SOME PROPERTIES OF SOFT β-COMPACT AND RELATED SOFT TOPOLOGICAL SPACES

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Abstract. Benchalli et al. [4] introduced the notion of soft β-compactness by using soft filter basis. In continuation, in this paper we further study some more properties of soft β-compactness in soft topological spaces. Furthermore we introduce and discuss, soft β-first countable, soft β-second countable spaces, soft β-closed spaces and soft generalized β-compact spaces in soft topological spaces.

Keywords: Soft sets, soft β-compact, soft β-first countable spaces, soft generalized β-compact spaces.

1. Introduction

Mathematics is based on exact concepts and there is not vagueness for mathematical theories. In the fields such as medicine, engineering, economics and sociology, the notions are vague, researchers need to define some modern methods for vagueness. To deal with the these problems in real life, researchers anticipated several methods such as fuzzy set theory, rough set theory and soft set theory. Fuzzy set theory [14] proposed by Zadeh in 1965 provides an appropriate framework for representing and processing vague concepts. The basic idea of fuzzy set theory hinges on fuzzy membership function. By fuzzy membership function, we can establish the belonging of an element to set to a degree. Rough set theory [13] which is proposed by Pawlak in 1982 is another mathematical approach to vagueness to catch the granularity induced by vagueness in information systems. It based on equivalence relation. The benefit of rough set method is that it does not need any additional information about data, like membership in fuzzy set theory.

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Theory of fuzzy sets and theory of rough sets can be considered as tools for dealing with vagueness but both of these theories have their own difficulties. The reason for these difficulties is possibly, the inadequacy of the parametrization tool of the theory as mentioned by Molodtsov [10] in 1999. Soft set theory was initiated by Molodtsov [10] as a completely new approach for modeling vagueness and uncertainty. According to Molodtsov [10][11], the soft set theory has been successfully applied to many areas, such as functions smoothness, game theory, Riemann integration, theory of measurement and so on. He also showed how soft set theory is free from the parametrization inadequacy syndrome of, rough set theory, game theory, fuzzy set theory and probability theory.

Recently, weak forms of soft open sets were studied by many researchers like [5] [1] [8] [9] in soft topological spaces respectively. Furthermore, Benchalli et al. [2][3][4] have studied soft β-separation axioms, soft β-compactness, soft β-connectedness in soft topological spaces.

In the present paper, we studied some more properties of soft β-compact spaces in soft topological spaces which are defined over an initial universe with a fixed set of parameters. We have set up a soft topology with the help of soft β-closed spaces. In addition to that the concept of soft β-first countable, soft β-second countable and soft β-Lindelöf spaces are studied. Furthermore we introduced the notion of the soft generalized β-compact spaces in soft topological spaces. Also, we have explored some basic properties of these concepts.

The organization of this paper is as follows: Section 2 briefly reviews some basic concepts about soft sets, definitions of some weaker forms of soft sets and related properties in soft topological spaces; Section 3 defines the concepts of soft β-compact spaces , soft β-spaces and studies some relative properties; Section 4 introduces the concepts of soft β-closed spaces and their properties; Section 5 we give the definition of soft β-first countable and soft β-second countable spaces and their related properties; Section 6 introduces about soft generalized β-compact spaces and section 7 is conclusion of the paper.

2. Preliminary

Through-out this paper (X, τ, E) will be a soft topological space.

Definition 2.1 ([12]). *Let X be an initial universe and let E be a set of parameters. Let $P(X)$ denote the power set of X and let A be a nonempty subset of E. A pair (F, A) is called a soft set over $\tilde{X}$, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over $\tilde{X}$ is a parameterized family of subsets of the universe $\tilde{X}$. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.*

Definition 2.2 ([12]). *Let τ be the collection of soft sets over X; then τ is said to be a soft topology on X if it satisfies the following axioms:*

(a) Φ, X belong to τ.

(b) The union of any number of soft sets in $\tau \in \tau$.

(c) *The intersection of any two soft sets in* $\tau \in \tau$.

The triplet (X, τ, E) *is called a soft topological space over* X. *The relative complement of a soft set* (F, A) *is denoted by* $(F, A)^c$ *and is defined by* $(F, A)^c = (F^c, A)$.

