# SOFT ROUGH GROUPS AND CORRESPONDING DECISION MAKING

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**Abstract.** In this article, we apply soft rough sets (briefly, $SR$-sets) to the special algebraic structure-group and give the concepts of soft rough groups (briefly, normal subgroups) (briefly, $SR$-groups, $SRN$-subgroups), which is an extended definition of rough groups. Further, we use the terminologies of $C$-soft sets and $CC$-soft sets to research soft rough algebraic structures. Moreover, the roughness in groups w.r.t. $MSR$-approximation spaces are investigated. At the same time, we study some soft rough operations over groups. Specially, upper and lower $SR$-groups ($SRN$-subgroups) are explored. Finally, we raise a kind of decision making method (DM-method) for $SR$-groups and give an actual example to illustrate.
**Keywords:** $SR$-set, $SR$-group, $SRN$-subgroup, $MSR$-set, decision making.

## 1. Introduction

The basic logical thinking methods of human understanding of things and the establishment of knowledge are the classification. A classification method in mathematics is a partition of objects. Each partition only corresponds to an equivalent relationship of the domain. In this sense, each equivalent relationship on the universe is a knowledge, each equivalence class is a basic concept of this knowledge, a family of equivalence relations are a knowledge base (briefly, KB). Therefore, the study of knowledge base will be translated into the study of equivalence relations. In the context of this logic, Pawlak [20] proposed rough set theory (briefly, RST) to deal with ambiguous uncertainty problems in 1982. Since then, research on RST has emerged in many fields. However, the equivalence relation can not be used effectively in many practical problems, thus restricts the application and development of RST to a certain extent. Therefore, the rough set models [24, 28, 29] based on general binary relations were proposed, which greatly enriched and developed the Pawlak RST. In particular, Zhang et al. [29] explored the constructed methods of RA-operations and multigranuation rough sets. At the same time, the combinations of rough sets and algebraic

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structures are also found in many literatures. For examples, Biswas and Nanda [5] applied RST to groups, proposed rough groups (subgroups) and obtained some related properties. Later on, Kuroki [13] put forth the concept of a rough ideal in semigroups and studied some properties of such notion. Ali [3] and Davvaz et al. [7] researched the roughness in rings and hemirings, respectively. Recently, Bağırmaz [4] investigated rough prime ideals on approximation spaces. In recent years, rough sets were widely used in DM problems.

In 1999, a novel theory such as soft set theory (briefly, SST) was put forth by Russian scholar–Molodtsov [18], which is mainly used to solve the uncertainty problem. Compared with the traditional mathematical approaches, for examples, probability theory, fuzzy analytic method and RST, This theory has the unique advantage in dealing with the problem of uncertainty. Maji et al. [16] defined equality of two soft sets and complement of a soft set with examples. We know that some soft binary operations like $AND$, $OR$ and the operations of union and intersection were defined in [16]. Further, Ali [2] first pointed out that some assertions in [16] et al. are not true in general and gave some new operations of two soft sets. Based on this novel idea, the research on SST was wisely used in recent years. Several authors researched the combinations of soft sets and algebraic structures. In particular, Aktaş and N. Çağman [1] introduced the basic properties of soft sets and gave the notion of soft groups. Later on, Sezgin and A.O. Atagün [22] first corrected some of problematic cases in [1] and proposed the concepts of normalistic soft groups (briefly, NS-groups) and normalistic soft group homomorphism (briefly, NSG-homomorphisms), and studied some structures that are preserved under NSG-homomorphisms. In 2008, the definitions of some types of soft semirings, soft ideals and soft semiring homomorphism were given by Feng et al. [10]. In addition, SST in DM has become a hot topic. Firstly, Maji et al. [17] applied SST to solve a DM problem using SST. In particular, Çağman and N. Çağman [6] investigated the products of soft sets and constructed a uni-int DM-method by means of uni-int decision functions to solve the actual problems.

We know that uncertainties have many different performance, hence we can not catch hold of them by a single uncertain tool. Some researchers proposed some hybrid soft computing models, such as fuzzy soft sets (briefly, $FS$-sets), fuzzy rough sets (briefly, $FR$-sets), rough soft sets (briefly, $RS$-sets) and $SR$-sets, and so on. In 2010, Feng et al. [8] proposed $RS$-sets by means of RST and SST. Recently, based on the idea in [8], Zhan et al. [25] firstly applied $RS$-sets to hemirings, and investigated some vital properties of rough soft hemirings (briefly, $RS$-hemirings), and investigated some related properties. Moreover, Ma [14] and Pan and Zhan [19] applied $RS$-sets to $BCI$-algebras and groups, respectively. The most important criticism on RST is that it lacks parameterization tools and approaches. To tackle this problem, the notion of $SR$-sets was first proposed in [9]. Moreover, Qin et al. [21] researched $SR$-sets based on similarity measures. However, its shortcoming is that the soft set must be full in [9]. At the same time, Shabir et al. [23] showed clearly that there exist two problems on Feng's approach in [9], such as, (1) an upper approximation of a given set may be empty; (2) the upper approximation of the given set may not contain itself on Feng's $SR$-sets. In view of this reason, they proposed the concept of $MSR$-sets to resolve this problem. Later on, Kumar and Inbarani[12] investigated modified $SR$-sets by means of $ECG$ signal classification for cardiac arrhythmias. As a result, the combination of SST and RST is more effective when dealing with uncertain problems. In particular, Inbarani [11] studied $SR$-sets for heart value disease diagnosis. Based on the idea of [23], the $MSR$-set was applied to the algebraic structure. Zhan et al. [26] firstly applied $SR$-sets to hemirings. And so, the roughness in hemirings w.r.t. $MSR$-approximation spaces was studied. Further, Zhan and Zhu [27] proposed he concept of $Z$-SRF-sets and studied $Z$-SRF-ideals in hemirings by using three uncertain soft set models. Recently, Ma et al. [15] reviewed some types of DM-methods based on some kinds of hybrid soft set models.

