ON INTUITIONISTIC PRODUCT FUZZY GRAPHS

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Abstract. In this paper, we provide three new operations on intuitionistic product fuzzy graphs; namely direct product, semi-strong product and strong product.We discuss which of these operations preseverses the notions of strong and complete. We also give some neaw properties of balanced intuitionistic fuzzzy graphs.
Keywords: Fuzzy product graph, fuzzy intuitionistic product graph, balanced intuitionistic product fuzzy graph.

1. Background

In 1736, Euler introduced the concept of graph theory while trying to find a solution to the well known Konigsberg bridge problem. This subject is now considered as a branch of combinatorics. The theory of graph is an extremely useful tool in solving combinatorial problems in areas such as geometry, algebra, number theory, topology, operations research, optimization and computer science. The notion of fuzzy set was first introduced in [19] and the notion of fuzzy graph was first introduced in [4]. Sense then, several authors explored this type of graphs. Since the notions of degree, complement, completeness, regularity, connectedness and many others play very important rules in the crisp graph case, the idea is to see what corresponds to these notions in the case of fuzzy graphs. Several authors introduced and studied what they called product fuzzy graphs, see for example [41]. AL-Hawary [32] introduced the concept of balanced fuzzy graphs. He defined three new operations on fuzzy graphs and explored what classes of fuzzy graphs are balanced. Since then, many authors have studied the idea of balanced on distinct kinds of fuzzy graphs, see for example [8, 9, 18, 24, 25, 27]. In 1983, Atanassov [16] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [17]. Atanassov added a new component in the definition of fuzzy set. The fuzzy sets give the degree of

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membership of an element in a given set and the nonmembership degree equals one minus the degree of membership, while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other. The only requirement is that the sum of these two degrees is not greater than 1. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [15, 28]. Parvathy and Karunambigai [29] introduced the concept of Intuitionistic fuzzy graph (IFG) elaborately and analyzed its components. Articles [16, 20, 22, 28] motivated us to analyze balanced IFGs and their properties. Several authors have studied balanced IFGs, see for example [24, 25] and balanced product IFGs (PIFGs) were studied by [27, 42]. The idea of balanced came from matroids, see [31, 33, 34, 35, 36, 37, 38, 40, 39].

This paper deals with the significant properties of balanced IFGs. The basic definition and theorems needed are discussed in section 2. In Section 3, we define three new operations on PIFGs and we discuss which of these operations preserves the notions of strong and complete. Section 4 is devoted to providing more new results on PIFGs. We remark that the results in this paper were done in Bayan Hourani masters thesis titled "On complete and balanced fuzzy graphs" under the supervision of Talal Al-Hawary at Yarmouk University in 2015.

2. Background

A fuzzy subset of a non-empty set V is a mapping $\sigma : V \to [0,1]$ and a fuzzy relation μ on a fuzzy subset σ, is a fuzzy subset of $V \times V$. All throughout this paper, we assume that σ is reflexive, μ is symmetric and V is finite.

Definition 1 ([4]). A *fuzzy graph* with V as the underlying set is a pair $G : (\sigma, \mu)$ where $\sigma : V \to [0,1]$ is a fuzzy subset and $\mu : V \times V \to [0,1]$ is a fuzzy relation on σ such that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y), \forall\ x,y \in V$, where $\wedge$ stands for minimum. The underlying crisp graph of G is denoted by $G^* : (\sigma^*, \mu^*)$ where $\sigma^* = \sup p(\sigma) = \{x \in V : \sigma(x) > 0\}$ and $\mu^* = \sup p(\mu) = \{(x,y) \in V \times V : \mu(x,y) > 0\}$. $H = (\sigma', \mu')$ is a *fuzzy subgraph* of G if there exists $X \subseteq V$ such that $\sigma' : X \to [0,1]$ is a fuzzy subset and $\mu' : X \times X \to [0,1]$ is a fuzzy relation on σ' such that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y), \forall\ x,y \in X$.

Definition 2 ([41]). *An* intuitionistic fuzzy graph (*simply, IFG*) *is of the form* $G : (V,E)$ *where*

(i) $V = \{\nu_0, \nu_1, ..., \nu_n\}$ *such that* μ_{1}:$V \to [0,1]$ *and* $\gamma_1 : V \to [0,1]$, *denotes the degree of membership and non-membership of the element* $v_i \in V$, *respectively and* $0 \leq \mu_1(\nu_i) + \gamma_1(\nu_i) \leq 1, \forall\ \nu_i \in V, (i = 1, 2, ..., n)$,

(ii) $E \subseteq V \times V$ *where* $\mu_2 : V \times V \to [0,1]$ *and* $\gamma_2 : V \times V \to [0,1]$ *are such that*

$$\mu_2(\nu_i, \nu_j) \leq \min[\mu_1(\nu_i), \mu_1(\nu_j)],$$
$$\gamma_2(\nu_i, \nu_j) \leq \max[\gamma_1(\nu_i), \gamma_1(\nu_j)]$$

and $0 \leq \mu_2(\nu_i, \nu_j) + \gamma_2(\nu_i, \nu_j) \leq 1$, *for every* $(\nu_i, \nu_j) \in E$, $(i, j = 1, 2, ..., n)$.