Definition 2.3 ([12]). *Let* (X, τ, E) *be a soft topological space over* $\tilde{X}$ *and let* (F, A) *be a soft set over* X.

(a) *Soft Interior: The soft interior of* (F, A) *is the soft set* $int((F, A)) = \cup\{(O, A) : (O, A)$ *is soft open and* $(O, A)\tilde{\subset}(F, A)\}$.

(b) *Soft Closure : The soft closure of* (F, A) *is the soft set* $cl((F, A)) = \cap\{(F, E) : (F, E)$ *is soft closed and* $(F, A)\tilde{\subset}(F, E)\}$.

Definition 2.4 ([8]). *A soft set* (F, A) *of a soft topological space* (X, τ, E) *is said to be*

(a) *Soft open if its complement is soft closed.*

(b) *Soft* α*-open if* $(F, A)\tilde{\subset}int(cl(Int((F, A))))$.

(c) *Soft preopen if* $(F, A)\tilde{\subset}int(cl((F, A)))$.

(d) *Soft semiopen if* $(F, A)\tilde{\subset}cl(int((F, A)))$.

(e) *Soft* β*-open if* $(F, A)\tilde{\subset}cl(int(cl((F, A))))$.

The complement of soft open, (resp. soft α*-open, soft preopen, soft semiopen, soft* β*-open) sets are said to be soft closed (resp.soft* α*-closed, soft preclosed, soft semiclosed, soft* β*-closed). The intersection of soft closed(resp. soft* α*-closed, soft preclosed, soft semiclosed, soft* β*-closed)sets containing* (F, A) *is called the soft closure (resp. soft* α*-closure, soft pre-closure, soft semi-closure, soft* β*-closure) of* (F, A) *and is denoted by* $scl(F, A)$ *(resp.* $s\alpha cl(F, A)$, $sPcl(F, A)$, $sScl(F, A)$, $s\beta cl(F, A)$*). The soft interior of* (F, A) *is defined by the union of all soft open (resp.Soft* α*-open, soft preopen, soft semiopen, soft* β*-open) sets contained in* (F, A) *and is denoted by* $sint(F, A)$ *(resp.* $s\alpha int\,(F, A)$, $sPint(F, A)$, $sSint(F, A)$, $s\beta int(F, A)$*).*

Definition 2.5 ([2]). *Let* (X, τ, E) *be a soft topological space over* $X, (G, E)$ *be soft closed set in* X *and* $x \in X$ *such that* $x \notin (G, E)$. *If there exists soft* β*-open sets* (F_1, E) *and* (F_2, E) *such that* $x \in (F_1, E)$, $(G, E) \subseteq (F_2, E)$ *and* $(F_1, E) \cap (F_2, E) = \phi$, *then* (X, τ, E) *is called a soft* β*-regular space.*

Definition 2.6 ([2]). *A space* X *is said to be a soft* β*-normal if for any pair of disjoint soft closed sets* (F_1, E) *and* (F_2, E) *there exists disjoint soft* β*-open sets* (U, E) *and* $(V, E) \ni (F_1, E) \subset (U, E)$ *and* $(F_2, E) \subset (V, E)$.

Definition 2.7 ([7]). *Let $SS(X)_E$ and $SS(Y)_E$ be families of soft sets. $u : X \Rightarrow Y$ and $p : E \Rightarrow E'$ be mappings. Then a mapping $f_{pu} : SS(X)_E \Rightarrow SS(Y)_{E'}$ defined as:*

(a) Let (F, E) be a soft set in $SS(X)_E$. The image of (F, E) under f_{pu}, written as $f_{pu}(F, E) = (f_{pu}(F), p(E))$, is a soft set in $SS(Y)_E$ such that

$$f_{pu}(F, E) = \begin{cases} \bigcup\limits_{x \in p'(y) \cap A} u(F(x)), & p'(y) \cap A \neq \phi \\ \phi, & otherwise \end{cases}$$

for all $y \in E'$

(b) Let (G, E') be a soft set in $SS(V)_{E'}$. Then the inverse image of (G, E') under f_{pu}, written as $f_{pu}^{-1}(G, E') = (f_{pu}^{-1}(G).p^{-1}(E'))$, is a soft set in $SS(U)_E$ such that

$$f_{pu}^{-1}(G, E') = \begin{cases} u^{-1}(G(p(x))), & p(x) \in E' \\ \phi, & otherwise \end{cases}$$

for all $x \in E$.