Based on the above idea, in this present article, we apply the $MSR$-sets to groups. Referring to Shabir et al. [23] and Zhan et al. [26], we divide this paper into four parts. In section 2, we point out some basic terminologies, such as, rough sets and soft sets. In section 3, we give some operations of modified $SR$-sets in goups. In section 4, we investigate some characterizations of $SR$-groups and $SRN$-subgroups. Finally, an efficient approach for DM problem based on $SR$-groups is given in section 5.

## 2. Basic terminologies

Some useful terminologies about soft sets and rough sets are given.

**Definition 2.1** ([18]). The $\mathfrak{S} = (\delta, A)$ is said to be a soft set over $U$, where $\delta : A \to P(U)$ is a set-valued mapping.

**Definition 2.2** ([8]). A soft set $\mathfrak{S} = (\delta, A)$ over $U$ is called full if $\bigcup_{a\in A} \delta(a) = U$.

**Definition 2.3** ([1, 22]). (1) A soft set $\mathfrak{S} = (\delta, A)$ over group $G$ is said to be a soft group over $G$ if and only if $\delta(x) < G$ for all $x \in Supp(\delta, A)$,

(2) A soft set $\mathfrak{S} = (\delta, A)$ over group $G$ is called a normalisitic soft group (briefly, NS-group) over $G$ if $\delta(x)$ is a normal subgroup of $G$ for all $x \in Supp(\delta, A)$.

**Definition 2.4** ([9]). Let $\mathfrak{S} = (\delta, A)$ be a soft set over $U$. We define two basic operations:

$$\underline{apr}_P(X) = \{u \in U | \exists a \in A : u \in \delta(a) \subseteq X\},$$

$$\overline{apr}_P(X) = \{u \in U | \exists a \in A : u \in \delta(a), \delta(a) \cap X \neq \emptyset\},$$

assigning to every subset $X \subseteq U$, two sets $\underline{apr}_P(X)$ and $\overline{apr}_P(X)$ are called the lower and upper $SR$-approximations of $X$ in $P$, respectively. If $\underline{apr}_P(X) = \overline{apr}_P(X)$, $X$ is named as soft definable; if not $X$ is named as a $SR$-set. In what follows, we name it Feng-$SR$-set.

**Definition 2.5** ([23])**.** Put $\xi : U \to P(A)$ be a set-valued mapping shown as $\xi(x) = \{a|x \in \delta(a)\}$ and $(U, \xi)$ be an $MS$-approximation space. For any $V \subseteq U$, the lower $MSR$-approximation and upper $MSR$-approximation of $V$ are denoted by $\underline{V}_\xi$ and $\overline{X}_\xi$, resp., which are shown as

$$\underline{V}_\xi = \{x \in V|\xi(x) \neq \xi(z) \text{ for any } z \in V^c\}$$

and

$$\overline{V}_\xi = \{x \in U|\xi(x) = \xi(z) \text{ for some } z \in V\},$$

where $V^c = U - V$ is the complement of $V$.

Here, lower MSR-approximation can also be regarded as

$$\underline{V}_\xi = \{x \in U|\forall\ z \in V^c[\xi(x) \neq \xi(z)]\}.$$

If $\underline{V}_\xi = \overline{V}_\xi$, then $V$ is called $MS$-definable, if not $V$ is named as a Shabir-SR-set.

## 3. Soft rough approximations

Section 3 investigates some fundamental properties of modified soft rough sets over groups.

**Definition 3.1.** Assume that $\mathfrak{S} = (\delta, A)$ is a soft set over a group $G$ and $\xi : G \to P(A)$ is a map defined as $\xi(x) = \{a \in A|x \in \delta(a)\}$, $\mathfrak{S}$ is named as a $C$-soft set over $G$ if $\xi(a_1) = \xi(a_2)$ and $\xi(b_1) = \xi(b_2)$ imply $\xi(a_1 \cdot b_1) = \xi(a_2 \cdot b_2)$ for all $a_1, a_2, b_1, b_2 \in G$.