Definition 3 ([41]). *An IFG* $H = (\acute{V}, \acute{E})$ *is said to be an* Intuitionistic fuzzy subgraph *(IFSG) of the IFG* $G : (\gamma_1; \mu_1, \gamma_2; \mu_2)$ *if*

(i) $\acute{V} \subseteq V$, *where* $\acute{\mu}_{1_i} = \mu_{1_i}$, $\acute{\gamma}_{1i} = \gamma_{1i}$, $\forall \nu_i \in \acute{V}$, $i = 1, 2, 3, ..., n$.

(ii) $\acute{E} \subseteq E$, *where* $\acute{\mu}_{2_{ij}} = \mu_{2_{ij}}$, $\acute{\gamma}_{2_{ij}} \geq \gamma_{2_{ij}}$, $\forall (\upsilon_i, \nu_j) \in \acute{E}$, $i, j = 1, 2, ..., n$.

Corollary 1 ([30]). *The* complement *of an IFG,* $G : (V, E)$ *is an IFG,* $G^c : (V^c, E^c)$, *where*

(i) $V^c = V$,

(ii) $\mu^c_{1i} = \mu_{1i}$ *and* $\gamma^c_{1i} = \gamma_{1i}$, $\forall i = 1, 2, ..., n$,

(iii) $\mu^c_{2ij} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij}$ *and* $\gamma^c_{2ij} = \max(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij}$, $\forall i, j = 1, 2, ..., n$.

Definition 4 ([30]). *An IFG,* $G : (V, E)$ *is said to be* complete IFG *if* $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, $\forall \nu_i, \nu_j \in V$.

Definition 5 ([30]). *An IFG,* $G : (V, E)$ *is said to be* strong *IFG if* $\mu_{2ij} = min(\mu_{1i}, \mu_{1j})$ and $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, $\forall \nu_i, \nu_j \in E$.

Definition 6 ([24]). Let $G_1 : (V_1, E_1)$ *and* $G_2 : (V_2, E_2)$ *be two IFG's. An isomorphism between* G_1 *and* G_2 *(denoted by* $G_1 \simeq G_2$*) is a bijective map* $h : V_1 \to V_2$ *which satisfies*

$$\mu_1(\nu_i) = \acute{\mu}_1(h(\nu_i)), \nu_1(\nu_i) = \acute{\nu}_1(h(\nu_i)), \mu_2(\nu_i, \nu_j) = \acute{\mu}_2(h(\nu_i), h(\nu_j))$$

and

$$\gamma_2(\nu_i, \nu_j) = \acute{\gamma}_2(h(\nu_i), h(\nu_j)), \forall \nu_i, \upsilon_j \in V.$$

Definition 7 ([24]). *The* density *of an IFG,* $G : (V, E)$ *is*

$$D(G) = (D_\mu(G), D_\gamma(G))$$

where

$$D_\mu(G) = \frac{2 \sum_{(\nu_i, \nu_j) \in V} (\mu_2(\nu_i, \nu_j))}{\sum_{(\nu_i, \nu_j) \in E} (\mu_1(\nu_i) \wedge \mu_1(\nu_j))}$$

and

$$D_\gamma(G) = \frac{2 \sum_{(\nu_i, \nu_j) \in V} (\gamma_2(\nu_i, \nu_j))}{\sum_{(\nu_i, \nu_j) \in E} (\gamma_1(\nu_i) \vee \gamma_1(\nu_j))}.$$

Definition 8 ([24]). *An IFG, $G:(V,E)$ is* balanced *if $D(H)\leq D(G)$, that is $D_\mu(H)\leq D_\mu(G)$, $D_\gamma(H)\leq D_\gamma(G)$,$\forall$ non-empty subgraphs H of G.*

Definition 9 ([25]). *Let $G_1:(V_1,E_1)$ and $G_2:(V_2,E_2)$ be an IFG'S, where $V=V_1\times V_2$ and*

$$E=\{(x_1,y_1)(x_2,y_2):(x_1,x_2)\in E_1,\ (y_1,y_2)\in E_2\}.$$

Then the direct product *of G_1 and G_2 is an IFG denoted by $G_1\sqcap G_2=(V,E)$, where*

$$(\mu_1\sqcap\acute{\mu}_1)(x_1,y_1)=\mu_1(x_1)\wedge\acute{\mu}_1(y_1),\ \forall(x_1,y_1)\in V_1\times V_2,$$
$$(\gamma_1\sqcap\acute{\gamma}_1)(x_1,y_1)=\gamma_1(x_1)\vee\acute{\gamma}_1(y_1),\ \forall(x_1,y_1)\in V_1\times V_2,$$
$$(\mu_2\sqcap\acute{\mu}_2)(x_1,y_1)(x_2,y_2)=\mu_2(x_1,x_2)\wedge\acute{\mu}_2(y_1,y_2),\ \forall(x_1,x_2)\in E_1,(y_1,y_2)\in E_2,$$
$$(\gamma_2\sqcap\acute{\gamma}_2)(x_1,y_1)(x_2,y_2)=\gamma_2(x_1,x_2)\vee\acute{\gamma}_2(y_1,y_2),\ \forall(x_1,x_2)\in E_1,(y_1,y_2)\in E_2.$$

