# FIXED POINT RESULT FOR NEW RATIONAL TYPE CONTRACTION ON CLOSED BALL FOR MULTIVALUED MAPPING

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**Abstract.** The purpose of this paper is to introduce the idea of new rational type contractive condition on multivalued mapping to find the fixed point results for such mapping on a closed ball in complete metric space. Example has been given to demonstrate the variety of our result. Our results combine, extend and infer several comparable results in the existing literature.

**Keywords:** Fixed point, complete metric space, closed ball, multivalued mapping, new rational type contractive condition.

## 1. Introduction and preliminaries

Fixed point theory plays an important role in functional and non linear analysis. Banach proved significant result for contraction mappings. Afterward, a large number of fixed point results have been established by various authors and they showed different generalizations of the Banach's result. In literature, there are many concerning results about the fixed point of mappings which are contractive

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over the whole space. It is very simple to show that $K : G \longrightarrow G$ is not a contraction but $K : D \longrightarrow G$ is a contraction, where $D$ is a subset of $G$. It is possible to get fixed point for such mappings if they satisfy a certain condition. It has been shown by Hussain et al. [9], the presence of fixed point for such mappings that fulfill a certain condition on a closed ball. For further results on closed ball (see also [2, 3, 4, 5, 8, 12, 13, 14, 15, 16, 17, 18, 19]).

Nadler [11], initiated the study of fixed point theorems for the multivalued mappings (see also [1, 6, 7]). In this paper, the concept of new type of multivalued rational contraction has been introduced. Common fixed point results such contraction on a closed ball in complete metric space have been established. Example has been given. We give the following definitions and results which will be needed in the sequel.

**Definition 1.1** ([10])**.** Let $X$ be a nonempty set. A mapping $d : X \times X \to [0, \infty)$ is called a metric if the following conditions hold. for any $x, y, z \in X$ :

(i) $d(x, y) \geq 0$,

(ii) $d(x, y) = 0$ , iff $x = y$,

(iii) $d(x, y) = d(y, x)$,

(iv) $d(x, y) \leq d(x, z) + d(z, y)$.

Then $d$ is called metric on $X$, and the pair $(X, d)$ is called metric space.

**Definition 1.2** ([10])**.** Let $(X, d)$ be a metric space.

(i) A sequence $\{x_n\}$ in $(X, d)$ is called Cauchy sequence if given $\varepsilon > 0$, there corresponds $n_0 \in N$ such that for all $n, m \geq n_0$ we have $d(x_m, x_n) < \varepsilon$ or $\lim_{n,m\to\infty} d(x_n, x_m) = 0$.

(ii) A sequence $\{x_n\}$ converges (for short $d$-converges) to $x$ if $\lim_{n\to\infty} d(x_n, x) = 0$. In this case $x$ is called a $d$-limit of $\{x_n\}$.

(iii) $(X, d)$ is called complete if every Cauchy sequence in $X$ converges to a point $x \in X$ such that $d(x, x) = 0$.

**Definition 1.3** ([14])**.** Let $K$ be a nonempty subset of metric space $X$ and let $x \in X$. An element $y_0 \in K$ is called a best approximation in $K$ if

$$d(x, K) = d(x, y_0), \text{ where } d(x, K) = \inf_{y \in K} d(x, y).$$

If each $x \in X$ has at least one best approximation in $K$, then $K$ is called a proximinal set. The set of all proximinal subsets of $X$ is denoted by $P(X)$.

**Example 1.4.** Let $M = \{y \in C[-1, 1] : \int_0^1 y(t)dt = 0\}$. Then $M$ is a closed bounded subset of $C[-1, 1]$ that is not proximinal, where $C[-1, 1]$ is the set of all continuous and bounded functions from $[-1, 1]$ to $\mathbb{R}$ and

$$d(f, g) = \sup_{x \in [-1,1]} |f(x) - g(x)|.$$

**Definition 1.5** ([14])**.** The function $H_d : P(X) \times P(X) \to R^+$, defined by

$$H_d(A, B) = \max\{\sup_{a \in A} d(a, B), \ \sup_{b \in B} d(A, b)\}$$

is called Hausdorff-Pompeiu metric on $P(X)$.

**Lemma 1.6** ([14])**.** Let $(X, d)$ be a metric space. Let $(P(X), H_d)$ is a Hausdorff-Pompeiu metric space on $P(X)$. Then, for all $A, B \in P(X)$ and for each $a \in A$ there exists $b_a \in B$ satisfies $d(a, B) = d(a, b_a)$ then $H_d(A, B) \geq d(a, b_a)$.