**Example 3.2.** We can consider that $G = \{\pm 1, \pm i\}$ is a group with $i^2 = -1$ and can define a soft set $\mathfrak{S} = (\delta, A)$ over $G$ shown by Table 1.

Table 1 table for soft set $\mathfrak{S}$

| | $1$ | $-1$ | $i$ | $-i$ |
|---|---|---|---|---|
| $e_1$ | 0 | 0 | 1 | 1 |
| $e_2$ | 1 | 1 | 1 | 1 |
| $e_3$ | 0 | 0 | 0 | 0 |

Then $\xi : G \to P(A)$ of SRA-space $(G, \xi)$ is shown by $\xi(1) = \xi(-1) = \{e_2\}$, $\xi(i) = \xi(-i) = \{e_1, e_2\}$.

It is easy to check that $\mathfrak{S}$ is a $C$-soft set over $G$.

Let $H, K$ be any two non-empty subsets in any group, denote $H \cdot K = \{h \cdot k|\forall h \in H, k \in K\}$.

**Theorem 3.3.** *We can suppose that* $\mathfrak{S} = (\delta, A)$ *is a* $C$*-soft set over* $G$ *and* $(G, \xi)$ *is an* $MS$*-approximation space. For any two non-empty subsets* $H, K$ *in* $G$*. Then* $\overline{H}_\xi \cdot \overline{K}_\xi \subseteq \overline{H \cdot K}_\xi$.

**Proof.** Let $x \in \overline{H}_\xi \cdot \overline{K}_\xi$, then $x = a \cdot b$, where $a \in \overline{H}_\xi$ and $b \in \overline{K}_\xi$, and so there exist $h \in H$ and $k \in K$ .t. $\xi(a) = \xi(h)$ and $\xi(b) = \xi(k)$. By the hypothesis, $\mathfrak{S}$ is a $C$-soft set, so $\xi(a \cdot b) = \xi(h \cdot k)$ for $h \cdot k \in H \cdot K$. Hence $x = a \cdot b \in \overline{H \cdot K}_\xi$. That is, $\overline{H}_\xi \cdot \overline{K}_\xi \subseteq \overline{H \cdot K}_\xi$. □

**Example 3.4.** Assume that $G = \{1, a, b, c\}$ is a group in the given Table 2:

Table 2 table for group $G$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | $a$ | $b$ | $c$ |
| $a$ | $a$ | 1 | $c$ | $b$ |
| $b$ | $b$ | $c$ | 1 | $a$ |
| $c$ | $c$ | $b$ | 1 | $a$ |

Define $\mathfrak{S} = (\delta, A)$ over $G$ which is given by Table 3.

Table 3 table for soft set $\mathfrak{S}$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 0 | 0 | 0 | 1 |
| $e_2$ | 1 | 1 | 1 | 0 |
| $e_3$ | 0 | 0 | 0 | 0 |

Suppose that the mapping $\xi : G \to P(A)$ of SRA-space $(G, \xi)$ is shown by $\xi(1) = \xi(a) = \xi(b) = \{e_2\}$, $\xi(c) = \{e_1\}$. Then it is clear that $\mathfrak{S}$ is a $C$-soft set over $G$.

Let $H = \{1, c\}$ and $K = \{c\}$, then $H \cdot K = \{1, c\}$, $\overline{H}_\xi = \{1, a, b, c\}$ and $\overline{K}_\xi = \{c\}$, so $\overline{H}_\xi \cdot \overline{K}_\xi = \{1, c\}$ and $\overline{H \cdot K}_\xi = \{1, a, b, c\}$. Thus $\overline{H}_\xi \cdot \overline{K}_\xi \subset \overline{H \cdot K}_\xi$.

**Definition 3.5.** Suppose that $\mathfrak{S} = (\delta, A)$ is a $C$-soft set over $G$ and $(G, \xi)$ is an $MS$-approximation space, then $\mathfrak{S}$ is named as a $CC$-soft set over $G$ if for all $c \in G$, $\xi(c) = \xi(x \cdot y)$, $\exists a, b \in G$ s.t. $\xi(x) = \xi(a)$ and $\xi(y) = \xi(b)$ with $c = a \cdot b$.

**Remark 3.6.** From Definition 3.5, we can obtain that a $CC$-soft set over $G$ is a $C$-soft set, but the converse do not hold in general.

**Example 3.7.** In Example 3.2, $\mathfrak{S}$ is a $C$-soft set over $G$, let $c = i \in G$ and $\xi(i) = \xi(1 \cdot i)$, from Table 1, we can see that $\xi(1) = \xi(-1)$ and $\xi(i) = \xi(i)$, but $c = i \neq (-i) = (-1) \cdot i$, so $\mathfrak{S}$ is not a $CC$-soft set over $G$.

