

## THE ANALYSIS OF A VARIABLE-VISCOSITY FLUID FLOW BETWEEN PARALLEL POROUS PLATES WITH NON-UNIFORM WALL TEMPERATURE

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**Abstract.** This paper examined the effectiveness of porosity on a variable-viscosity channel flow with non-uniform wall temperature. The flow is considered to be a steady, incompressible and the fluid viscosity varies linearly with temperature. The analytic expressions are obtained seeking asymptotic solutions for fluid velocity and temperature and these expressions are used to derive and obtain solutions for entropy generation rate and Bejan numbers with variations in other physical parameters present in the fluid flow.

**Keywords:** variable-viscosity, non-uniform wall temperature, entropy generation rate, Bejan number and porous medium.

### 1. Introduction

Studies involving fluid flow between parallel porous plates have been investigated by a number of researchers [1]–[11] because of its wide range of engineering applications such as electronic cooling, thermal insulation, crude oil extraction and nuclear reactor. Others, for example, include [12] which considered the unsteady viscous fluid flow with porous medium in the presence of radiation and chemical reaction, [13] conducted the experimental investigation of the permeability and inertia effect on fluid flow through homogeneous porous media and [14] presented a theoretical study of the fluid flow and transfer through a porous medium channel bounded by a permeable parallel walls with equal suction or equal injection.

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In all these studies, it is worthy to note the influence and effect of porosity on velocity, temperature, pressure distribution and other physical properties within the flow channel.

Meanwhile, [15] in his entropy-generation analysis for variable-viscosity channel flow with non-uniform temperature analyzed that most fluids used in engineering and industrial systems can be subjected to extreme conditions, such as high temperature, pressure and shear rate without considering the effect of porosity. In addition to that, [16] also pointed out that fluid flow is often accompanied with heat transfer which is an integral part of natural convection flow and belongs to the class of problems in boundary layer theory which occurs in various physical phenomena such as fire engineering, combustion modeling, nuclear reactor, heat exchangers, etc. without considering such fluid flowing through porous medium.

But there is need to find out the property of the porous medium which measures the capacity and ability of the formation to transmit fluids. This is a very important property that controls the directional movement and the flow rate of the fluid in any formation. As a result of that, there is need to investigate and examine the effectiveness of porosity on a variable-viscosity channel flow with non-uniform wall temperature which was not considered in [15]. Hence, the analytic expressions for fluid velocity and temperature profiles are determined by seeking asymptotic solutions and these expressions are used to derive and obtain solutions for entropy generation rate and Bejan numbers with variations in other physical parameters present in the fluid flow regime.

In the rest of this paper, the problem is formulated in Section 2. The governing equations are solved using ADM and the entropy generation rate was determined in Section 3. A presentation of analytical results of the problem are shown in tables and graphs in Section 4 and Section 5 gives the concluding remarks.

## 2. Mathematical model

Consider the steady flow of an incompressible fluid flowing in the  $x$  – direction through parallel porous plates of width, ( $a$ ) and length, ( $L$ ) with a nonuniform wall temperature under the action of a constant pressure gradient as shown in Figure 1.

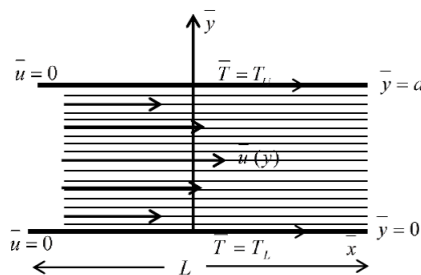


Figure 1: The geometry of the problem

The temperature dependent viscosity ( $\bar{\mu}$ ) as described in [15] can be expressed as

$$(1) \quad \bar{\mu} = \mu_0 [1 - \beta (\bar{T} - T_L)],$$

where  $\mu_0$  is the fluid dynamic viscosity at the lower wall temperature  $T_l$ ,  $\beta$  is the viscosity - variation parameter and  $T$  is the fluid temperature. Neglecting the consumption of the reactant, the continuity, momentum (along x and y axes) and energy equations governing the fluid flow in nondimensionless form may be written following [15], [17] as:

