SOME RESULTS ON SLOWLY CHANGING FUNCTION ORIENTED RELATIVE ORDER, RELATIVE TYPE AND RELATIVE WEAK TYPE OF DIFFERENTIAL MONOMIALS

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Abstract. In this paper, we establish the relationship between the relative L-order (relative L^*-order), relative L-type (relative L^*-type) and relative L-weak type (relative L^*-weak type) of a transcendental meromorphic function f with respect to an transcendental entire function g and that of monomial generated by the meromorphic f and entire g.

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1. Introduction, definitions and notations

We denote by $\mathbb{C}$ the set of all finite complex numbers. Let f be an entire function defined on $\mathbb{C}$. The function $M_f(r) = \max_{|z|=r} |f(z)|$ known as maximum modulus function corresponding to f.

When f is meromorphic, $M_f(r)$ can not be defined as f is not analytic. In this situation one may define another function $T_f(r)$ known as Nevanlinna's Characteristic function of f, playing the same role as $M_f(r)$ in the following manner:

$$T_f(r) = N_f(r) + m_f(r)$$

where the function $N_f(r)$ and $m_f(r)$ are respectively the enumerative function and the proximity function corresponding to f. If f is an entire function, then the Nevanlinna's Characteristic function $T_f(r)$ of f reduces to $m_f(r)$. Also for a non-constant entire f, $T_f(r)$ is strictly increasing and continuous function of r and its inverse $T_f^{-1} : (T_f(0), \infty) \to (0, \infty)$ exists and is such that $\lim_{s\to\infty} T_g^{-1}(s) = \infty$.

Furthermore, we called the function $N_f(r,a)$ $\left(\overline{N}_f(r,a)\right)$ as counting function of a-points (distinct a-points) of f. We put

$$N_f(r,a) = \int_0^r \frac{n_f(t,a) - n_f(0,a)}{t} dt + \overline{n}_f(0,a)\log r ,$$

where we denote by $n_f(r,a)$ $(\overline{n}_f(r,a))$ the number of a-points (distinct a-points) of f in $|z| \le r$ and an ∞ -point is a pole of f. In many occasions $N_f(r,\infty)$ and $\overline{N}_f(r,\infty)$ are denoted by $N_f(r)$ and $\overline{N}_f(r)$ respectively. Also we denote by $n_{f|=1}(r,a)$,the number of simple zeros of $f-a$ in $|z| \le r$. Accordingly, $N_{f|=1}(r,a)$ is defined in terms of $n_{f|=1}(r,a)$ in the usual way and we set

$$\delta_1(a;f) = 1 - \limsup_{r\to\infty} \frac{N(r,a;f \mid= 1)}{T_f(r)} \ \{\text{cf. [9]}\},$$

the deficiency of 'a' corresponding to the simple a-points of f, i.e., simple zeros of $f-a$. In this connection, Yang [8] proved that there exists at most a denumerable number of complex numbers $a \in \mathbb{C} \cup \{\infty\}$ for which

$$\delta_1(a;f) > 0 \text{ and } \sum_{a\in\mathbb{C}\cup\{\infty\}} \delta_1(a;f) \le 4 .$$

Further, a meromorphic function $b = b(z)$ is called small with respect to f if $T_b(r) = S_f(r)$ where $S_f(r) = o\{T_f(r)\}$ i.e., $\frac{S_f(r)}{T_f(r)} \to 0$ as $r \to \infty$. Moreover for any transcendental meromorphic function f, we call $P[f] = bf^{n_0}(f^{(1)})^{n_1}...(f^{(k)})^{n_k}$, to be a differential monomial generated by it where $\sum_{i=0}^{k} n_i \ge 1$ (all $n_i \mid i = 0,1,...,k$ are non-negative integers) and the meromorphic function b is small with respect to f. In this connection, the numbers $\gamma_{P[f]} = \sum_{i=0}^{k} n_i$ and $\Gamma_{P[f]} = \sum_{i=0}^{k}(i+1)n_i$ are called the degree and weight of $P[f]$ respectively {cf. [1]}.