## 2. Main result

Let $(X, d)$ be a complete metric space, $x_0 \in X$ and $S : X \to P(X)$ be the multivalued mapping on $X$. Then there exists $x_1 \in Sx_0$ be an element such that $d(x_0, Sx_0) = d(x_0, x_1)$. Let $x_2 \in Sx_1$ be such that $d(x_1, Sx_1) = d(x_1, x_2)$. Let $x_3 \in Sx_2$ be such that $d(x_2, Sx_2) = d(x_2, x_3)$. Continuing this process, we construct a sequence $x_n$ of points in $X$ such that $x_{n+1} \in Sx_n$, $d(x_n, Sx_n) = d(x_n, x_{n+1})$. We denote this iterative sequence by $\{XS(x_n)\}$. We say that $\{XS(x_n)\}$ is a sequence in $X$ generated by $x_0$.

**Theorem 2.1.** *Let $(X, d)$ be a complete metric space and $x_0$ be any arbitrary point in $X$. Let the mapping $S : X \to P(X)$ satisfy:*

$$\begin{aligned} H_d(Sx, Sy) \leq a_1 \ d(x, y) + a_2 \frac{(a + d\,(x, Sx)).d\,(y, Sy)}{(a + d\,(x, y))} \\ + a_3 \left[d\,(x, Sx) + d\,(y, Sy)\right], \end{aligned} \tag{2.1}$$

*for all $x, y \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$ and $a, r, a_1, a_2, a_3 > 0$, with $a_1 + a_2 + 2a_3 < 1$. Also*

$$d(x_0, Sx_0) \leq (1 - \lambda)r, \tag{2.2}$$

*where $\lambda = (\frac{a_1 + a_3}{1 - a_2 - a_3})$. Then $\{XS(x_n)\}$ is a sequence in $\overline{B_d(x_0, r)}$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{XS(x_n)\} \to h \in \overline{B_d(x_0, r)}$. Also, if inequality* (2.1) *holds for $h$, then $S$ has fixed point $h$ in $\overline{B_d(x_0, r)}$.*

**Proof.** Let $\{XS(x_n)\}$ is a sequence in $X$ generated by $x_0$, then, we have $x_{n+1} \in Sx_n$ where $n = 0, 1, 2, ....$ By Lemma 1.6, we have

$$\begin{aligned} d\,(x_1, x_2) &= d\,(x_1, Sx_1) \leq H_d(Sx_0, Sx_1) \\ d\,(x_1, x_2) &\leq a_1 d\,(x_0, x_1) + a_2 \frac{(a + d\,(x_0, Sx_0)).d\,(x_1, Sx_1)}{(a + d\,(x_0, x_1))} \\ &\quad + a_3 \left[d\,(x_0, Sx_0) + d\,(x_1, Sx_1)\right] \end{aligned}$$

$$\begin{aligned}
&\leq a_1 d(x_0,x_1) + a_2 \frac{(a+d(x_0,x_1)).d(x_1,x_2)}{(a+d(x_0,x_1))} \\
&+ a_3 [d(x_0,x_1) + d(x_1,x_2)] \\
&\leq a_1 d(x_0,x_1) + a_2 d(x_1,x_2) + a_3 [d(x_0,x_1) + d(x_1,x_2)] \\
&\leq \left(\frac{a_1+a_3}{1-a_2-a_3}\right) d(x_0,x_1) \\
&\leq \lambda(1-\lambda)r.
\end{aligned}$$

Now,

$$\begin{aligned}
d(x_0,x_2) &\leq d(x_0,x_1) + d(x_1,x_2) \\
&\leq (1-\lambda)r + \lambda(1-\lambda)r \\
&\leq (1-\lambda^2)r \leq r \\
d(x_0,x_2) &\leq r.
\end{aligned}$$

This implies that $x_2 \in \overline{B_d(x_0,r)}$. Similarly, by repeating the same process for

$$\begin{aligned}
d(x_2,x_3) &= d(x_2,Sx_2) \\
d(x_2,x_3) &\leq H_d(Sx_1,Sx_2) \text{ by Lemma 1.6}
\end{aligned}$$

we get

$$d(x_2,x_3) \leq \lambda^2 d(x_0,x_1).$$

Consequently, $x_3, x_4, \dots, x_j \in \overline{B_d(x_0,r)}$, for some $j \in N$. If $j = 2i+1$, where $i = 0,1,2,\cdots,\frac{j-1}{2}$ we get