$$(2) \quad \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0$$

$$(3) \quad \rho \left[ \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right] = - \frac{\partial \bar{p}}{\partial \bar{x}} + 2 \frac{\partial}{\partial \bar{x}} \left( \bar{\mu} \frac{\partial \bar{u}}{\partial \bar{x}} \right) + \frac{\partial}{\partial \bar{y}} \left[ \bar{\mu} \left( \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right) \right] - \frac{\bar{\mu}}{K} \bar{u}$$

$$(4) \quad \rho \left[ \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right] = - \frac{\partial \bar{p}}{\partial \bar{y}} + 2 \frac{\partial}{\partial \bar{y}} \left( \bar{\mu} \frac{\partial \bar{v}}{\partial \bar{y}} \right) + \frac{\partial}{\partial \bar{x}} \left[ \bar{\mu} \left( \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right) \right]$$

$$(5) \quad \begin{aligned} & \rho c_p \left[ \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right] \\ & = k \left[ \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right] + \bar{\mu} \left[ 2 \left( \frac{\partial \bar{u}}{\partial \bar{x}} \right)^2 + 2 \left( \frac{\partial \bar{v}}{\partial \bar{y}} \right)^2 + \left( \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \right)^2 \right] + \frac{\bar{\mu}}{K} \bar{u}^2 \end{aligned}$$

where  $\rho$  is the fluid density,  $K$  is Darcy's permeability constant,  $k$  is the thermal conductivity,  $p$  is the pressure,  $c_p$  is the specific heat at constant pressure,  $u$  represent the axial velocity,  $v$  is the normal velocity, also,  $x$  and  $y$  are distances measured in the streamline and normal direction respectively. The additional last term in (3) and (5) is due to the effect of porosity and modification of Brinkman form of Darcys law of porosity as described in [18]

Introducing the following non dimensional variables and parameters in equations (1)–(5) as follows:

$$(6) \quad \begin{aligned} y &= \frac{\bar{y}}{\epsilon L}, & x &= \frac{\bar{x}}{L}, & u &= \frac{\bar{u}}{U}, & v &= \frac{\bar{v}}{\epsilon U}, & \epsilon &= \frac{a}{L}, & \mu &= \frac{\bar{\mu}}{\mu_0}, \\ T &= \frac{\bar{T} - T_l}{T_u - T_l}, & p &= \frac{\epsilon^2 L \bar{p}}{\mu_0 U}, & \alpha &= \beta (T_u - T_l) & Br &= \frac{\mu_0 U^2}{k (T_u - T_l)}, \\ -\frac{\partial p}{\partial x} &= G, & Pe &= \frac{\rho c_p L U}{k}, & Re &= \frac{\rho U L}{\mu_0} & \text{and } \gamma &= \frac{a^2}{K}, \end{aligned}$$

where  $\epsilon$  is the channel aspect ratio, ( $a$ ) is channel width, ( $L$ ) is the channel characteristic length,  $T_u$  is the upper wall temperature,  $T_l$  is the lower wall temperature,  $\alpha$  represents viscosity - variation parameter,  $p$  is the fluid pressure,  $\mu_0$  is the fluid dynamic viscosity,  $G$  is the constant pressure gradient,  $U$  is the velocity scale, and  $Br$  is the Brinkmann number. Also,  $Re$  is the Reynolds number,  $Pe$  is the Peclet number and  $\gamma$  represents porous permeability parameter.

With the introduction of (6) into (1)–(5), the governing equations for the fluid flow in dimensionless form as in [15] may be written as:

$$(7) \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$(8) \quad \epsilon^2 Re \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = G + 2\epsilon^2 \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial u}{\partial y} + \epsilon^2 \frac{\partial v}{\partial x} \right) \right] - \gamma \mu u$$

$$(9) \quad \epsilon^4 Re \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + 2\epsilon^2 \left( \mu \frac{\partial v}{\partial y} \right) + \epsilon^2 \frac{\partial}{\partial x} \left[ \mu \left( \frac{\partial u}{\partial y} + \epsilon^2 \frac{\partial v}{\partial x} \right) \right]$$

$$(10) \quad \epsilon^2 Pe \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \epsilon^2 \frac{\partial^2 T}{\partial x^2} + \mu Br [\varphi + \gamma u^2]$$

where

$$(11) \quad \varphi = 2\epsilon^2 \left( \frac{\partial u}{\partial x} \right)^2 + 2\epsilon^2 \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \epsilon^2 \frac{\partial v}{\partial x} \right)^2.$$