Definition 10 ([25]). *Let $G_1:(V_1,E_1)$ and $G_2:(V_2,E_2)$ be an IFG'S, where $V=V_1\times V_2$ and*

$$E=\{(x,x_2)(x,y_2):x\in V_1,\ (x_2,y_2)\in E_2\}$$
$$\cup\{(x_1,y_1)(x_2,y_2):(x_1,x_2)\in E_1,(y_1,y_2)\in E_2\}.$$

Then the semi-strong product *of G_1 and G_2 is an IFG denoted by $G_1\odot G_2=(V,E)$, where*

$$(\mu_1\odot\acute{\mu}_1)(x_1,x_2)=\mu_1(x_1)\wedge\acute{\mu}_1(x_2),\ \forall(x_1,x_2)\in V_1\times V_2,$$
$$(\gamma_1\odot\acute{\gamma}_1)(x_1,x_2)=\gamma_1(x_1)\vee\acute{\gamma}_1(x_2),\ \forall(x_1,x_2)\in V_1\times V_2,$$
$$(\mu_2\odot\acute{\mu}_2)(x,x_2)(x,y_2)=\mu_1(x)\wedge\acute{\mu}_2(x_2,y_2),\ \forall x\in V_1,(x_2,y_2)\in E_2,$$
$$(\gamma_2\odot\acute{\gamma}_2)(x,x_2)(x,y_2)=\gamma_1(x)\vee\acute{\gamma}_2(x_2,y_2),\ \forall x\in V_1,(x_2,y_2)\in E_2,$$
$$(\mu_2\odot\acute{\mu}_2)(x_1,x_2)(y_1,y_2)=\mu_2(x_1,x_2)\wedge\acute{\mu}_2(y_1,y_2),\ \forall(x_1,x_2)\in E_1,(y_1,y_2)\in E_2,$$
$$(\gamma_2\odot\acute{\gamma}_2)(x_1,x_2)(y_1,y_2)=\gamma_2(x_1,x_2)\vee\acute{\gamma}_2(y_1,y_2),\ \forall(x_1,x_2)\in E_1,(y_1,y_2)\in E_2.$$

Definition 11 ([25]). *Let $G_1:(V_1,E_1)$ and $G_2:(V_2,E_2)$ be an IFG'S, where* $V=V_1\times V_2$ and $E=\{(x,y_1)(x,y_2):x\in V_1,(y_1,y_2)\in E_2\}\cup\{(x_1,y)(x_2,y):y\in V_2,(x_1,x_2)\in E_1\}\cup\{(x_1,y_1)(x_2,y_2):(x_1,x_2)\in E_1,(y_1,y_2)\in E_2\}$ *Then the* strong product *of G_1 and G_2 is an IFG denoted by $G_1*G_2:(V,E)$*, where

$$\mu_1*\acute{\mu}_1)(x_1,x_2)=\mu_1(x_1)\wedge\acute{\mu}_1(x_2)forevery(x_1,x_2)\in V_1\times V_2,$$
$$(\gamma_1*\acute{\gamma}_1)(x_1,x_2)=\gamma_1(x_1)\vee\acute{\gamma}_1(x_2)forevery(x_1,x_2)\in V_1\times V_2,$$
$$(\mu_2*\acute{\mu}_2)(x,x_2)(x,y_2)=\mu_1(x)\wedge\acute{\mu}_2(x_2,y_2),\ \forall x\in V_1,(x_2,y_2)\in E_2,$$
$$(\gamma_2*\acute{\gamma}_2)(x,x_2)(x,y_2)=\gamma_1(x)\vee\acute{\gamma}_2(x_2,y_2),\ \forall x\in V_1,(x_2,y_2)\in E_2,$$
$$(\mu_2*\acute{\mu}_2)(x_1,x_2)(y_1,y_2)=\mu_2(x_1,x_2)\wedge\acute{\mu}_2(y_1,y_2),\ \forall(x_1,x_2)\in E_1,(y_1,y_2)\in E_2,$$
$$(\gamma_2*\acute{\gamma}_2)(x_1,x_2)(y_1,y_2)=\gamma_2(x_1,x_2)\vee\acute{\gamma}_2(y_1,y_2),\ \forall(x_1,x_2)\in E_1,(y_1,y_2)\in E_2,$$
$$(\mu_2*\acute{\mu}_2)(x_1,y)(x_2,y)=\mu_2(x_1,x_2)\wedge\acute{\mu}_2(y),\ \forall(x_1,x_2)\in E_1,y\in V_2,$$
$$(\gamma_2*\acute{\gamma}_2)(x_1,y)(x_2,y)=\gamma_2(x_1,x_2)\vee\acute{\gamma}_2(y),\ \forall(x_1,x_2)\in E_1,y\in V_2.$$

3. Product intuitionistic fuzzy graph (PIFG)

The first definition of product intuitionistic fuzzy graph was introduced by Vinoth and Geetha in [27]. In this section, we recall some necessary definitions related to our work.