Definition 2.8 ([3]). *A cover of a soft set is said to be a soft β-open cover if every member of the cover is a soft β-open set.*

Definition 2.9 ([3]). *A soft topological space (X, τ, E) is said to be soft β-compact space if each soft β-open cover of $\tilde{X}$ has a finite subcover.*

3. Some properties of soft β-compact spaces in soft topological spaces

In this section, the concept of soft β-space is introduced and studied the concept of soft β-compactness in terms of soft β-compact spaces. Some more properties of soft β-compactness spaces are studied in detail. Soft β-compactness can be infinite as H. Mahamood[6] introduced and discussed the concept of soft Heine Borel theorem for infinite soft compactness.

Definition 3.1. *A soft topological space (X, τ, E) is said to be soft β-space if every soft β-open set of $\tilde{X}$ is soft open in $\tilde{X}$.*

Example 3.2. Every soft descrete topology is a soft β-space.

Example 3.3. Every soft indescrete topology is a soft β-space.

Corollary 3.4. *If a soft topological space (X, τ, E) is a soft β-compact space and soft β-space, then (X, τ, E) is soft compact space.*

Proof. Let $\{(G_k)_i : i \in I\}$ be a soft open cover of $\tilde{X}$. Since any soft open set is soft β-open set, therefore $\{(G_k)_i : i \in I\}$ is soft β-open cover of $\tilde{X}$. Since $\tilde{X}$ is soft β-compact space and soft β-space, there exists a soft finite subset I_m of I such that $\tilde{X} \subseteq \cup\{(G_k)_i : i \in I\}$. Hence (X, τ, E) is soft compact space. □

Corollary 3.5. *If $f : X \to Y$ is a soft β-continuous function and soft β-space, then f is soft continuous function.*

Proof. Let us consider soft β-open set $\{(F_E)_i : i \in I\}$ of $\tilde{Y}$. For f is soft β-continuous function, $\{f^{-1}((F_E)_i) : i \in I\}$ is a soft β-open set of X and for $\tilde{X}$ is soft β-space, $\{f^{-1}((F_E)_i) : i \in I\}$ form a soft open set of $\tilde{X}$. Thus, f is a soft continuous function. □

Corollary 3.6. *Let (X, σ, E) be a soft topological space. If (X, σ_e) is a soft β-compact space, for each $e \in E$, then (X, σ, E) is a soft β-compact space.*

Proof. Let (X, σ_e) is a soft β-compact space and let $E = \{e_1, e_2, ...e_n\}$ be a set of parameters, for every $i = 1, 2, 3, ...n$. Let $\{(G_k)_i : i \in I\}$ be a soft β-open cover of $\tilde{X}$. For $\bigcup_{i \in I_m} (G_k)_i(e) = \tilde{X}$, for every $e \in E$, and (X, σ_e) is soft β-compact space, there is a finite subset I_m of I such that $\bigcup_{i \in I_m} (G_k)_i(e) = \tilde{X}$. So $\bigcup_{i \in I_m} (G_k)_i(e) = \tilde{X}$, hence $\{(G_k)_i : i \in I_m\}$ is a soft finite subcover of $\{(G_k)_i : i \in I\}$. Therefore, (X, σ, E) is soft β-compact space. □

Definition 3.7. *A soft mapping $f : X \to Y$ is said to be soft β^*-open if the image of each soft β-open set of $\tilde{X}$ is soft β-open in $\tilde{Y}$.*

Theorem 3.8. *Let (F, A) and (G, B) be the soft subsets of a soft topological space (X, τ, E), such that (F,A) is soft β-compact in $\tilde{X}$ and (G, B) is soft β-closed set in $\tilde{X}$. Then $(F, A) \cap (G, B)$ is soft β-compact in $\tilde{X}$.*