**Theorem 3.8.** *Consider $\mathfrak{S} = (\delta, A)$ be a $CC$-soft set over $G$ and $(G, \xi)$ an $MS$-approximation space. For any two non-empty subsets $H, K$ in $G$. Then $\overline{H}_\xi \cdot \overline{K}_\xi = \overline{H \cdot K}_\xi$.*

**Proof.** By Theorem 3.3, we have $\overline{H}_\xi \cdot \overline{K}_\xi \subseteq \overline{H \cdot K}_\xi$. So we only need to prove $\overline{H \cdot K}_\xi \subseteq \overline{H}_\xi \cdot \overline{K}_\xi$.

Now let $c \in \overline{H \cdot K}_\xi$, so $\xi(c) = \xi(x \cdot y)$ for some $x \in H$ and $y \in K$. Since $\mathfrak{S}$ is a $CC$-soft set over $G$, there exist $a, b \in G$ s.t. $\xi(a) = \xi(x)$ and $\xi(b) = \xi(y)$ with $c = a \cdot b$. Thus $a \in \overline{H}_\xi$ and $b \in \overline{K}_\xi$. Hence $c = a \cdot b \in \overline{H}_\xi \cdot \overline{K}_\xi$. That is $\overline{H \cdot K}_\xi \subseteq \overline{H}_\xi \cdot \overline{K}_\xi$. So $\overline{H}_\xi \cdot \overline{K}_\xi = \overline{H \cdot K}_\xi$. □

**Theorem 3.9.** *Consider* $\mathfrak{S} = (\delta, A)$ *be a* $CC$*-soft set over* $G$ *and* $(G, \xi)$ *an* $MS$*-approximation space. For any two non-empty subsets* $H, K$ *in* $G$*. Then* $\underline{H}_{\xi} \cdot \underline{K}_{\xi} \subseteq \underline{H \cdot K}_{\xi}$.

**Proof.** Suppose that $\underline{H}_{\xi} \cdot \underline{K}_{\xi} \subseteq \underline{H \cdot K}_{\xi}$ does not hold, then there exists $c \in \underline{H}_{\xi} \cdot \underline{K}_{\xi}$, but $c \notin \underline{H \cdot K}_{\xi}$. This means that $\xi(a) \neq \xi(x)$ and $\xi(b) \neq \xi(y)$ for all $x \in H^c$ and $y \in K^c$. (♠)

On the other hand, $c \notin \underline{H \cdot K}_{\xi}$, then we may have the following two conditions:

(1) $c \notin H \cdot K$, but $c \in \underline{H}_{\xi} \cdot \underline{K}_{\xi} \subseteq H \cdot K$. This is a contradiction.

(2) $c \in H \cdot K$ and $\xi(c) = \xi(x^{'} \cdot y^{'})$ for some $x^{'} \cdot y^{'} \in (H \cdot K)^c$. Thus $x^{'} \in H^c$ or $y^{'} \in K^c$. Indeed, if $x^{'} \notin H^c$ and $y^{'} \notin K^c$, we have $x^{'} \cdot y^{'} \in H \cdot K$, a contradiction. By the hypothesis, $\mathfrak{S}$ is a $CC$-soft set over $G$, so $\exists a^{'}, b^{'} \in G$ such that $\xi(a^{'}) = \xi(x^{'})$ and $\xi(b^{'}) = \xi(y^{'})$ with $a^{'} \cdot b^{'} = c$ for some $x^{'} \in H^c$ and $y^{'} \in K^c$. Which contradicts to (♠). Hence $\underline{H}_{\xi} \cdot \underline{K}_{\xi} \subseteq \underline{H \cdot K}_{\xi}$. □

If $\mathfrak{S} = (F, A)$ is not a $CC$-soft set over $G$, Theorem 3.9 is not true.

**Example 3.10.** Let $G = \{1, 3, 5, 7\} \subseteq Z_8$ be a group where the operation is the ordinary multiplication.

We consider a soft set $\mathfrak{S} = (F, A)$ over $G$ which is given by Table 4.

Table 4 table for soft set $\mathfrak{S}$

| | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| $e_1$ | 1 | 1 | 1 | 0 |
| $e_2$ | 1 | 1 | 1 | 0 |
| $e_3$ | 0 | 0 | 0 | 1 |

We can consider the mapping $\xi : G \to P(A)$ of SRA-space $(G, \xi)$ is given by $\xi(1) = \xi(3) = \xi(5) = \{e_1, e_2\}$, $\xi(7) = \{e_3\}$.

Let $H = \{1, 3, 7\}$ and $K = \{7\}$, Then $\underline{H}_{\xi} = \{1, 3\}$ and $\underline{K}_{\xi} = \{7\}$, so $\underline{H}_{\xi} \cdot \underline{K}_{\xi} = \{5, 7\}$. Also we have $\underline{H \cdot K}_{\xi} = \{7\}$, so $\underline{H}_{\xi} \cdot \underline{K}_{\xi} \nsubseteq \underline{H \cdot K}_{\xi}$.

## 4. Characterizations of soft rough groups

Section 4 discusses the operations of lower and upper $MSR$-approximations of $SR$-groups. In order to investigate the roughness of group $G$ with respect to soft rough approximation space over groups, we first give the notions of $SR$-groups and $SRN$-subgroups.