$$d(x_{2i+1},x_{2i+2}) \leq \lambda d(x_{2i},x_{2i+1}). \tag{2.3}$$

Similarly, if $j = 2i+2$, where $i = 0,1,2,\cdots,\frac{j-2}{2}$, we have

$$d(x_{2i+2},x_{2i+3}) \leq \lambda d(x_{2i+1},x_{2i+2}). \tag{2.4}$$

Now, (2.3) implies that

$$d(x_{2i+1},x_{2i+2}) \leq \lambda^{2i+1} d(x_0,x_1). \tag{2.5}$$

Also, (2.4) implies that

$$d(x_{2i+2},x_{2i+3}) \leq \lambda^{2i+2} d(x_0,x_1). \tag{2.6}$$

Now, by combining (2.5) and (2.6), we have

$$d(x_j,x_{j+1}) \leq \lambda^j d(x_0,x_1) \text{ for all } j \in N. \tag{2.7}$$

Now,

$$\begin{aligned} d(x_0,x_{j+1}) &\leq d(x_0,x_1)+d(x_1,x_2)+\cdots+d(x_j,x_{j+1}) \\ &\leq d(x_0,x_1)+\lambda d(x_0,x_1)+\cdots+\lambda^j d(x_0,x_1) \text{ by (2.7)} \\ &\leq (1+\lambda+\lambda^2+\cdots+\lambda^j)d(x_0,x_1) \\ &\leq \frac{1(1-\lambda^j)}{1-\lambda}(1-\lambda)r \text{ as } j\to\infty \\ &\leq r. \end{aligned}$$

Thus, $x_{j+1}\in\overline{B_d(x_0,r)}$ . Hence $x_n\in\overline{B_d(x_0,r)}$ for all $n\in\mathbb{N}\cup\{0\}$, therefore $\{XS(x_n)\}$ is a sequence in $\overline{B_d(x_0,r)}$. Now, the inequality (2.7) can be written as

$$d(x_n,x_{n+1})\leq\lambda^n d(x_0,x_1) \text{ for all } n\in N. \tag{2.8}$$

Hence for any $m>n$,

$$\begin{aligned} d(x_n,x_m) &\leq d(x_n,x_{n+1})+d(x_{n+1},x_{n+2})+\cdots+d(x_{m-1},x_m) \\ &\leq \left(\lambda^n+\lambda^{n+1}+\cdots+\lambda^{m-1}\right)d(x_0,x_1) \text{ by using (2.8)} \\ &< \frac{\lambda^n}{1-\lambda}d(x_0,x_1)\longrightarrow 0, \text{ as } m,n\longrightarrow\infty. \end{aligned}$$

Thus we proved that $\{XS(x_n)\}$ is a Cauchy sequence in $(\overline{B_d(x_0,r)},d)$. As every closed ball in a complete metric space is complete, so there exists $h\in\overline{B_d(x_0,r)}$ such that $\{XS(x_n)\}\to h$, it follows that $h\in Sh$, otherwise $d(h,Sh)=z>0$, that is

$$\lim_{n\to\infty} d(x_n,h)=0. \tag{2.9}$$

Now,

$$\begin{aligned} d(h,Sh) &\leq d(h,x_{2n+2})+d(x_{2n+2},Sh) \\ &\leq d(h,x_{2n+2})+H_d(Sx_{2n+1},Sh), \text{ by Lemma 1.6} \\ d(h,Sh) &\leq d(h,x_{2n+2})+H_d(Sh,Sx_{2n+1}) \\ &\leq d(h,x_{2n+2})+a_1 d(h,x_{2n+2})+a_2\frac{(a+d(h,Sh)).d(x_{2n+1},Sx_{2n+1})}{(a+d(h,x_{2n+1}))} \\ &+a_3[d(h,Sh)+d(x_{2n+1},Sx_{2n+1})] \\ &\leq d(h,x_{2n+2})+a_1 d(h,x_{2n+2})+a_2\frac{(a+d(h,Sh)).d(x_{2n+1},x_{2n+2})}{(a+d(h,x_{2n+1}))} \\ &+a_3[d(h,Sh)+d(x_{2n+1},x_{2n+2})]. \end{aligned}$$

This implies that

$$\begin{aligned} z &\leq d(h,x_{2n+2})+a_1 d(h,x_{2n+2})+a_2\frac{(a+z).d(x_{2n+1},x_{2n+2})}{(a+d(h,x_{2n+1}))} \\ &+a_3[z+d(x_{2n+1},x_{2n+2})]. \end{aligned}$$

Letting $n \to \infty$ , it fallows that

$$\begin{aligned} (1-a_3)z &\leq 0 \\ (1-a_3) &\neq 0 \\ &\Rightarrow z \leq 0. \end{aligned}$$