In this connection, the following definitions are well known:

Definition 1 The order ρ_f and lower order λ_f of a meromorphic function f are defined as

$$\rho_f = \limsup_{r\to\infty} \frac{\log T_f(r)}{\log r} \text{ and } \lambda_f = \liminf_{r\to\infty} \frac{\log T_f(r)}{\log r} .$$

Somasundaram and Thamizharasi [6] introduced the notions of L-order and L-lower order for entire functions where $L \equiv L(r)$ is a positive continuous function increasing slowly i.e.,$L(ar) \sim L(r)$ as $r \to \infty$ for every positive constant "a". Their definitions are as follows:

Definition 2 [6] The L-order ρ_f^L and the L-lower order λ_f^L of a meromorphic function f are defined as follows:

$$\rho_f^L = \limsup_{r\to\infty} \frac{\log T_f(r)}{\log[rL(r)]} \text{ and } \lambda_f^L = \liminf_{r\to\infty} \frac{\log T_f(r)}{\log[rL(r)]} .$$

The more generalised concept of L-order and L-lower order of meromorphic functions are L^*-order and L^*-lower order respectively which are as follows:

Definition 3 The L^*-order $\rho_f^{L^*}$ and the L^*-lower order $\lambda_f^{L^*}$ of a meromorphic function f are defined by

$$\rho_f^{L^*} = \limsup_{r\to\infty} \frac{\log T_f(r)}{\log[re^{L(r)}]} \text{ and } \lambda_f^{L^*} = \liminf_{r\to\infty} \frac{\log T_f(r)}{\log[re^{L(r)}]} .$$

Lahiri and Banerjee [5] introduced the definition of relative order of a meromorphic function with respect to an entire function which is as follows:

Definition 4 [5] Let f be meromorphic and g be entire. The relative order of f with respect to g denoted by $\rho_g(f)$ is defined as

$$\begin{aligned} \rho_g(f) &= \inf\{\mu > 0 : T_f(r) < T_g(r^\mu) \text{ for all sufficiently large } r\} \\ &= \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} . \end{aligned}$$

The definition coincides with the classical one [5] if $g(z) = \exp z$.

Similarly, one can define the relative lower order of a meromorphic function f with respect to an entire g denoted by $\lambda_g(f)$ in the following manner:

$$\lambda_g(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log r} .$$

Datta and Biswas [2] gave the definition of relative type and relative weak type of a meromorphic function with respect to an entire function g which are as follows:

Definition 5 [2] The relative type $\sigma_g(f)$ of a meromorphic function f with respect to an entire function g are defined as

$$\sigma_g(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}}, \quad \text{where } 0 < \rho_g(f) < \infty .$$

Similarly, one can define the lower relative type $\overline{\sigma}_g(f)$ in the following way

$$\overline{\sigma}_g(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{r^{\rho_g(f)}}, \quad \text{where } 0 < \rho_g(f) < \infty .$$

Definition 6 [2] The relative weak type $\tau_g(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative lower order $\lambda_g(f)$ is defined by

$$\tau_g(f) = \liminf_{r\to\infty} \frac{T_g^{-1}T_f(r)}{r^{\lambda_g(f)}} .$$

Analogously, one can define the growth indicator $\overline{\tau}_g(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative lower order $\lambda_g(f)$ as

$$\overline{\tau}_g(f) = \limsup_{r\to\infty} \frac{T_g^{-1}T_f(r)}{r^{\lambda_g(f)}} .$$

In order to prove our results we require the following definitions:

Definition 7 The relative L-order $\rho_g^L(f)$ and the relative L-lower order $\lambda_g^L(f)$ of a meromorphic function f with respect to an entire function g are defined as follows:

$$\rho_g^L(f) = \limsup_{r\to\infty} \frac{\log T_g^{-1}T_f(r)}{\log[rL(r)]} \text{ and } \lambda_g^L(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1}T_f(r)}{\log[rL(r)]} .$$

Definition 8 The relative L-type $\sigma_g^L(f)$ and the relative L-lower type $\overline{\sigma}_g^L(f)$ of a meromorphic function f with respect to an entire function g are defined as follows:

$$\sigma_g^L(f) = \limsup_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\rho_g^L(f)}} \text{ and } \overline{\sigma}_g^L(f) = \liminf_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\rho_g^L(f)}},$$

where $0 < \rho_g^L(f) < \infty$.