**Definition 4.1.** We can consider that $\mathfrak{S} = (\delta, A)$ is a soft set over $G$ and $(G, \xi)$ is an $MS$-approximation space. For any $X \subseteq G$, the lower and upper $MSR$-approximations of $X$ are given by $\underline{X}_{\xi}$ and $\overline{X}_{\xi}$, resp., which two operations are defined as

$$\underline{X}_{\xi} = \{x \in X | \forall y \in X^c : \xi(x) \neq \xi(y)\}$$

and

$$\overline{X}_{\xi} = \{x \in G | \exists y \in X : \xi(x) = \xi(y)\}$$

If $\underline{X}_\xi \neq \overline{X}_\xi$, then
(1) $X$ is called a lower (upper) $SR$-group (resp., $SRN$-subgroup) over $G$, if $\underline{X}_\xi$ ($\overline{X}_\xi$) is a subgroup (resp., normal subgroup) of $G$;
(2) $X$ is called an $SR$-group (resp., $SRN$-subgroup) over $G$, if $\underline{X}_\xi$ and $\overline{X}_\xi$ are subgroups (resp., normal subgroups) of $G$.

**Example 4.2.** *Let $G = \{\pm 1, \pm i\}$ be a group with $i^2 = -1$ and define a soft set $\mathfrak{S} = (\delta, A)$ over $G$ which is given by Table 5:*

*Table 5 table for soft set $\mathfrak{S}$*

| | 1 | −1 | $i$ | $-i$ |
|---|---|---|---|---|
| $e_1$ | 0 | 0 | 1 | 1 |
| $e_2$ | 1 | 1 | 1 | 1 |
| $e_3$ | 1 | 1 | 0 | 0 |

*We can consider $\xi : G \to P(A)$ of SRA-space $(G, \xi)$ is shown by $\xi(1) = \xi(-1) = \{e_2, e_3\}$ and $\xi(i) = \xi(-i) = \{e_1, e_2\}$.*

*Let $X = \{\pm 1, i\}$, then $\underline{X}_\xi = \{\pm 1\}$ and $\overline{X}_\xi = \{\pm 1, \pm i\} = G$. We can check that $\underline{X}_\xi$ and $\overline{X}_\xi$ are normal subgroups of $G$. This shows that $X$ is an $SRN$-subgroup over $G$.*

**Proposition 4.3.** *We can consider that $(G, \xi)$ is an $MS$-approximation space. If $H$ and $K$ are lower $SR$-groups ($SRN$-subgroups) over $G$, then so is $H \cap K$.*

**Proof.** If $H$ and $K$ are lower $SR$-groups ($SRN$-subgroups) over $G$, then $\underline{H}_\xi$ and $\underline{K}_\xi$ are subgroups (normal subgroups) of $G$, so $\underline{H}_\xi \cap \underline{K}_\xi$ is a subgroup (normal subgroup) of $G$. By Definition 4.1, it is easy to verity $\underline{H \cap K}_\xi = \underline{H}_\xi \cap \underline{K}_\xi$. So $\underline{H \cap K}_\xi$ is also a subgroup (normal subgroup) of $G$. Hence $H \cap K$ is a lower $SR$-group ($SRN$-subgroup) of $G$. □

In general, $H \cap K$ is not an upper $SR$-group of $G$, if $H$ and $K$ are upper $SR$-groups of $G$. Actually we give the following example to illustrate.

**Example 4.4.** Consider the group $G$ and the soft set $\mathfrak{S} = (F, A)$ in Example 4.2. Now let $H = \{1, -i\}$ and $K = \{-1, -i\}$. By calculation, we have $\overline{H}_\xi = \overline{K}_\xi = \{\pm 1, \pm i\}$. It is obvious that $\overline{H}_\xi$ and $\overline{K}_\xi$ are subgroups of $G$, so $H$ and $K$ are upper $SR$-groups of $G$. But $\overline{H \cap K}_\xi = \overline{\{-i\}}_\xi = \{\pm i\}$ is not a subgroup of $G$, so $H \cap K$ is not an upper $SR$-groups of $G$.

Next, the example shows that $H \cup K$ is also not a lower (an upper) $SR$-group of $G$, if $H$ and $K$ are $SR$-groups of $G$.

**Example 4.5.** We can consider that $G = \{1, a, b, c\}$ is a set with a multiplication operation $(\cdot)$ as follow:

| $\cdot$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | $a$ | $b$ | $c$ |
| $a$ | $a$ | 1 | $c$ | $b$ |
| $b$ | $b$ | $c$ | 1 | $a$ |
| $c$ | $c$ | $b$ | $a$ | 1 |

Then $G$ is a group. We can consider that $\mathfrak{S} = (\delta, A)$ is a soft set over $G$ which is defined as Table 6.