Proof. Let $\{(H_E)_i : i \in I\}$ be a cover of $(F, A) \cap (G, B)$ consisting of soft β-open subsets of $\tilde{X}$. Since $(G, B)^c$ is a soft β-open set, $\{(H_E)_i : i \in I\} \cup (G, B)^c$ is soft β-open cover of (F,A). Since (F,A) is soft β-compact in $\tilde{X}$, there exists a soft finite subset $I_m \subset I$ such that $(F, A) \subset \{(H_E)_i : i \in I_m\} \cup (G, B)^c$. Therefore $(F, A) \cap (G, B) \subset \{(H_E)_i : i \in I_m\}$. As a consequence, $(F, A) \cap (G, B)$ is soft β-compact in $\tilde{X}$. □

Theorem 3.9. *Let $f : X \to Y$ be a soft β-open, soft β-continuous and injective mapping. If a soft subset (H, E) of Y is soft β-compact in Y, then $f^{-1}((H, E))$ is soft β-compact in $\tilde{X}$.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a soft β-open cover of $f^{-1}((H, E))$ in $\tilde{X}$. Then $f^{-1}((H, E)) \subset \cup\{(F_E)_i : i \in I\}$ and hence $(H, E) \subset \cup f(f^{-1}((H, E))) \subset f(\cup\{(F_E)_i : i \in I\} = \cup\{f(F_E)_i : i \in I\}$. Since (H,E) is soft β-compact in $\tilde{Y}$ there is a soft finite subset $I_m \subset I$ such that $(H, E) \subset \cup\{(F_E)_i : i \in I_m\}$. So $f^{-1}((H, E)) \subset f^{-1}(\cup\{f((F_E))_i : i \in I_m\} = \cup\{f^{-1}(f(F_E)_i : i \in I_m\} = \cup\{(F_E)_i : i \in I_m\}$. □

Theorem 3.10. *The preimage of soft β-compact space under soft β^*-open bijective mapping is soft β-compact space.*

Theorem 3.11. *If a function $f : X \to Y$ is soft β^*-open bijective mapping and $\tilde{Y}$ be soft β-compact space, then $\tilde{X}$ is soft β-compact space.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a collection of soft β-open covering of $\tilde{X}$. Then let $\{f(F_E)_i : i \in I\}$ be a soft β-open cover is a collection of soft β-open sets covering $\tilde{Y}$. For $\tilde{Y}$ is soft β-compact space, by definition there exists a finite family $I_m \subset I$ such that $\{f(F_E)_i : i \in I_m\}$ covers $\tilde{Y}$. Since f is soft bijective, we have $\tilde{X} = f^{-1}(Y) = f^{-1}(f(\bigcup_{i\in I_m}(F_E)_i)) = \bigcup_{i\in I_m}(F_E)_i$. Thus $\tilde{X}$ is soft β-compact space. □

4. Soft β-closed spaces

This section deals the concept of soft β-closed spaces and its related concepts in detail.

Definition 4.1. *Let (X, τ, E) be a soft topological space and it is said to be soft β-closed space if and only if for every soft family $\{(F_E)_i : i \in I\}$ of soft β-open set such that $\bigcup_{i\in I}(F_E)_i = \tilde{X}$ there is a soft finite subfamily $I_m \subset I$ such that $\bigcup_{i\in I_m} s\beta cl(F_E)_i = \tilde{X}$.*

Definition 4.2. *A soft set (F,E) in a soft topological space $\tilde{X}$ is said to be soft β-closed relative to $\tilde{X}$ if and only if for every family $\{(H_E)_i : i \in I\}$ of soft β-open set such that $\bigcup_{i\in I_m}(H_E)_i = (F, E)$ there is a soft finite subfamily $I_m \subset I$ such that $\bigcup_{i\in I_m} s\beta cl(H_E)_i = (F, E)$.*

Remark 4.3. Every soft β-compact space is soft β-closed but the converse is not true.

Example 4.4. The following example shows that soft β-closed space need not to be soft β-compact space.

Let $\tilde{X} \neq \phi$ be a soft set and $(F, E)_n = 1-1/n$ for every $e_x \in \tilde{X}$ and $n \in N^+$. The collection $\{(F, E)_n : n \in N^+\}$ is a soft β-base for a soft topology on $\tilde{X}$. The collection $\{(F, E)_n : n \in N^+\}$ is obviously a soft β-open cover of $\tilde{X}$. On the other hand we have $s\beta cl(F, E)_n = \tilde{X}$ for every $n \geq 3$. Hence (X,τ,E) is soft β-closed but not soft β-compact space.