As $z = d(h, Sh) \leq 0$, rise a contradiction so that $h \in Sh$. □

**Example 2.2.** Let $X = \mathbb{R}^+ \cup \{0\}$ and let $d : X \times X \to X$ be the complete metric on $X$ defined by

$$d(x, y) = |x - y| \text{ for all } x, y \in X.$$

Define the multivalued mapping, $S : X \times X \to P(X)$ by,

$$Sx = \begin{cases} [\frac{x}{3}, \frac{2}{3}x], & \text{if } x \in [0, 9] \\ [x, x+1], & \text{if } x \in (9, \infty) \end{cases}$$

Considering, $x_0 = 1, r = 8$, then $\overline{B_d(x_0, r)} = [0, 9]$. Now $d(x_0, Sx_0) = d(1, S1) = d(1, \frac{2}{3}) = \frac{1}{3}$. $d(\frac{2}{3}, S\frac{2}{3}) = d(\frac{2}{3}, \frac{4}{9}) = \frac{2}{9}$. So we obtain a sequence $\{XS(x_n)\} = \{1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, ....\}$ in $X$ generated by $x_0$. Now, $\overline{B_d(x_0, r)} \cap \{XS(x_n)\} = \{1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, ....\}$. Let $10, 11 \in X$, then $H_d(S10, S11) = 1$. Let $a = 1$, $x = 10$, $y = 11$, $a_1 = \frac{1}{3}$, $a_2 = \frac{1}{4}$, and $a_3 = \frac{1}{7}$. Then

$$\begin{aligned} H_d(S10, S11) &\geq \frac{1}{3}d(10, 11) + \frac{1}{4}\frac{(1 + d(10, 10)).d(11, 11)}{(1 + d(10, 11))} \\ &\quad + \frac{1}{7}[d(10, 10) + d(11, 11)] = \frac{1}{3} \end{aligned}$$

So, the contractive condition does not hold for whole space. Now, for $1, \frac{2}{3} \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$, we have

$$\begin{aligned} H_d(S1, S\frac{2}{3}) &= [\max\{\sup_{a \in S1} d(a, S\frac{2}{3}), \sup_{b \in S\frac{2}{3}} d(S1, b)\}] \\ &= \max\left\{\sup_{a \in S1} d(a, [\frac{\frac{2}{3}}{3}, \frac{2\frac{2}{3}}{3}]), \sup_{b \in S\frac{2}{3}} d([\frac{1}{3}, \frac{2}{3}], b)\right\} \\ &= \max\left\{d(\frac{2}{3}, [\frac{2}{9}, \frac{4}{9}]), d([\frac{1}{3}, \frac{2}{3}], \frac{2}{9})\right\} \\ &= \max\left\{d(\frac{2}{3}, \frac{4}{9}), d(\frac{1}{3}, \frac{2}{9})\right\} \\ &= \max\left\{\left|\frac{2}{3} - \frac{4}{9}\right|, \left|\frac{1}{3} - \frac{2}{9}\right|\right\} \\ &= \max\left\{\frac{2}{9}, \frac{1}{9}\right\} = \frac{2}{9} \end{aligned}$$

Now,

$$\begin{aligned}
&a_1 d(1,\frac{2}{3}) + a_2 \frac{(a + d\,(1, S1)).d\left(\frac{2}{3}, S\frac{2}{3}\right)}{(a + d(1,\frac{2}{3}))} + a_3 \left[ d(1, S1) + d(\frac{2}{3}, S\frac{2}{3}) \right] \\
&= \frac{1}{3}\left|1 - \frac{2}{3}\right| + \frac{1}{4}\frac{(1+\frac{1}{3}).\frac{2}{9}}{1 + \left|1 - \frac{2}{3}\right|} + \frac{1}{7}\left[\frac{1}{3} + \frac{2}{9}\right] \\
&= \frac{1}{9} + \frac{2}{36} + \frac{5}{63} = \frac{62}{252}.
\end{aligned}$$

As $\frac{2}{9} < \frac{62}{252}$. So, the contractive condition holds for $1, \frac{2}{3} \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$. Similarly, the contractive condition holds for all $x, y \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$. Also,

$$\begin{aligned}
\frac{1}{3} &\leq (1 - \frac{40}{51}) \times 8 \\
d(x_0, Sx_0) &\leq (1 - \lambda)r.
\end{aligned}$$

Hence, all the conditions of Theorem 2.1 are satisfied. Now, we have $\{XS(x_n)\}$ is a sequence in $\overline{B_d(x_0, r)}$, for all $n \in \mathbb{N} \cup \{0\}$. Moreover, 0 is a fixed point of $S$.