Since the channel aspect ratio is very small, that is,  $0 < \epsilon \leq 1$ , the lubrication approximation based on asymptotic simplification of the governing equations (7)–(11) is invoked and obtain the following:

$$(12) \quad \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) - \gamma \mu u + G + \mathcal{O}(\epsilon^2) = 0$$

$$(13) \quad -\frac{\partial p}{\partial y} + \mathcal{O}(\epsilon^2) = 0$$

$$(14) \quad \frac{\partial^2 T}{\partial y^2} + \mu Br \left[ \left( \frac{\partial u}{\partial y} \right)^2 + \gamma \mu u^2 \right] \mathcal{O}(\epsilon^2) = 0$$

where  $\mu = 1 - \alpha T$  with the following boundary conditions at the upper wall of the channel as:

$$(15) \quad u = 0, \quad T = 1 \quad \text{at } y = 1$$

and at the lower wall of the channel as:

$$(16) \quad u = 0, \quad T = 0 \quad \text{at } y = 0$$

It is worthy to note that (15) and (16) indicate that the temperatures at both upper and lower walls are fixed and different.

### 3. Method of solution

Following [15], we solve equations (12)–(14) subject to the boundary conditions in (15) and (16), based on assumption that the variations in the fluid viscosity is very small and seek an asymptotic solutions for the velocity and temperature of the fluid flow within porous medium in this form:

$$(17) \quad u = u_0 + \alpha u_1, \quad T = T_0 + \alpha T_1$$

Substituting (17) into (12) - (14), the following equations are obtained in the following orders:

Order zero ( $\alpha^0$ )

$$(18) \quad \frac{\partial^2 u_0}{\partial y^2} - \gamma u_0 + G = 0, \quad \frac{\partial^2 T_0}{\partial y^2} + Br \left[ \left( \frac{\partial u_0}{\partial y} \right)^2 + \gamma u_0^2 \right] = 0$$

with the boundary conditions

$$(19) \quad u_0 = 0, \quad T_0 = 1 \quad \text{at } y = 1 \quad \text{and} \quad u_0 = 0, \quad T_0 = 0 \quad \text{at } y = 0$$

Order one ( $\alpha^1$ )

$$(20) \quad \frac{\partial^2 u_1}{\partial y^2} - T_0 \frac{\partial^2 u_0}{\partial y^2} - \frac{\partial u_0}{\partial y} \frac{\partial T_0}{\partial y} - \gamma (u_1 - u_0 T_0) = 0,$$

$$(21) \quad \frac{\partial^2 T_1}{\partial y^2} + Br \left[ 2 \frac{\partial u_0}{\partial y} \frac{\partial u_1}{\partial y} - T_0 \left( \frac{\partial u_0}{\partial y} \right)^2 + \gamma (2u_0 u_1 - T_0 u_0^2) \right] = 0$$

subject to the boundary conditions

$$(22) \quad u_1 = 0, \quad T_1 = 0 \quad \text{at } y = 1 \quad \text{and} \quad u_1 = 0, \quad T_1 = 0 \quad \text{at } y = 0.$$

Equations (18)–(22) are thereby coded on Mathematica software package, then substituting the results back into (17) to obtain the solutions for the velocity and temperature profiles which are hereby discussed in the next section because of the large volume of outputs.

#### 4. Entropy generation

Entropy generation is a measure of the account of irreversibility associated with the real process. It is a measure of disorderliness of a system. In order to preserve the quality of energy in a fluid flow process or at least to reduce the entropy generation, it is also important to study the distribution of the entropy generation within the fluid volume. The entropy production is due to heat transfer and the combined effects of fluid friction and Joules dissipation. Following [15], [16], [19], [20], the general equation for the entropy generation per unit volume in the presence of porous medium is given by:

$$(23) \quad S^m = \frac{k}{T_l^2} \left( \frac{\partial \bar{T}}{\partial \bar{y}} \right)^2 + \frac{\bar{\mu}}{T_l} \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 + \frac{\bar{\mu}}{T_l K} \bar{u}^2.$$