Theorem 4.5. *Soft topological space (X, τ, E) is a soft β-closed if and only if for every soft finite intersection property ψ in $\tilde{X}$ then $\cap(H, B) \in \psi s\beta cl(H, B) \neq \phi$.*

Proof. Let $\{(G_E)_i : i \in I\}$ be a soft β-open cover of $\tilde{X}$ and let for every finite collection of $\{(G_E)_i : i \in I\}$, $\bigcup_{i\in I_m}(G_A)_i \subset \tilde{X}$ for some $i \in I_m$. Then $\bigcap_{i\in I_m}((H, B)^c)_i \supset \phi$, for some $i \in I_m$. Thus $\{s\beta cl((G_A)^c)_i : i \in I\} = \psi$ forms a soft β-open finite intersection property in $\tilde{X}$, then $\bigcap_{i\in I}(G_A)_i = \phi$ which implies $\bigcap_{i\in I} s\beta cl(s\beta cl(H, B))^c = \phi$, which is contradiction. Then every soft β-open $\{(G_E)_i : i \in I\}$ of $\tilde{X}$ has a soft finite subfamily I_m such that $\bigcup_{i\in I_0} s\beta cl(G_A)_i = \tilde{X}$ for every $i \in I_m$. Hence $\tilde{X}$ is soft β-closed space.

Conversely, assume there exists a soft β-open finite intersection property ψ in $\tilde{X}$ such that $\bigcap_{(H,B)\in\psi} s\beta cl(H, B) = \phi$. That implies $\bigcup_{(H,B)\in\psi}(s\beta cl(H, B))^c = \tilde{X}$ for every $i \in I$ and hence $\{(G_E)_i : i \in I\} = \{s\beta cl(H, B) : (H, B) \in \psi\}$ is soft

β-open set cover of $\tilde{X}$. For X is soft β-closed, by definition $\{(G_E)_i : i \in I\}$ has a finite subfamily I_m such that $\bigcup_{i\in I_m} s\beta cl(s\beta cl(H,B))^c = \tilde{X}$ for every $I \in I_m$ and hence $\bigcap_{I\in I_0}(s\beta cl(s\beta cl(H,B))^c)^c = \phi$. Thus $\cap(H,B) \in I_m(H,B) \neq \phi$ is a contradiction. Therefore $\cap(H,B) \in \psi s\beta cl(H,B) \neq \phi$. □

Theorem 4.6. *Let $f : X \to Y$ be a soft β-irresolute function and if $\tilde{X}$ is soft β-closed space, then $\tilde{Y}$ is soft β-closed space.*

Proof. Let $\{(F_E)_i : i \in I\}$ be a soft β-open cover of Y. Since the function f is soft β-irresolute surjection, $\{f^{-1}(F_E)_i : i \in I\}$ is soft β-open cover of $\tilde{X}$. By the hypothesis, there exists a soft finite subset I_m of ψ such that $\bigcup_{i\in I_m} s\beta cl(f^{-1}(F_E)_i) = \tilde{X}$. For f is surjective and by the theorem $\tilde{Y} = f(\tilde{X}) = f(\bigcup_{i\in I_m} s\beta cl(f^{-1}(F_E)_i)) \subset \bigcup_{i\in I_0} s\beta cl(f(f^{-1}(F_E)_i)) = \bigcup_{i\in I_m} s\beta cl(F_E)_i$. Consequently, $\tilde{Y}$ is soft β-closed space. □

5. Soft β-first countable and soft β-second countable spaces

In this section, we proposed the concept of soft β-first countable and soft β-second countable spaces. Their properties are studied with suitable example.