Definition 9 The relative L-weak type $\tau_g^L(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative L-lower order $\lambda_g^L(f)$ is defined by

$$\tau_g^L(f) = \liminf_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\lambda_g^L(f)}} .$$

Similarly, one can define the growth indicator $\overline{\tau}_g^L(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative L-lower order $\lambda_g^L(f)$ as

$$\overline{\tau}_g^L(f) = \limsup_{r\to\infty} \frac{T_g^{-1}T_f(r)}{[rL(r)]^{\lambda_g^L(f)}} .$$

The more generalised concept of relative L-order (relative L-lower order), relative L-type (relative L-lower type) and relative L-weak type of meromorphic function with respect to an entire function are relative L^*-order (relative L^*-lower order), relative L^*-type (relative relative L^*-lower type) and relative L^*-weak type respectively which are as follows:

Definition 10 The L^*-order $\rho_f^{L^*}$ and the L^*-lower order $\lambda_f^{L^*}$ of a meromorphic function f are defined by

$$\rho_g^{L^*}(f) = \limsup_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log\left[re^{L(r)}\right]} \text{ and } \lambda_g^{L^*}(f) = \liminf_{r\to\infty} \frac{\log T_g^{-1} T_f(r)}{\log\left[re^{L(r)}\right]} .$$

Definition 11 The relative L^*-type $\sigma_g^{L^*}(f)$ and the relative L^*-lower type $\overline{\sigma}_g^{L^*}(f)$ of a meromorphic function f with respect to an entire function g are defined as follows:

$$\sigma_g^{L^*}(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{\left[re^{L(r)}\right]^{\rho_g^{L^*}(f)}} \text{ and } \overline{\sigma}_g^{L^*}(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{\left[re^{L(r)}\right]^{\rho_g^{L^*}(f)}},$$

where $0 < \rho_g^{L^*}(f) < \infty$.

Definition 12 The relative L^*-weak type $\tau_g^{L^*}(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative L^*-lower order $\lambda_g^{L^*}(f)$ is defined by

$$\tau_g^{L^*}(f) = \liminf_{r\to\infty} \frac{T_g^{-1} T_f(r)}{\left[rL(r)\right]^{\lambda_g^{L^*}(f)}} .$$

Similarly, one can define the growth indicator $\overline{\tau}_g^{L^*}(f)$ of a meromorphic function f with respect to an entire function g with finite positive relative L^*-lower order $\lambda_g^{L^*}(f)$ as

$$\overline{\tau}_g^{L^*}(f) = \limsup_{r\to\infty} \frac{T_g^{-1} T_f(r)}{\left[rL(r)\right]^{\lambda_g^{L^*}(f)}} .$$

In this paper, we wish to establish the relationship between the relative L-order (relative L^*-order), relative L-type (relative L^*-type) and relative L-weak type (relative L^*-weak type) of a transcendental meromorphic function f with respect to a transcendental entire function g and that of monomial generated by the transcendental meromorphic f and transcendental entire g. We use the standard notations and definitions in the theory of entire and meromorphic functions which are available in [4] and [7].

2. Lemmas

In this section, we present two lemmas which will be needed in the sequel.

Lemma 1 [3] *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and $\sum_{a\in\mathbb{C}\cup\{\infty\}} \delta_1(a; f) = 4$ and g be a transcendental entire function with regular growth and non zero finite order. Also let $\sum_{a\in\mathbb{C}\cup\{\infty\}} \delta_1(a; g) = 4$. Then*

$$\lim_{r\to\infty} \frac{\log T_{P[g]}^{-1} T_{P[f]}(r)}{\log T_g^{-1} T_f(r)} = 1 .$$

Lemma 2 [3] *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function with regular growth and non zero finite type. Also let* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then*

$$\lim_{r\to\infty}\frac{T^{-1}_{P[g]}T_{P[f]}\left(r\right)}{T^{-1}_{g}T_{f}\left(r\right)}=\left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}$$

where$\Theta(\infty;f)=1-\limsup\limits_{r\to\infty}\frac{\overline{N}_f(r)}{T_f(r)}$ *and* $\Theta(\infty;g)=1-\limsup\limits_{r\to\infty}\frac{\overline{N}_g(r)}{T_g(r)}$.

3. Theorems

In this section, we present the main results of the paper.

Theorem 1 *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function with regular growth and non zero finite order. Also let* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then the relative L-order and relative L-lower order order of $P[f]$ with respect to $P[g]$ are same as those of f with respect to g.*