The first term in (23) is the irreversibility due to heat transfer, the second term is the entropy generation due to viscous dissipation and the last term is the local entropy generation due to the effects of porosity. We express the entropy generation number in dimensionless form using the existing dimensionless variables and parameter in (6) as:

$$(24) \quad N_s = \frac{S^m a^2 T_l^2}{k(T_u - T_l)^2} = \left( \frac{\partial T}{\partial y} \right)^2 + \frac{Br}{\Omega} \left[ \mu \left( \frac{\partial u}{\partial y} \right)^2 + \gamma u^2 \right].$$

We let the first term,  $\left(\frac{\partial T}{\partial y}\right)^2$  be assigned  $N_1$  which is the irreversibility due to heat transfer and the second term,  $\frac{Br}{\Omega} \left[ \mu \left(\frac{\partial u}{\partial y}\right)^2 + \gamma u^2 \right]$  be referred to as  $N_2$ , which is the entropy generation due to the effects of viscous dissipation and porosity of the flow regime. Also,  $\Omega = \frac{T_u - T_l}{T_l}$  is the wall temperature parameter. We defined

$$(25) \quad \phi = \frac{N_1}{N_2}$$

as the irreversibility distribution ratio. The expression (25) shows that heat transfer dominates when  $0 < \phi < 1$  and fluid friction dominates when  $\phi > 1$ . This is used to determine the contribution of heat transfer in many engineering designs. As an alternative to irreversibility parameter, the Bejan number ( $Be$ ) is defined as

$$(26) \quad Be = \frac{N_1}{N_s} = \frac{1}{1 + \phi} \quad \text{where } 0 \leq Be \leq 1.$$

## 5. Discussion of results

In this section, the effectiveness of porosity on a variable-viscosity fluid flow with non-uniform wall temperature together with other important flow parameters are presented and discussed.

Tables (1) and (2) show the effects of porosity on velocity and temperature profiles respectively. It is noticed that there is reduction in the fluid velocity as the porosity parameter ( $\gamma$ ) increases with the maximum at the centreline of the fluid flow while the fluid temperature increases generally from lower plate to upper plate but an increment is noticed around the centreline as the porosity parameter increases but a reduction is noticed around both upper and lower surfaces as less efficient packing occur in the interior centreline of the channel.

Table 1: Effect of porosity on velocity profile

$G = 1, Br = 10, \alpha = 0.1$			
$y$	$u(y) _{\gamma=0.1}$	$u(y) _{\gamma=0.5}$	$u(y) _{\gamma=1}$
0	0	0	0
0.1	0.0457074812	0.0441201095	0.0422986555
0.2	0.0818163056	0.0787948482	0.0753293076
0.3	0.1080885602	0.1039304097	0.0991628394
0.4	0.1243223076	0.1194280309	0.1138177099
<b>0.5</b>	<b>0.1303270245</b>	<b>0.1251614367</b>	<b>0.1192404521</b>
0.6	0.1259088886	0.1209633679	0.1152939695
0.7	0.1108657006	0.1066202469	0.1017520109
0.8	0.0849913686	0.0818746647	0.0782992783
0.9	0.0480900265	0.0464359858	0.0445376403
1	0	0	0

Table 2: Effect of porosity on temperature profile

$G = 1, Br = 10, \alpha = 0.1$			
$y$	$T(y) _{\gamma=0.1}$	$T(y) _{\gamma=0.5}$	$T(y) _{\gamma=1}$
0	0	0	0
0.1	0.1324173788	0.1314741222	0.1303639983
0.2	0.2481120524	0.2473560560	0.2463829314
0.3	0.3540722466	0.3540473065	0.3538154817
<b>0.4</b>	<b>0.4553874610</b>	<b>0.4560804967</b>	<b>0.4565928550</b>
<b>0.5</b>	<b>0.5552065540</b>	<b>0.5562102886</b>	<b>0.5570476095</b>
<b>0.6</b>	<b>0.6546952371</b>	<b>0.6554518644</b>	<b>0.6560340920</b>
0.7	0.7529839977	0.7530596886	0.7529383744
0.8	0.8471031944	0.8464430053	0.8455749361
0.9	0.9319078047	0.9310179742	0.9299661899
1	1	1	1

Figures 2 to 4 display the velocity profile respectively with variations in porous permeability parameter ( $\gamma$ ), viscosity variation parameter ( $\alpha$ ) and pressure gradient ( $G$ ).