Definition 5.1. *Let (X, τ, E) be a soft topological space and let (U, E) be a family of soft β-neighbourhood of some soft point $e_x \in \tilde{X}$. If for each soft β-neighbourood (F, E) of e_x, there exists (H, C) in (U, E) such that $e_x \in (H, C) \subseteq (F, E)$ then we say that (U, E) is a soft neighbourhood base at e_x.*

Definition 5.2. *Let (X, τ, E) be a soft topological space, and let e_x be a soft point in $\tilde{X}$. If e_x has a soft β-countable neighbourhood base, then we say that (X, τ, E) is soft β-first countable at the soft point e_x. If (X, τ, E) is soft β-first countable at each of its soft points, then we say that (X, τ, E) is soft β-first countable.*

Definition 5.3. *Let (X, τ, E) be a soft topological space is soft β-second countable if it has a soft β-countable base $\mho$ for its soft topology say $\mho = \{(F, A_1), (F, A_2), (F, A_3)...\}$. That is given any open set (F, A) and point $e_x \in (F, A)$ there is $(F, A_n) \in \mho$ such that $(F, A_n) \subseteq (F, A)$ with $e_x \in (F, A_n)$.*

Theorem 5.4. *Every soft β-second countable space is soft β-first countable.*

Proof. Let (X, τ, E) be a soft β-second countable space and suppose
$\mho = \{(F, A_1), (F, A_2), (F, A_3)...\}$ be the soft β-countable base. We can take for a soft β-basis at e_x the sequence of all (F, A_n) which contain e_x, call this collection as $\mho'$. Then $\mho'$ is soft β-countable as it is soft subset of the β-countable basis $\mho$ and since $\mho$ is soft β-basis, for any soft β-neighbourhood (G,B) of e_x, there exists $(F, A_n) \in \mho'$ such that $e_x \in (F, A_n) \subseteq (G, B)$. This implies that (X, τ, E) is soft β-first countable. □

Theorem 5.5. *A subpace of soft β-first countable space is soft β-first countable and same holds for soft β-second countability.*

Proof. We prove the results for soft β-second countability and first result follows from second. Let (X, τ, E) is soft β-second countable space and (G, τ', A) be a soft β-subspace of (X, τ, E). Suppose $\mho = \{(G, A_1), (G, A_2), (G, A_3), ..\}$ is soft β-countable basis for the space (X, τ, E). Then take the soft β-basis for subspace (G, τ', A) as $\mho' = \{(G, A) \cap (K, B) : (K, B) \in \mho\}$, which is soft β-countable. Therefore (G, τ', A) is soft β-second countable. □

Remark 5.6. There exist a soft β-first countable space which is not soft β-second countable.

Example 5.7. If we take soft descrete topology τ' on $\tilde{X}$, over the parameter set E, then each soft set in $\tilde{X}$ is soft β-open with respect to soft descrete topology. Take $\mho_{e_x} = \{(F, A)(e)\}$ a soft β-neighbourhood base at each soft point e_x. Then $\mho_{e_x}$ is soft countable and for each soft β-neighbourhood (G,B) of e_x, there is always $\{e_x\} \in \mho_{e_x}$ such that $e_x \in \{e_x\} \subseteq (G, B)$. Therefore (X, τ', E) is soft β-first countable space but it is not soft β-second countable space.

Theorem 5.8. *The image of soft β-first countable space under a soft open continuous map are soft β-first countable.*

Proof. Let (X, τ_1, E) and (Y, τ_2, E) be two soft topological spaces over the parameter set E, and suppose (X, τ_1, E) is soft β-first countable, and let $f : (X, \tau_1, E) \Rightarrow (Y, \tau_2, E)$ be a soft onto and continuous open mapping. Since f is onto, for any soft point e_y in $\tilde{Y}$, there exists a soft point e_x in $\tilde{X}$, such that $f[e_x] = e_y$. Since (X, τ_1, E) is soft β-first countable, there exists a soft β-countable neighbourhood base $\{(F, A_n)\}_{n \in N}$ at e_x. Then it is easy to see that $\{f(F, A_n)\}_{n \in N}$ is soft β-neighbourhood base at e_y. □

Definition 5.9. *A soft topological space (X, τ, E) is said to be soft β-Lindelöf if each soft β-open covering $(G, E_i)_{i \in N}$ of $\tilde{X}$ has a soft β-countable subcover.*