**Corollary 2.3.** Let $(X, d)$ be a complete metric space and let the mapping $S : X \to P(X)$ satisfy:

$$d(Sx, Sy) \leq a_1\; d(x, y) + a_2 \frac{(a + d\,(x, Sx)).d\,(y, Sy)}{(a + d\,(x, y))},$$

for all $x, y \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$ there exist $a$, $r > 0$,

$$d(x_0, Sx_0) \leq (1 - \lambda)r,$$

where $\lambda = (\frac{a_1}{1-a_2})$ and $a_1, a_2$ are non negative reals with $a_1 + a_2 < 1$. Then $\{XS(x_n)\}$ is a sequence in $\overline{B_d(x_0, r)}$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{XS(x_n)\} \to h \in \overline{B_d(x_0, r)}$. Then $S$ has fixed point $h$ in $\overline{B_d(x_0, r)}$.

**Proof.** By using $a_3 = 0$ in theorem (2.1), we get the required result. □

**Corollary 2.4.** Let $(X, d)$ be a complete metric space and let the mapping $S : X \to P(X\ )$ satisfy:

$$d(Sx, Sy) \leq a_1\; d(x, y) + a_3 \left[d\,(x, Sx) + d\,(y, Sy)\right],$$

for all $x, y \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$ there exist $r > 0$,

$$d(x_0, Sx_0) \leq (1 - \lambda)r,$$

where $\lambda = (\frac{a_1+a_3}{1-a_3})$ and $a_1, a_3$ are non negative reals with $a_1 + 2a_3 < 1$. Then $\{XS(x_n)\}$ is a sequence in $\overline{B_d(x_0, r)}$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{XS(x_n)\} \to h \in \overline{B_d(x_0, r)}$. Then $S$ has fixed point $h$ in $\overline{B_d(x_0, r)}$.

**Proof.** By using $a_2 = 0$ in theorem (2.1), we get the result. □

**Corollary 2.5.** Let $(X, d)$ be a complete metric space and $x_0$ be any arbitrary point on $X$ let the mapping $S : X \to P(X)$ satisfy:

$$d(Sx, Sy) \leq a_2 \frac{(a + d(x, Sx)).d(y, Sy)}{(a + d(x, y))} + a_3 [d(x, Sx) + d(y, Sy)],$$

for all $x, y \in \overline{B_d(x_0, r)} \cap \{XS(x_n)\}$ there exist $a, r > 0$,

$$d(x_0, Sx_0) \leq (1 - \lambda)r,$$

where $\lambda = (\frac{a_3}{1-a_2-a_3})$ and $a_2, a_3$ are non negative reals with $a_2 + 2a_3 < 1$. Then $\{XS(x_n)\}$ is a sequence in $\overline{B_d(x_0, r)}$ for all $n \in \mathbb{N} \cup \{0\}$ and $\{XS(x_n)\} \to h \in \overline{B_d(x_0, r)}$. Then $S$ has fixed point $h$ in $\overline{B_d(x_0, r)}$.

**Proof.** By using $a_1 = 0$ in theorem (2.1), we get the result. □

**Corollary 2.6.** Let $(X, d)$ be a complete metric space and let the mapping $S : X \to X$ satisfy:

$$d(Sx, Sy) \leq a_1 \ d(x, y) + a_2 \frac{(a + d(x, Sx)).d(y, Sy)}{(a + d(x, y))} + a_3 [d(x, Sx) + d(y, Sy)],$$

for all $x, y \in X$, there exist $a > 0$, where $a_1, a_2, a_4$ are non negative reals with $a_1 + 2a_3 < 1$. Then $S$ has a fixed point.

**Corollary 2.7.** Let $(X, d)$ be a complete metric space and let the mapping $S : X \to X$ satisfy:

$$d(Sx, Sy) \leq a_2 \frac{(a + d(x, Sx)).d(y, Sy)}{(a + d(x, y))} + a_3 [d(x, Sx) + d(y, Sy)],$$

for all $x, y \in X$, there exist $a > 0$, where $a_2, a_3$ are non negative reals with $a_2 + 2a_3 < 1$. Then $S$ has a fixed point.

**Corollary 2.8.** Let $(X, d)$ be a complete metric space and let the mapping $S : X \to X$ satisfy:

$$d(Sx, Sy) \leq a_1 \ d(x, y) + a_3 [d(x, Sx) + d(y, Sy)],$$

for all $x, y \in X$, where $a_1, a_3$ are non negative reals with $a_1 + 2a_3 < 1$. Then $S$ has a fixed point.

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