Proof. By Lemma 1 we obtain that,

$$\begin{aligned}\rho^{L}_{P[g]}\left(P[f]\right) &= \limsup_{r\to\infty}\frac{\log T^{-1}_{P[g]}T_{P[f]}\left(r\right)}{\log\left[rL\left(r\right)\right]}\\ &= \limsup_{r\to\infty}\left\{\frac{\log T^{-1}_{g}T_{f}\left(r\right)}{\log\left[rL\left(r\right)\right]}\cdot\frac{\log T^{-1}_{P[g]}T_{P[f]}\left(r\right)}{\log T^{-1}_{g}T_{f}\left(r\right)}\right\}\\ &= \limsup_{r\to\infty}\frac{\log T^{-1}_{g}T_{f}\left(r\right)}{\log\left[rL\left(r\right)\right]}\cdot\lim_{r\to\infty}\frac{\log T^{-1}_{P[g]}T_{P[f]}\left(r\right)}{\log T^{-1}_{g}T_{f}\left(r\right)}\\ &= \rho^{L}_{g}\left(f\right)\cdot 1\\ &= \rho^{L}_{g}\left(f\right)\ .\end{aligned}$$

In a similar manner, $\lambda^{L}_{P[g]}\left(P[f]\right)=\lambda^{L}_{g}\left(f\right)$. This proves the theorem. ∎

Theorem 2 *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function with regular growth and non zero finite order. Also let* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then the relative L^*-order and relative L^*-lower order order of $P[f]$ with respect to $P[g]$ are same as those of f with respect to g.*

We omit the proof of Theorem 2 because it can be carried out in the line of Theorem 1.

Theorem 3 *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function of regular growth having non zero finite type and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then the relative L-type and relative L-lower type of $P[f]$ with respect to $P[g]$ are* $\left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}$ *times that of f with respect to g if $\rho_g^L(f)$ is positive finite.*

Proof. From Lemma 2 and Theorem 1 we get that

$$\begin{aligned}\sigma^L_{P[g]}(P[f]) &= \limsup_{r\to\infty}\frac{T^{-1}_{P[g]}T_{P[f]}(r)}{[rL(r)]^{\rho_{P[g]}(P[f])}}\\ &= \lim_{r\to\infty}\frac{T^{-1}_{P[g]}T_{P[f]}(r)}{T_g^{-1}T_f(r)}\cdot\limsup_{r\to\infty}\frac{T_g^{-1}T_f(r)}{[rL(r)]^{\rho_g^L(f)}}\\ &= \left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}\cdot\sigma_g^L(f)\ .\end{aligned}$$

Similarly,

$$\overline{\sigma}^L_{P[g]}(P[f]) = \left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}\cdot\overline{\sigma}_g^L(f)\ .$$

Thus the theorem is established. ∎

Theorem 4 *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function of regular growth having non zero finite type and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then the relative L^*-type and relative L^*-lower type of $P[f]$ with respect to $P[g]$ are* $\left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}$ *times that of f with respect to g if $\rho_g^{L^*}(f)$ is positive finite.*

We omit the proof of Theorem 4 because it can be carried out in the line of Theorem 3.

Now, we state the following two theorems without proof because it can be carried out in the line of Theorem 3 and Theorem 4 respectively.

Theorem 5 *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function of regular growth having non zero finite type and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then* $\tau^L_{P[g]}(P[f])$ *and* $\overline{\tau}^L_{P[g]}(P[f])$ *are* $\left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}$ *times that of* *f with respect to g i.e.,* $\tau^{L}_{P[g]}(P[f]) = \left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}} \cdot \tau^{L}_{g}(f)$ *and* $\overline{\tau}^{L}_{P[g]}(P[f]) = \left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}} \cdot \overline{\tau}^{L}_{g}(f)$ *when* $\lambda^{L}_{g}(f)$ *is positive finite.*

Theorem 6 *Let f be a transcendental meromorphic function of finite order or of non-zero lower order and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;f)=4$ *and g be a transcendental entire function of regular growth having non zero finite type and* $\sum_{a\in\mathbb{C}\cup\{\infty\}}\delta_1(a;g)=4$. *Then* $\tau^{L^*}_{P[g]}(P[f])$ *and* $\overline{\tau}^{L^*}_{P[g]}(P[f])$ *are* $\left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}}$ *times that of f with respect to g i.e.,* $\tau^{L^*}_{P[g]}(P[f]) = \left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}} \cdot \tau^{L^*}_{g}(f)$ *and* $\overline{\tau}^{L^*}_{P[g]}(P[f]) = \left(\frac{\Gamma_{P[f]}-(\Gamma_{P[f]}-\gamma_{P[f]})\Theta(\infty;f)}{\Gamma_{P[g]}-(\Gamma_{P[g]}-\gamma_{P[g]})\Theta(\infty;g)}\right)^{\frac{1}{\rho_g}} \cdot \overline{\tau}^{L^*}_{g}(f)$ *when* $\lambda^{L^*}_{g}(f)$ *is positive finite.*

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