorphism. Let $f(x), f(y) \in f(V)$. We have $\phi(f(x) + f(y)) = \phi(f(x + y)) = [x + y] = [x] \oplus [y]$. For all $r \in K$ and $f(x) \in f(V)$, we have

$$\phi(r \circ' f(x)) = \phi(f(r \circ x)) = \{[z] \ : \ z \in r \circ x\} = r \odot [x] = r \odot \phi(f(x)).$$

Moreover, if $\phi(f(x)) = \phi(f(y)$, then $[x] = [y]$, so $xR^f y$. It implies that $f(x) = f(y)$. Hence ϕ is an injective map. One can easily check that ϕ is a surjective map. □

Remark 2.17. According to Theorem 2.15, we see that if $(V, +, \circ)$ is a p-topological hypervector space, then $(V/R^f, \oplus, \odot)$ is a p-topological hypervector space with respect to quetiont topology.

Proposition 2.18. *Let $(V, +, \circ)$ be a hypervector space over the field K and H be a subspace of V. Define an equivalence relation ρ^H on V as follows:*

$$a\rho^H b \Longleftrightarrow a - b \in H.$$

If $(V, +, \circ)$ is a strongly right distributive hypervector space, then ρ^H is a regular equivalence relation.

Proof. Let $x\rho^H y$ and r be an arbitrary element of K. So $x-y\in H$. Since V is strongly right distributive, we have $r\circ(x-y)=r\circ x-r\circ y\subseteq H$. Hence ρ^H is regular. □

Corollary 2.19. *Let V and ρ^H are similar to Proposition 2.18 and $V/\rho^H = V/H=\{[x]=x+H \quad | \quad x\in V\}$. Then V/H with respect to actions*

$$\oplus : V/H\times V/H\to V/H,$$

where $[x]\oplus[y]=[x+y], \forall x,y\in V$, and

$$\odot : K\times V/H\to V/H,$$

where $k\odot[x]=\{[z] \quad : \quad z\in k\circ x\}$, is a hypervector space.

Proof. According to Propositions 2.12 and 2.18, the proof is clear. □

Proposition 2.20. *Let H be a subspace of a strongly right distributive hypervector space $(V,\circ,+)$, the following statements are valid:*

1) *$\pi : V\to V/H$ such that $\pi(x)=[x]$ is a strong homomorphism map.*
2) *If V is a p-topological hypervector space, then $(V/H,\oplus,\odot)$ is a p-topological hypervector space.*

3. τ_*-topological hypervector space

In this section, by using a topology on the power set of a hypervector space, we present a new topological hypervector space and investigate some of its properties.

Definition 3.1. Let $(V,+,\circ)$ be a hypervector space over the field K and also (V,τ) and $(\mathcal{P}^*(V),\tau_*)$ be two topological spaces. $\circ : K\times V\to\mathcal{P}^*(V)$ is called τ_*-continuous, if for every open set U of $\mathcal{P}^*(V)$ (the member of τ_*) the set $\{(k,x) \ : \ k\circ x\in U\}$ is an open set in $K\times V$.

Definition 3.2. Let $(V,+,\circ)$ be a hypervector space over the field K and also (V,τ) and $(\mathcal{P}^*(V),\tau_*)$ be two topological spaces. V is called a τ_*- topological hypervector space, if the mapping $+ : V\times V\to V$ is continuous and the mapping $\circ : K\times V\to\mathcal{P}^*(V)$ is τ_*-contiuous.

Lemma 3.3. *Let (H,τ) be a topological space and*

$$\mathcal{S}_V=\{U\in\mathcal{P}^*(H) \ : \ U\subseteq V\},$$

where $V\in\tau$. Then $\Lambda=\{\mathcal{S}_V \ : \ V\in\tau\}$ is a basis for topology τ_Λ on $\mathcal{P}^*(H)$.

Proof. Let $\mathcal{S}_{V_1},\mathcal{S}_{V_2}\in\Lambda$, where $V_1,V_2\in\tau$. So we have $U\in\mathcal{S}_{V_1\cap V_2}=S_{V_1}\cap S_{V_2}$, where $V_1\cap V_2\in\tau$. Since $H\in\tau$, so we have $\mathcal{S}_H=\mathcal{P}^*(H)$, thus $U\in\mathcal{S}_H$, $\forall\, U\in\mathcal{P}^*(H)$. □

In above Lemma, τ_Λ that induced by τ is called the upper-topology on $\mathcal{P}^*(H)$.

Theorem 3.4. *Let $(H,+,\circ)$ be a hypervector space over the field K and (H,τ) be a topological space and τ_Λ be the upper-topology on $\mathcal{P}^*(H)$. Then $(H,+,\circ)$ is a p-topological hypervector space if and only if $(H,+,\circ)$ is a τ_Λ- topological hypervector space.*

Proof. Let $\circ : K \times H \to \mathcal{P}^*(H)$, where $\circ(r,x) = r \circ x$. Then we have

$$\begin{aligned}\circ^{-1}(\mathcal{S}_V) &= \{(r,x) \in K \times H \ : \ r \circ x \in \mathcal{S}_V\} \\ &= \{(r,x) \in K \times H \ : \ r \circ x \subseteq V\} = V^*.\end{aligned}$$

So the map $\circ$ is τ_Λ-continuous if and only if the map $\circ$ is p-continuous. □

Lemma 3.5. *Let (H,τ) be a topological space and*

$$\mathcal{I}_V = \{U \in \mathcal{P}^*(H) \ : \ U \cap V \neq \phi\},$$

where $V \in \tau$. Then $\Gamma = \{\mathcal{I}_V, \quad V \in \tau\}$ is a subbasis for topology τ_Γ on $\mathcal{P}^*(H)$.

Proof. It is enough to show that

$$\bigcup_{V \in \tau} \mathcal{I}_V = \mathcal{P}^*(H).$$

This proof is trivial, because $\mathcal{I}_H = \mathcal{P}^*(H)$. □

In above Lemma, τ_Γ that induced by τ is called the lower-topology on $\mathcal{P}^*(H)$.

Theorem 3.6. *Let $(V,+,\circ)$ be a hypervector space over the field K, (V,τ) be a topological space and τ_Γ be the lower-topology on $\mathcal{P}^*(H)$. Then $(V,+,\circ)$ is a strongly p-topological hypervector space if and only if $(V,+,\circ)$ is a τ_Γ-topological hypervector space.*

Proof. Let $\circ : K \times H \to \mathcal{P}^*(H)$, where $\circ(r,x) = r \circ x$. Then we have

$$\begin{aligned}\circ^{-1}(\mathcal{I}_V) &= \{(r,x) \in K \times H \ : \ r \circ x \in \mathcal{S}_V\} \\ &= \{(r,x) \in K \times H \ : \ r \circ x \cap V \neq \phi\} = V_*.\end{aligned}$$

So $\circ$ is a τ_Γ-continuous map if and only if $\circ$ is a strongly p-continuous map. □

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Accepted: 27.06.2017