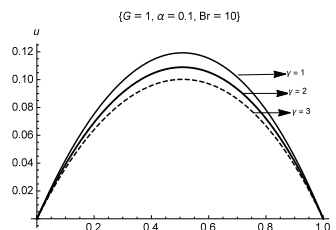


Figure 2: Velocity profile for variations in  $\gamma$

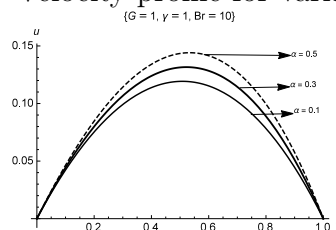


Figure 3: Velocity profile for variations in  $\alpha$

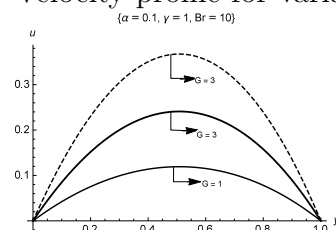


Figure 4: Velocity profile for variations in  $G$

The general observation shows that the velocity of the fluid flow reduces as the porous permeability parameter ( $\gamma$ ) increases while the velocity increases as both the viscosity variation parameter ( $\alpha$ ) and pressure gradient ( $G$ ) increase. In Figure 2, the presence of porous medium in the flow has the tendency to increase the resistance of the fluid motion in the channel. In Figure 3, an increase in the viscosity variation parameter ( $\alpha$ ) also leads to greater viscosity distribution and hence increases the fluid velocity and the greater the pressure gradient, the greater the discharge rate of fluid depending on the porous material as shown in Figure 4.

The effects of porous permeability parameter ( $\gamma$ ), viscosity variation parameter ( $\alpha$ ), viscous dissipation parameter ( $Br$ ) and pressure gradient ( $G$ ) on the temperature profiles are respectively displayed in Figures 5 to 8. A reduction is noticed as the porous permeability parameter ( $\gamma$ ) increases while an increase in temperature is observed with increasing values of viscosity variation parameter ( $\alpha$ ), viscous dissipation parameter ( $Br$ ) and pressure gradient ( $G$ ).

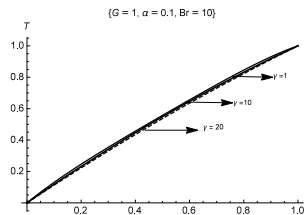


Figure 5: Temperature Profile for variations in  $\gamma$

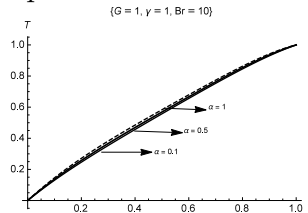


Figure 6: Temperature Profile for variations in  $\alpha$

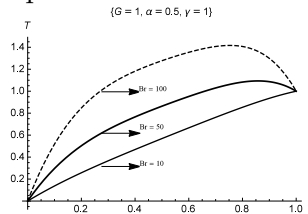


Figure 7: Temperature Profile for variations in  $Br$

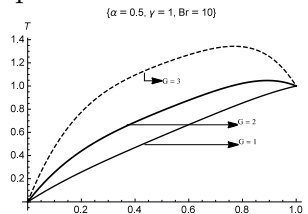


Figure 8: Temperature Profile for variations in  $G$



Figures 9 to 13 respectively display the entropy generation rate with variations in porous permeability parameter ( $\gamma$ ), wall temperature parameter ( $\Omega$ ), viscosity variation parameter ( $\alpha$ ), viscous dissipation parameter ( $Br$ ) and pressure gradient ( $G$ ).