Definition 12 ([27]). *Let $G:(V,E)$ be an IFG. If $\mu_2(\nu_i,\nu_j)\leq\mu_1(\nu_i)\mu_1(\nu_j)$ and $\gamma_2(\nu_i,\nu_j)\leq\gamma_1(\nu_i)\gamma_1(\nu_j)$, $\forall(\nu_i,\nu_j)\in V$, the intuitionistic fuzzy graph is called* product intuitionistic fuzzy graph *of G (simply, PIFG).*

Definition 13 ([24]). *A PIFG $G:(V,E)$ is said to be complete product if $\mu_2(\nu_i,\nu_j)=\mu_1(\nu_i)\mu_1(\nu_j)$ and*

$$\gamma_2(\nu_i,\nu_j)=\gamma_1(\nu_i)\gamma_1(\nu_j),\forall\nu_i,\nu_j\in V.$$

Definition 14 ([24]). *The complement of a PIFG $G:(V,E)$ is $G^c:(V^c,E^c)$ whereV^c=V, $\mu_{1i}^c=\mu_{1i}$ and $\gamma_{1i}^c=\gamma_{1i}$, $\forall i=1,2,....n$, $\mu_{2ij}^c=\mu_1(\nu_i)\mu_1(\nu_j)-\mu_2(\nu_i,\nu_j)$ and*

$$\gamma_{2ij}^c=\gamma_1^c(\nu_i)\gamma_1^c(\nu_j)-\gamma_2^c(\nu_i,\nu_j),\ \forall i,j=1,2,...n.$$

Definition 15 ([25]). *The* density *of a PIFG $G:(V,E)$ is defined by $D(G)=(D_\mu(G),D_\gamma(G))$ where*

$$D_\mu(G)=\frac{2\sum_{\nu_i,\nu_j\in V}(\mu_2(\nu_i,\nu_j))}{\sum_{\nu_i,\nu_j\in E}(\mu_1(\nu_i)\mu_1(\nu_j))},$$
$$D_\gamma(G)=\frac{2\sum_{\nu_i,\nu_j\in V}(\gamma_2(\nu_i,\nu_j))}{\sum_{\nu_i,\nu_j\in E}(\gamma_1(\nu_i)\gamma_1(\nu_j))}.$$

Definition 16 ([25]). *A PIFG $G:(V,E)$ is* balanced *if $D(H)\leq D(G)$, that is, $D_\mu(H)\leq D_\mu(G)$, $D_\gamma(H)\leq D_\gamma(G)$,,$\forall$ subgraph H of G.*

4. Operations on PIFGs

In this section, we define the operations of direct product, semi-strong product and strong product on PIFG'S and we study some results related to balanced PIFG'S.

Definition 17. *Let $G_1:(V_1,E_1)$ and $G_2:(V_2,E_2)$ be PIFG'S, where $V=V_1\times V_2$ and $E=\{(u_1,v_1)(u_2,v_2):(u_1,u_2)\in E_1,\ (v_1,v_2)\in E_2\}$. Then the* direct product *of G_1 and G_2 is a PIFG denoted by $G_1\sqcap G_2:(V,E)$ where*

$$(\mu_1\sqcap\acute{\mu}_1)(u_1,v_1)=\mu_1(u_1)\acute{\mu}_1(v_1),\ \forall(u_1,v_1)\in V_1\times V_2,$$
$$(\gamma_1\sqcap\acute{\gamma}_1)(u_1,v_1)=\gamma_1(u_1)\acute{\gamma}_1(v_1),\ \forall(u_1,v_1)\in V_1\times V_2,$$
$$(\mu_2\sqcap\acute{\mu}_2)(u_1,v_1)(u_2,v_2)=\mu_2(u_1,u_2)\acute{\mu}_2(v_1,v_2),\ \forall(u_1,u_2)\in E_1,(v_1,v_2)\in E_2$$

and

$$(\gamma_2 \sqcap \acute{\gamma}_2)(u_1, v_1)(u_2, v_2) = \gamma_2(u_1, u_2)\acute{\nu}_2(v_1, v_2),\ \forall (u_1, u_2) \in E_1, (v_1, v_2) \in E_2.$$

Definition 18. *Let $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ be PIFG'S, where $V = V_1 \times V_2$ and*

$$E = \{(u, u_2)(u, v_2) : u \in V_1,\ (u_2, v_2) \in E_2\} \cup \{(u_1, v_1)(u_2, v_2) : (u_1, u_2) \in E_1, (v_1, v_2) \in E_2\}.$$