Table 6 table for soft set $\mathfrak{S}$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 0 | 0 | 1 | 1 |
| $e_2$ | 1 | 1 | 0 | 0 |
| $e_3$ | 1 | 1 | 0 | 1 |

We can consider $\xi : G \to P(A)$ of SRA-space $(G, \xi)$ is given by $\xi(1) = \xi(a) = \{e_2, e_3\}$ and $\xi(b) = \{e_1\}$, $\xi(b) = \{e_1, e_3\}$.

If we take $H = \{1, a\}$ and $K = \{1, b\}$, then $\underline{H}_\xi = \{1, a\}$, $\underline{K}_\xi = \{1, b\}$, $\overline{H}_\xi = \{1, a\}$, $\overline{K}_\xi = \{1, b\}$, so $H$ and $K$ are $SR$-groups of $G$. Moreover, $\underline{H \cup K}_\xi = \underline{\{1, a, b\}}_\xi = \{1, a, b\}$ and $\overline{H \cup K}_\xi = \overline{\{1, a, b\}}_\xi = \{1, a, b\}$. That is, $H \cup K$ is not a lower (an upper) $SR$-group of $G$.

In the following, we investigate the upper and lower $SR$-groups.

**Theorem 4.6.** *We can consider $\mathfrak{S} = (\delta, A)$ be a $C$-soft set over $G$ and $H$ a subgroup of $G$. Then $\overline{H}_\xi$ is a subgroup of $G$.*

**Proof.** Since $H \subseteq \overline{H}_\xi$, then $e \in H \subseteq \overline{H}_\xi$. For all $a, b \in \overline{H}_\xi$, by Definition 4.1, $\xi(a) = \xi(x)$ and $\xi(b) = \xi(y)$ for some $x, y \in H$. By the hypothesis, $\mathfrak{S} = (F, A)$ is a $C$-soft set over $G$, so $\xi(a \cdot b) = \xi(x \cdot y)$ for $x \cdot y \in H \cdot H \subseteq H$, then $a \cdot b \in \overline{H}_\xi$. Since $H$ is a subgroup of $G$, we have $a^{-1} \in H$ for all $a \in H$. That is, for all $a \in \overline{H}_\xi$, we have $a^{-1} \in \overline{H}_\xi$. Hence $\overline{H}_\xi$ is a subgroup of $G$. □

**Theorem 4.7.** *We can consider $\mathfrak{S} = (\delta, A)$ be a $CC$-soft set over $G$ and $H$ a subgroup of $G$. Then $\underline{H}_\xi$ is a subgroup of $G$ if $\underline{H}_\xi \neq \emptyset$.*

**Proof.** Let $\underline{H}_\xi \neq \emptyset$, for all $x, y \in \underline{H}_\xi$. Suppose that $x \cdot y \notin \underline{H}_\xi$. Then we have $\xi(x) \neq \xi(a)$ for all $a \in H^c$ and $\xi(y) \neq \xi(b)$ for all $b \in H^c$.

On the other hand, it may have the following two conditions, if $x \cdot y \notin \underline{H}_\xi$.

(1) $x \cdot y \notin H$, which contradicts with $x \cdot y \in \underline{H}_\xi \cdot \underline{H}_\xi \subseteq H \cdot H \subseteq H$;

(2) $x \cdot y \in H$ and $\xi(c) = \xi(m \cdot n)$ for some $c \in H^c$. By the hypothesis, $\mathfrak{S} = (F, A)$ is a $CC$-soft set over $G$, then $\exists m, n \in G$ such that $\xi(x) = \xi(m)$ and $\xi(y) = \xi(n)$ satisfying $m \cdot n = c \in H^c$. Thus, $m \in H^c$ or $n \in H^c$. If $m \notin H^c$ and $n \notin H^c$, we have $m \cdot n \in H \cdot H \subseteq H$, a contradiction. That is, $\exists m \in H^c$ s.t. $\xi(x) = \xi(m)$ or $n \in H^c$ with $\xi(y) = \xi(n)$. This is contradicts to $x \cdot y \notin \underline{H}_\xi$. Thus $x \cdot y \in \underline{H}_\xi$. Similarly, we have $x^{-1}, y^{-1} \in \underline{H}_\xi$. This implies $\underline{H}_\xi$ is a subgroup of $G$. □

**Theorem 4.8.** *We can consider that* $\mathfrak{S} = (F, A)$ *is a* $C$*-soft set over* $G$ *and* $H$ *is a normal subgroup of* $G$*. Then* $\overline{H}_\xi$ *is a normal subgroup of* $G$*.*

**Proof.** Let $H$ be a normal subgroup of $G$. By Theorem 4.6, we have $\overline{H}_\xi$ is a subgroup of $G$. If $x \in G$ and $y \in \overline{H}_\xi$, then $\xi(y) = \xi(a)$ for some $a \in H$, by the hypothesis, we have $x \cdot a \cdot x^{-1} \in H$. By the hypothesis, $\xi(x) = \xi(x)$, $\xi(x^{-1}) = \xi(x^{-1})$ and $\mathfrak{S} = (F, A)$ be a $C$-soft set, so $\xi(x \cdot y \cdot x^{-1}) = \xi(x \cdot a \cdot x^{-1})$ for some $x \cdot a \cdot x^{-1} \in H$, thus $x \cdot y \cdot x^{-1} \in \overline{H}_\xi$. Hence $\overline{H}_\xi$ is a normal subgroup of $G$. □