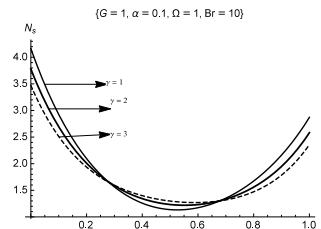


Figure 9: Entropy generation rate for variations in  $\gamma$

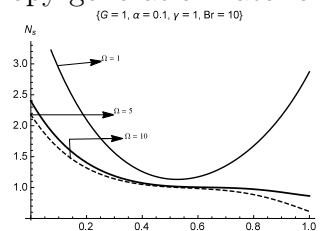


Figure 10: Entropy generation rate for variations in  $\Omega$

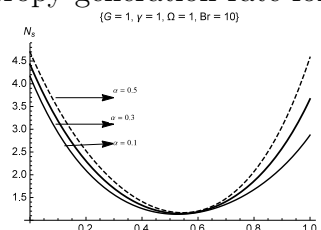


Figure 11: Entropy generation rate for variations in  $\alpha$

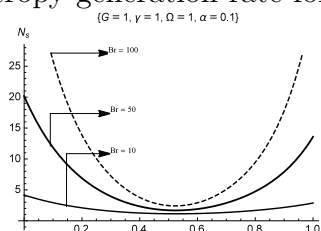


Figure 12: Entropy generation rate for variations in  $Br$

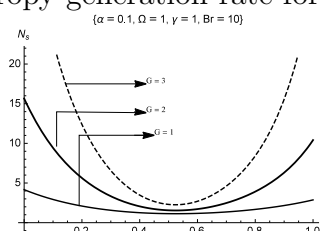


Figure 13: Entropy generation rate for variations in  $G$

Generally, the flow is active and maintains equilibrium around the core centreline region of the flow system. In Figure 9, the entropy generation rate decreases with increasing values of porous permeability parameter ( $\gamma$ ) near the walls and otherwise around the centreline region of the channel. Also, the entropy generation rate decreases with increasing values of wall temperature parameter ( $\Omega$ ) in Figure 10, while the entropy generation rate increases with increasing values of viscosity variation parameter ( $\alpha$ ), viscous dissipation parameter ( $Br$ ) and pressure gradient ( $G$ ) respectively in Figures 11, 12 and 13.

Bejan rates of the fluid flow are presented in Figures 14 to 16 for variations in porous permeability parameter ( $\gamma$ ), wall temperature parameter ( $\Omega$ ) and viscosity variation parameter ( $\alpha$ ).

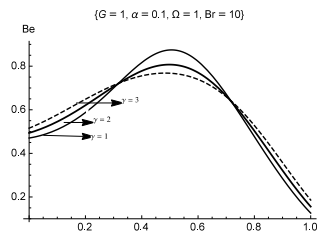


Figure 14: Bejan Number for variations in  $\gamma$

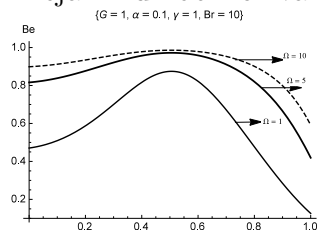


Figure 15: Bejan Number for variations in  $\Omega$

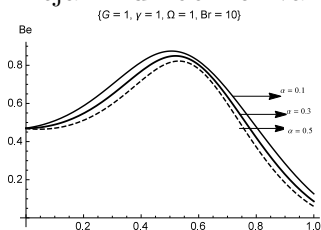


Figure 16: Bejan Number for variations in  $\alpha$

On a general note, it shows that heat-transfer irreversibility dominates around the centreline region of the channel while fluid friction irreversibility dominates at both upper and lower regions. The effects of porous permeability parameter ( $\gamma$ ) is interestingly shown in Figure 14 as increasing values result into an increase in the dominant effect of fluid friction irreversibility near the walls and a reduction in the dominant effect of the heat-transfer irreversibility around centreline region. Also, in Figure 15, as increasing values of wall temperature parameter ( $\Omega$ ) result into an increase in the dominant effect across the channel while an increasing values of viscosity variation parameter ( $\alpha$ ) result into a reduction in the dominant effect across the channel as shown in Figure 16.

## 6. Conclusion

A review of the effectiveness of porosity on a variable-viscosity channel flow with non-uniform wall temperature has been investigated. The analytic expressions are obtained seeking asymptotic solutions for fluid velocity and temperature and these expressions are used to derive and obtain solutions for entropy generation rate and Bejan numbers ( $Be$ ). The result shows that there is reduction in the fluid velocity as the porosity parameter ( $\gamma$ ) increases while the fluid temperature increases from lower plate to upper plates but an increment is noticed around the centreline as the porosity parameter increases. Also, the entropy generation rate decreases with increasing values of porous permeability parameter ( $\gamma$ ) near the walls and otherwise around the centreline region of the channel, and for Bejan number ( $Be$ ), increasing values of porous permeability parameter ( $\gamma$ ) result into increasing the dominant effect of fluid friction irreversibility near the walls and a reduction in the dominant effect of the heat-transfer irreversibility around centreline region.

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