Then the semi-strong product *of G_1 and G_2 is a PIFG denoted by $G_1 \odot G_2 = (V, E)$ where*

$$(\mu_1 \odot \acute{\mu}_1)(u_1, u_2) = \mu_1(u_1)\acute{\mu}_1(u_2), \forall (u_1, u_2) \in V_1 \times V_2,$$
$$(\gamma_1 \odot \acute{\gamma}_1)(u_1, u_2) = \gamma_1(u_1)\acute{\gamma}_1(u_2),\ \forall (u_1, u_2) \in V_1 \times V_2,$$
$$(\mu_2 \odot \acute{\mu}_2)(u, u_2)(u, v_2) = \mu_1^2(u)\acute{\mu}_2(u_2, v_2), \forall u \in V_1, (u_2, v_2) \in E_2,$$
$$(\gamma_2 \odot \acute{\gamma}_2)(u, u_2)(u, v_2) = \gamma_1^2(u)\acute{\gamma}_2(u_2, v_2), \forall u \in V_1, (u_2, v_2) \in E_2,$$
$$(\mu_2 \odot \acute{\mu}_2)(u_1, u_2)(v_1, v_2) = \mu_2(u_1, u_2)\acute{\mu}_2(v_1, v_2), \forall (u_1, u_2) \in E_1, (v_1, v_2) \in E_2$$

and

$$(\gamma_2 \odot \acute{\gamma}_2)(u_1, u_2)(v_1, v_2) = \gamma_2(u_1, u_2)\acute{\gamma}_2(v_1, v_2),\ \forall (u_1, u_2) \in E_1, (v_1, v_2) \in E_2.$$

Definition 19. *Let $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ be PIFG'S, where $V = V_1 \times V_2$ and $E = \{(u, v_1)(u, v_2) : u \in V_1,\ (v_1, v_2) \in E_2\} \cup \{(u_1, v)(u_2, v) : v \in V_2, (u_1, u_2) \in E_1\} \cup \{(u_1, v_1)(u_2, v_2) : (u_1, u_2) \in E_1, (v_1, v_2) \in E_2\}$. Then the* strong product *of G_1 and G_2 is a PIFG denoted by $G_1 * G_2 = (V, E)$ where*

$$(\mu_1 * \acute{\mu}_1)(u_1, u_2) = \mu_1(u_1)\acute{\mu}_1(u_2)\ \forall\ (u_1, u_2) \in V_1 \times V_2,$$
$$(\gamma_1 * \acute{\gamma}_1)(u_1, u_2) = \gamma_1(u_1)\acute{\gamma}_1(u_2)\ \forall\ (u_1, u_2) \in V_1 \times V_2,$$
$$(\mu_2 * \acute{\mu}_2)(u, u_2)(u, v_2) = \mu_1^2(u)\acute{\mu}_2(u_2, v_2)\ \forall\ u \in V_1, (u_2, v_2) \in E,$$
$$(\gamma_2 * \acute{\gamma}_2)(u, u_2)(u, v_2) = \gamma_1^2(u)\acute{\gamma}_2(u_2, v_2))\forall\ u \in V_1, (u_2, v_2) \in E_2,$$
$$(\mu_2 * \acute{\mu}_2)(u_1, u_2)(v_1, v_2) = \mu_2(u_1, u_2)\acute{\mu}_2(v_1, v_2)\ \forall\ (u_1, u_2) \in E_1,\ (v_1, v_2) \in E_2,$$
$$(\gamma_2 * \acute{\gamma}_2)(u_1, u_2)(v_1, v_2) = \gamma_2(u_1, u_2)\acute{\gamma}_2(v_1, v_2)\ \forall\ (u_1, u_2) \in E_1,\ (v_1, v_2) \in E_2,$$
$$(\mu_2 * \acute{\mu}_2)(u_1, v)(u_2, v) = \mu_2(u_1, u_2)\acute{\mu}_2^2(v)\ \forall\ (u_1, u_2) \in E_1,\ v \in V_2,$$

and

$$(\gamma_2 * \acute{\gamma}_2)(u_1, v)(u_2, v) = \gamma_2(u_1, u_2)\acute{\gamma}_2^2(v)\ \forall\ (u_1, u_2) \in E_1,\ v \in V_2.$$

Theorem 1. *Let $G_1 : (V_1, E_1)$ and $G_2 : (V_1, E_1)$ be strong PIFG'S. Then $G_1 \sqcap G_2$ is strong.*