**Theorem 4.9.** *We can consider that* $\mathfrak{S} = (\delta, A)$ *is a* $CC$*-soft set over* $G$ *and* $H$ *is a normal subgroup of* $G$*. Then* $\underline{H}_\xi$ *is a normal subgroup of* $G$ *if* $\underline{H}_\xi \neq \emptyset$*.*

**Proof.** By Theorem 4.7 and the hypothesis, it is easy to verify. □

The following example show that the converse of Theorems 4.8 and 4.9 are not true.

**Example 4.10.** Put the group $G$ in Example 4.5. $\mathfrak{S} = (\delta, A)$ is a soft set over $G$ shown by Table 7.

Table 7 table for soft set $\mathfrak{S}$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 0 | 0 | 1 | 1 |
| $e_2$ | 1 | 1 | 0 | 0 |
| $e_3$ | 1 | 1 | 0 | 0 |

We can consider that $\xi : G \to P(A)$ of SRA-space $(G, \xi)$ is given by $\xi(1) = \xi(a) = \{e_2, e_3\}$ and $\xi(b) = \xi(c) = \{e_1\}$. It is easy to check that $\mathfrak{S}$ is a $CC$-soft set over $G$.

Let $H = \{1, a, b\}$, then $\underline{H}_\xi = \{1, a\}$, $\overline{H}_\xi = \{1, a, b, c\}$. Thus we have $\underline{H}_\xi$ and $\overline{H}_\xi$ are normal subgroups of $G$, but $H$ is not a normal subgroup of $G$.

## 5. Soft rough groups in decision making methods

In recent years, both SST and RST have been applied to tackle the problems in decision making. Due to the particularity of the environment and strategy of different forms, every decision making method has own benefit and drawback. So it is not possible to decide which method is the most appropriate.

In this section, we praise a kind of DM-method to choose the optimal parameter of $\mathfrak{S} = (F, A)$ which is given. That is, $F(e)$ is the nearest accurate groups on $\mathfrak{S}$ based on another soft set $\mathfrak{E}$ over groups.

We can consider $G$ be a group and $E$ a set of parameters. Let $A = \{e_1, e_2, \cdots, e_n\} \subseteq E$, $\mathfrak{S} = (\delta, A)$ be an original description soft set over $G$ and $(G, \xi)$ be an $MS$-approximation space. Let $\mathfrak{E} = (X, B)$ be another soft set over $G$.

**The algorithm of DM-method:**

**Step I** Consider the group $G$, the given soft set $\mathfrak{S}$, the $MSR$-approximation space $(G,\xi)$ and define another soft set $\mathfrak{E}=(X,B)$ over $G$.

**Step II** Reckon the lower (upper) $SR$-approximation operators $\underline{(X,B)}_\xi$ and $\overline{(X,B)}_\xi$ w.r.t $\mathfrak{S}$, respectively.

**Step III** Reckon $\|X(e_i)\|$, where $\|X(e_i)\| = \frac{|\underline{X(e_i)}_\xi|}{|\overline{X(e_i)}_\xi|}$.

**Step IV** Find the maximum values $\|X(e_j)\|$ of $\|X(e_i)\|$, where $\|X(e_j)\| = \max_i \|X(e_i)\|$.

**Step V** The decision goal is $X(e_j)$.

**Example 5.1.** In order to find the nearest accurate group, we can consider $G=\{1,a,b,c\}$ be a group with the given Table 8:

Table 8 table for group $G$

| $\cdot$ | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| 1 | 1 | $a$ | $b$ | $c$ |
| $a$ | $a$ | 1 | $c$ | $b$ |
| $b$ | $b$ | $c$ | 1 | $a$ |
| $c$ | $c$ | $b$ | 1 | $a$ |

Define $\mathfrak{S}=(\delta,A)$ over $G$ which is given by Table 9.

Table 9 table for soft set $\mathfrak{S}$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 1 | 1 | 1 | 0 |
| $e_2$ | 1 | 1 | 0 | 1 |
| $e_3$ | 0 | 0 | 0 | 1 |
| $e_4$ | $c$ | 0 | 1 | 1 |

We can consider $\xi : G \to P(A)$ of SRA-space $(G,\xi)$ is given by $\xi(1)=\xi(a)=\{e_1,e_2\}$, $\xi(b)=\{e_1,e_4\}$, $\xi(b)=\{e_2,e_3,e_4\}$.

Define another soft set $\mathfrak{E}=(X,B)$ over $G$ which is given by Table 10.