Theorem 5.10. *Each soft β-second countable space is soft β-Lindelöf.*

Proof. Let (X, τ, E) be a soft β-countable space, and let $\mho$ be a soft β-countable open base of $\tilde{X}$. Let $(G, E_i)_{i \in N}$ be an arbitarary soft β-open covering of $\tilde{X}$. Put $\mho' = \{(F, A) \in \mho : \text{there is } (G, B) \in (G, E_i)_{i \in N} \text{ such that } (F, A) \subseteq (G, B)\}$. Then $\mho'$ is countable. Denote $\mho'$ by $\{(F, A_n) : n \in N\}$. For each $n \in N\}$, there exists an $(H, A_n) \in (G, E_i)_{i \in N}$ such that $(F, A_n)_{n \in N} \subseteq (H, A_n)_{n \in N}$. Then $\{(H, A_n) : n \in N\}$ is soft countable subfamily of $(G, E_i)_{i \in N}$. Next we shall prove that $\{(H, A_n) : n \in N\}$ is soft β-cover of (X, τ, E). We take a soft arbitary point e_x in (X, τ, E). Since $(G, E_i)_{i \in N}$ is soft β-covering of $\tilde{X}$, there exists a $(K, E) \in (G, E_i)_{i \in N}$ such that $e_x \in (K, E)$. Since (K,E) is soft β-open set, so there is a $(L, E) \in \mho$ such that $e_x \in \mho \subseteq (K, E)$ because $\mho$ is a soft β-base of $\tilde{X}$. Hence $(L, E) \in \mho'$ and therefore, there is $n \in N$ such that $(L, E) = (F, A_n)$. Thus $e_x \in (F, A_n)$. As a consequence, $\{(H, A_n) : n \in N\}$ is soft β-cover of (X, τ, E). □

Theorem 5.11. *Each soft β-regular and soft β-Lindelöf space is soft β-normal.*

Let (X,τ,E) be a soft β-regular and soft β-Lindelöf space. Let (K,E_1) and (K,E_2) be two disjoint soft β-closed sets over $\tilde{X}$. For each soft point $e_x \in (K,E_1) \subseteq (K,E_2)^c$, and since (X,τ,E) is soft β-regular, then there exists a soft β-open neighbourhood (G,B) of e_x such that $e_x \in \overline{(G,B)} \subseteq (F,A_2)^c$, that is $\overline{(G,B)} \cap (F,A_2) = \phi$. Let $\Psi = \{(G,B) : e_x \in (K,E_1)\}$ that is Ψ is the collection of soft β-neighbourhoods of $e_x \in (K,E_1)$, then $\Psi \cup (K,E_1)^c$ is soft β-open cover of (X,τ,E). Since (X,τ,E) is a soft β-Lindelöf, there exists soft β-countable subcover $\{(G,B_n) : n \in N\} \cup (K,E_1)^c$. Put $(Z,E_n) = (G,B_n)$ for each $n \in N$, then $(K,E_1) \subseteq \bigcup_{n\in N}(Z,E_n)$ and each $(Z,E_n) \cap (K,E_2) = \phi$. Similarly, there exists countably many soft β-open sets $\{(F,E_n) : n \in N\}$ such that $(K,E_2) \subseteq \bigcup_{n\in N}(F,E_n)$ and each $(F,E_n) \cap (K,E_1) = \phi$. For each $n \in N$ put $(Z,E_n)' = (Z,E_n) \cap [\bigcup_{i=1}^{n}(\overline{(F,E_i)}^c]$, $(F,E_n)' = (F,E_n) \cap [\bigcup_{i=1}^{n}(\overline{(Z,E_i)}^c]$.

Then for m,n$\in$ N, we have $(Z,E_n)' \cap (F,E_m)' = \phi$, put $(Z,E) = \bigcup_{n\in N}(Z,E_n)'$, $(F,E) = \bigcup_{n\in N}(F,E_n)'$. Then we have $(K,E_1) \subseteq (Z,E)$, $(K,E_2) \subseteq (F,E)$ and $(I,E) \cap (F,E) = \phi$. Therefore (X,τ,E) is soft β-normal.

6. Conclusion

In this paper, we have studied enriched soft topology. In the sequel, we have introduced soft β-spaces and soft β-compact spaces and obtained some results.

On the other hand, we have given the definition of soft β-closed spaces and studied their basic characteristics in soft topological spaces. We have introduced soft β-first countable and soft β-second countable spaces, and their properties are studied in detail. Finally, we have introduced soft generalized β-compact spaces in soft topological spaces. The results are helpful for the further research on soft topology.

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