Proof. If $(u_1, v_1)(u_2, v_2) \in E$, then since G_1 and G_2 are strong PIFG'S, then

$$\begin{aligned}(\mu_2 \sqcap \acute{\mu}_2)(u_1, v_1)(u_2, v_2) &= \mu_2(u_1, u_2)\acute{\mu}_2(v_1, v_2)\\ &= \mu_1(u_1)\mu_1(u_2)\acute{\mu}_1(v_1)\acute{\mu}_1(v_2)\\ &= (\mu_1 \sqcap \acute{\mu}_2)((u_1, v_1)(u_2, v_2)),\end{aligned}$$

and

$$\begin{aligned}(\gamma_2 \sqcap \acute{\gamma}_2)(u_1, v_1)(u_2, v_2) &= \gamma_2(u_1, u_2)\acute{\gamma}_2(v_1, v_2)\\ &= \gamma_1(u_1)\gamma_1(u_2)\acute{\gamma}_1(v_1)\acute{\gamma}_1(v_2)\\ &= (\gamma_1 \sqcap \acute{\gamma}_1)((u_1, v_1)(u_2, v_2)).\end{aligned}$$

Therefore, $G_1 \sqcap G_2$ *is strong.* □

Theorem 2. *Let $G_1 : (V_1, E_1)$ and $G_2 : (V_1, E_1)$ be strong PIFG's. Then $G_1 \odot G_2$ is strong.*

Proof. If $(u, v_1)(u, v_2) \in E$, then since G_1 and G_2 are strong, then

$$\begin{aligned}(\mu_2 \sqcap \acute{\mu}_2)(u, v_1)(u, v_2) &= \mu_1(u)^2\acute{\mu}_2(v_1, v_2)\\ &= \mu_1(u)\mu_1(u)\acute{\mu}_1(v_1)\acute{\mu}_1(v_2)\\ &= (\mu_1 \sqcap \acute{\mu}_2)((u, v_1)(u, v_2)),\end{aligned}$$

and

$$\begin{aligned}(\gamma_2 \sqcap \acute{\gamma}_2)(u, v_1)(u, v_2) &= \gamma_1(u)^2\gamma_2(v_1, v_2)\\ &= \gamma_1(u)\gamma_1(u)\acute{\gamma}_1(v_1)\acute{\gamma}_1(v_2)\\ &= (\gamma_1 \sqcap \acute{\gamma}_1)((u, v_1)(u, v_2)).\end{aligned}$$

If $(u_1, v_1)(u_2, v_2) \in E$, then since G_1 and G_2 are strong, then

$$\begin{aligned}(\mu_2 \sqcap \acute{\mu}_2)(u_1, v_1)(u_2, v_2) &= \mu_2(u_1, u_2)\acute{\mu}_2(v_1, v_2)\\ &= \mu_1(u_1)\mu_1(u_2)\acute{\mu}_1(v_1)\acute{\mu}_1(v_2)\\ &= (\mu_1 \sqcap \acute{\mu}_2)((u_1, v_1)(u_2, v_2)),\end{aligned}$$

and

$$\begin{aligned}(\gamma_2 \sqcap \acute{\gamma}_2)(u_1, v_1)(u_2, v_2) &= \gamma_2(u_1, u_2)\gamma_2(v_1, v_2)\\ &= \gamma_1(u_1)\gamma_1(u_2)\acute{\gamma}_1(v_1)\acute{\gamma}_1(v_2)\\ &= (\gamma_1 \sqcap \acute{\gamma}_1)((u_1, v_1)(u_2, v_2)).\end{aligned}$$

Hence $G_1 \odot G_2$ is strong. □

Theorem 3. Let $G_1 : (V_1, E_1)$ and $G_2 : (V_1, E_1)$ be strong PIFG'S. Then $G_1 \otimes G_2$ is strong.

Proof. If $(u,v_1)(u,v_2)\in E$, then since G_1 and G_2 are strong, then

$$\begin{aligned}(\mu_2\sqcap\acute{\mu}_2)(u,v_1)(u,v_2) &= \mu_1(u)^2\acute{\mu}_2(v_1,v_2)\\ &= \mu_1(u)\mu_1(u)\acute{\mu}_1(v_1)\acute{\mu}_1(v_2)\\ &= (\mu_1\sqcap\acute{\mu}_2)((u,v_1)(u,v_2)),\end{aligned}$$

and

$$\begin{aligned}(\gamma_2\sqcap\acute{\gamma}_2)(u,v_1)(u,v_2) &= \gamma_1(u)^2\gamma_2(v_1,v_2)\\ &= \gamma_1(u)\gamma_1(u)\acute{\gamma}_1(v_1)\acute{\gamma}_1(v_2)\\ &= (\gamma_1\sqcap\acute{\gamma}_1)((u,v_1)(u,v_2)).\end{aligned}$$