Table 10 table for soft set $\mathfrak{E}$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 1 | 0 | 0 | 0 |
| $e_2$ | 1 | 0 | 1 | 1 |
| $e_3$ | 1 | 1 | 1 | 0 |
| $e_4$ | 1 | 0 | 1 | 0 |
| $e_5$ | 0 | 1 | 0 | 1 |

That is, $X(e_1)=\{1\}$, $X(e_2)=\{1,b,c\}$, $X(e_3)=\{1,a,b\}$, $X(e_4)=\{b,c\}$, $X(e_5)=\{a,c\}$.

By calculations, we have that $\underline{X(e_1)}_\xi = \emptyset$, $\overline{X(e_1)}_\xi = \{1, a\}$, $\underline{X(e_2)}_\xi = \{b, c\}$, $\overline{X(e_2)}_\xi = \{1, a, b, c\}$, $\underline{X(e_3)}_\xi = \{1, a, b\}$, $\overline{X(e_3)}_\xi = \{1, a, b, c\}$, $\underline{X(e_4)}_\xi = \{b\}$, $\overline{X(e_4)}_\xi = \{1, a, b\}$, $\underline{X(e_5)}_\xi = \{c\}$ and $\overline{X(e_5)}_\xi = \{1, a, c\}$.

Then, we can obtain that $\|X(e_1)\| = 0$, $\|X(e_2)\| = 0.5$, $\|X(e_3)\| = 0.75$, $\|X(e_4)\| = 0.33$ and $\|X(e_5)\| = 0.33$. So the maximum values $\|X(e_j)\|$ is $\|X(e_3)\| = 0.75$. This means that $X(e_3) = \{1, a, b\}$ is the closest accurate group.

**Remark 5.2.** (1) For a given soft set $\mathfrak{S} = (F, A)$ which is the original description over $G$, due to the flexibility of another soft set $\mathfrak{E} = (X, B)$, by adjusting soft set $\mathfrak{E}$, we can obtain different result. In other word, the decision maker can find the optimal object by changing the soft set $\mathfrak{E}$. In Example 5.1, if we change the soft set $\mathfrak{E}$ as follow Table 11:

Table 11 table for soft set $\mathfrak{E}$

| | 1 | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| $e_1$ | 1 | 0 | 0 | 0 |
| $e_2$ | 1 | 0 | 1 | 1 |
| $e_3$ | 1 | 1 | 0 | 0 |
| $e_4$ | 1 | 0 | 1 | 0 |
| $e_5$ | 0 | 1 | 0 | 1 |

So $X(e_1) = \{1\}$, $X(e_2) = \{1, b, c\}$, $X(e_3) = \{a, b\}$, $X(e_4) = \{b, c\}$, $X(e_5) = \{a, c\}$. By calculations, we find the nearest accurate group is $X(e_2) = \{1, b, c\}$;

(2) By Steps III and IV in the algorithm of the DM-method, we have that: the larger the value of $\|X(e_i)\|$ is, the closer $|\underline{X(e_i)}_\xi|$ and $|\overline{X(e_i)}_\xi|$ are. No matter how to change the soft set $\mathfrak{E}$, $0 \leq \|X(e_j)\| = \max_i \|X(e_i)\| \leq 1$.

## 6. Conclusion

As far as known that Shabir et al. [23] showed clearly that there exist two problems on Feng et al.'s approach in [9], such as, (1) an upper approximation of a given set may be empty; (2) the upper approximation of the given set may not contain itself on Feng et al.'s $SR$-sets. Moreover, in order to remove the limited condition that full soft sets are needed in Feng-$SR$-sets, Shabir et al. proposed the concept of $MSR$-sets. Based on the idea of [8, 9], we can find that $RS$-sets and $SR$-sets are different theories, $RS$-sets are soft sets while $SR$-sets are classical sets. For the development of two theories, rough soft algebras ($RS$-algebras) can be seen in many articles, but few authors investigated the soft rough algebras ($SR$-algebras). Until now, only Zhan et al. [26] applied $SR$-sets to hemirings and the roughness in hemirings w.r.t. $MSR$-approximation space was studied.

In the present paper, based on the ideas of Shabir et al. [23] and Zhan et al. [26], we apply the $MSR$-sets to groups. In section 3, we first investigate some operations and fundamental properties of modified $SR$-set over groups which are different from the ordinary universe, it provides us the idea and theoretical basis for studying other algebraic structures. In section 4, by the characterization of groups, roughness such as lower and upper $SR$-groups ($SRN$-subgroups) in groups w.r.t. $MSR$-approximation space are explored. This can give us a correct idea for the subsequent studies on semigroups, $n$-ary groups $n$-ary semigroups and $(m, n)$-ary rings. In section 5, we put forward a kind of decision making method for $SR$-groups which are not the same to [14] and [26] and an actual example is given to illustrate the method.

In the future research, we can study the following topics:

(1) Applying this novel $SR$-sets to other different algebras, such as semigroups, $n$-ary groups, $n$-ary semigroups, $(m, n)$-ary rings and so on.

(2) Studying soft rough fuzzy groups by exchanging groups for fuzzy groups.

(3) Investigating DM-methods based on SRF-groups.

(4) Applying this novel $SR$-sets to some applied areas.

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