If $(u_1,v)(u_2,v)\in E$, then since G_1 and G_2 are strong, then

$$\begin{aligned}(\mu_2\sqcap\acute{\mu}_2)(u_1,v)(u_2,v) &= \mu_2(u_1,u_2)\acute{\mu}(v)^2\\ &= \mu_1(u_1)\mu_1(u_2)\acute{\mu}_1(v)\acute{\mu}_1(v)\\ &= (\mu_1\sqcap\acute{\mu}_2)((u_1,v)(u_2,v)),\end{aligned}$$

and

$$\begin{aligned}(\gamma_2\sqcap\acute{\gamma}_2)(u_1,v)(u_2,v) &= \gamma_1(u_1,u_2)\gamma_2(v,v)\\ &= \gamma_1(u_1)\gamma_1(u_2)\acute{\gamma}_1(v)\acute{\gamma}_1(v)\\ &= (\gamma_1\sqcap\acute{\gamma}_1)((u_1,v)(u_2,v)).\end{aligned}$$

If $(u_1,v_1)(u_2,v_2)\in E$, then since G_1 and G_2 are strong, then

$$\begin{aligned}(\mu_2\sqcap\acute{\mu}_2)(u_1,v_1)(u_2,v_2) &= \mu_2(u_1,u_2)\acute{\mu}_2(v_1,v_2)\\ &= \mu_1(u_1)\mu_1(u_2)\acute{\mu}_1(v_1)\acute{\mu}_1(v_2)\\ &= (\mu_1\sqcap\acute{\mu}_2)((u_1,v_1)(u_2,v_2)),\end{aligned}$$

and

$$\begin{aligned}(\gamma_2\sqcap\acute{\gamma}_2)(u_1,v_1)(u_2,v_2) &= \gamma_2(u_1,u_2)\gamma_2(v_1,v_2)\\ &= \gamma_1(u_1)\gamma_1(u_2)\acute{\gamma}_1(v_1)\acute{\gamma}_1(v_2)\\ &= (\gamma_1\sqcap\acute{\gamma}_1)((u_1,v_1)(u_2,v_2)).\end{aligned}$$

Hence $G_1\otimes G_2$ is strong. □

From Theorem 1 and Theorem 3, we get,

Corollary 2. *Let $G_1:(V_1,E_1)$ and $G_2:(V_1,E_1)$ be complete PIFG'S. Then* $G_1\sqcap G_2$ and $G_1\odot G_2$ *are strong.*

We remark that the above result need not be true for $G_1\otimes G_2$ since $G_1\otimes G_2$ is never complete by its definition.

5. Balanced PIFG'S

Balanced PIFGs were defined by Karunambigai, Sivasankar and Palanivel in [27] and some properties on PIFG were established. In this section, we examine many of the results that have not been studied.

Lemma 1. *Let $G:(V,E)$ be a self-complementary PIFG. Then*

$$\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)=\frac{1}{2}\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_2(\nu_j),$$
$$\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)=\frac{1}{2}\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j).$$

Proof. Let $G:(V,E)$ be a self-complementary PIFG. Then by Definition of G^c, $V^c=V$, $\mu^c_{1i}=\mu_{1i}$ *and* $\gamma^c_{1i}=\gamma_{1i}$, $\forall i=1,2,....,n$, $\mu^c_2(h(\nu_i),h(\nu_j))=(\mu_1(h(\nu_i)),\mu_1(h(\nu_j))-\mu_2(h(\nu_i),h(\nu_j)$ and $\gamma^c_2(h(\nu_i),h(\nu_j))=(\gamma_1(h(\nu_i),\gamma_1(h(\nu_j))-\gamma_2(h(\nu_i),h(\nu_j)),\forall\ i,\ j=1,2,...,n$, $\mu^c_2(h(\nu_i),h(\nu_j))=\mu_2(\nu_i,\nu_j)$. Now

$$\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)+\sum_{\nu_i,\nu_j\in V}\mu_2(h(\nu_i),h(\nu_j)=\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j).$$

Thus

$$2\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)=\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j).$$

So

$$\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)=\frac{1}{2}\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j).$$

Also,

$$\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)+\sum_{\nu_i,\nu_j\in V}\gamma_2(h(\nu_i),h(\nu_j)=\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j).$$

Thus

$$2\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)=\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j).$$

So

$$\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)=\frac{1}{2}\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j).$$

□

Theorem 4. *Let $G:(V,E)$ be a PIFG and $G^c:(V^c,E^c)$ be its complement. Then $D(G)+D(G^c)\geq(2,2)$.*

Proof. Let $G:(V,E)$ be a PIFG and $G^c:(V^c,E^c)$ be its complement. Let H be a non-empty fuzzy subgraph of G. Since G is PIFG

$$\mu_2^c(\nu_i,\nu_j)=\mu_1(\nu_i)\mu_1(\nu_j)-\mu_2(\nu_i,\nu_j),$$

Thus

$$\sum_{\nu_i,\nu_j\in V^C}\mu_2^c(\nu_i,\nu_j)+\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)=\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j).$$

So

$$\frac{\sum_{\nu_i,\nu_j\in V^C}\mu_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V^C}\mu_1(\nu_i)\mu_1(\nu_j)}+\frac{\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j)}=1.$$

Hence

$$\frac{2\sum_{\nu_i,\nu_j\in V^C}\mu_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V^C}\mu_1(\nu_i)\mu_1(\nu_j)}+\frac{2\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j)}=2.$$

Therefore,

$$\begin{aligned}&\frac{2\sum_{\nu_i,\nu_j\in V^C}\mu_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V^C}\mu_1(\nu_i)\mu_1(\nu_j)}+\frac{2\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V}\mu_1(\nu_i)\mu_1(\nu_j)}\\&\leq\frac{2\sum_{\nu_i,\nu_j\in V^C}\mu_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in E^C}\mu_1(\nu_i)\mu_1(\nu_j)}+\frac{2\sum_{\nu_i,\nu_j\in V}\mu_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in E}\mu_1(\nu_i)\mu_1(\nu_j)}.\end{aligned}$$

Also

$$\begin{aligned}\gamma_2^c(\nu_i,\nu_j)&=\gamma_1(\nu_i)\gamma_1(\nu_j)-\gamma_2(\nu_i,\nu_j),\\\mu_2^c(\nu_i,\nu_j)+\mu_2(\nu_i,\nu_j)&=\mu_1(\nu_i)\mu_1(\nu_j).\end{aligned}$$

So

$$\sum_{\nu_i,\nu_j\in V^C}\gamma_2^c(\nu_i,\nu_j)+\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)=\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j).$$

Hence

$$\frac{\sum_{\nu_i,\nu_j\in V^C}\gamma_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V^C}\gamma_1(\nu_i)\gamma_1(\nu_j))}+\frac{\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j)}=1$$

and

$$\frac{2\sum_{\nu_i,\nu_j\in V^C}\gamma_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V^C}\gamma_1(\nu_i)\gamma_1(\nu_j)}+\frac{2\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j)}=2$$

Therefore,

$$\begin{aligned}&\frac{2\sum_{\nu_i,\nu_j\in V^C}\gamma_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V^C}\gamma_1(\nu_i)\gamma_1(\nu_j)}+\frac{2\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in V}\gamma_1(\nu_i)\gamma_1(\nu_j)}\\&\leq\frac{2\sum_{\nu_i,\nu_j\in V^C}\gamma_2^c(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in E^C}\gamma_1(\nu_i)\gamma_1(\nu_j)}+\frac{2\sum_{\nu_i,\nu_j\in V}\gamma_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in E}\gamma_1(\nu_i)\gamma_1(\nu_j)}.\end{aligned}$$

□

Theorem 5. *Let $G:(V,E)$ be a self-complementary PIFG. Then $D(G)\geq(1,1)$.*

Proof.

$$\begin{aligned} D(G) &= (D_\mu(G), D_\gamma(G)) \\ &= ((\frac{2\sum_{\nu_i,\nu_j\in v}\mu_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in E}(\mu_1(\nu_i)\mu_2(\nu_j))}, \frac{2\sum_{\nu_i,\nu_j\in v}\gamma_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in E}(\gamma_1(\nu_i)\gamma_1(\nu_j))})) \\ &\geq (\frac{2\sum_{\nu_i,\nu_j\in v}\mu_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in v}(\mu_1(\nu_i)\mu_2(\nu_j))}, \frac{2\sum_{\nu_i,\nu_j\in v}\gamma_2(\nu_i,\nu_j)}{\sum_{\nu_i,\nu_j\in v}(\gamma_1(\nu_i)\gamma_1(\nu_j))}). \end{aligned}$$

But by Lemma 1, we get

$$\begin{aligned} D(G) &\geq (\frac{2\frac{1}{2}\sum_{\nu_i,\nu_j\in v}(\mu_1(\nu_i)\mu_2(\nu_j))}{\sum_{\nu_i,\nu_j\in v}(\mu_1(\nu_i)\mu_2(\nu_j))}, \frac{2\frac{1}{2}\sum_{\nu_i,\nu_j\in v}(\gamma_1(\nu_i)\gamma_1(\nu_j))}{\sum_{\nu_i,\nu_j\in v}(\gamma_1(\nu_i)\gamma_1(\nu_j))}) \\ &= (1,1). \end{aligned}$$

□

Theorem 6. *Let $G_1 : (V_1, E_1)$ and $G_2 : (V_1, E_1)$ be two complete PIFG'S Then $D(G_i) \leq D(G_1 \sqcap G_2)$ for $i = 1, 2$ if and only if $D(G_1) = D(G_2) = D(G_1 \sqcap G_2)$.*

Proof. If $D(G_i) \leq D(G_1 \sqcap G_2)$ for $i = 1, 2$, since $G_1$and G_2 are complete PIFG'S,

$$D(G_1) = D(G_2) = 2.$$

and by Corollary 2, $G_1 \sqcap G_2$ is strong and hence $D(G_1 \sqcap G_2) = 2$.Thus $D(G_i) \geq D(G_1 \sqcap G_2)$ for $i = 1, 2$ and so $D(G_1) = D(G_2) = D(G_1 \sqcap G_2)$.

The converse is trivial. □

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