THE IMPACT OF NONLINEAR INCIDENCE RATE AND REMOVABLE STORAGE MEDIA ON VIRAL PREVALENCE

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Abstract. In this paper, a new computer virus propagation model is proposed by introducing a generalized nonlinear incidence rate into the generic $SLBRS$ model. Theoretical analysis and numerical simulations show that, under some moderate conditions, the proposed model admits a globally asymptotically stable viral equilibrium. The impact of nonlinear incidence rate and removable storage media on viral prevalence is also illustrated.

Keywords: computer virus; propagation model; nonlinear incidence rate; equilibrium; global stability.

1. Introduction

Since the appearance of the first computer virus, human beings have blown the horn of a prolonged battle between the viruses and human intelligence. Nowadays, the endless war has been increasingly cruel. For one thing, multifarious computer viruses, as well as the various ways they attack computers, pose unprecedented threat to information security. For another, due to the rapid and widespread popularization of the Internet, the spreading ability of viruses has been highly enhanced. Currently, antivirus software is the major means of resisting computer viruses. Yet, in view of the serious situation and the fact that the development of new antivirus software always lags behind the emergence of new viruses, it is desperately necessary to explore the spreading behavior of viruses over the Internet. Thus, based on the appealing analogy between computer viruses and their biological counterparts, quite a few epidemic models of computer viruses, ranging from conventional models such as SIS models [1], [2], SIR models [3], [4], $SIRS$ models [5], [6], $SLBS$ models [7]-[12], to unconventional models such as delayed models [13]-[20], impulsive models [7], [21], stochastic models [7], [22] and the network-based models [12], [23], [24] have been proposed.

That the $SLBS$ model [9] was firstly presented by Yang et al based on the fact that a latent computer possesses infectivity marked a significant breakthrough in exploring the laws governing the spread of computer viruses. Afterwards, a considerable amount of work is done to extend the $SLBS$ model. Considering that an infected computer would have temporary immunity when it is cured by installing the latest version of antivirus software, Yang et al proposed an $SLBRS$ model [25]. However, this model not only ignored the influence of reinstalling operating system, but also neglected the impact of antivirus software. What'more, the assumption that all newly connected computers are all virus-free is unrealistic. Thus, a more reasonable $SLBRS$ model is studies in Ref. [26].

Unfortunately, the above-mentioned model assumed a bilinear infection rate which should be amended and overlooked the impact of removable storage media. About modifying the incidence rate, there are good reasons. On one hand, with the number of infected computers increasing and the countermeasures being strengthened, the contacts between infected computers and susceptible ones would tend to saturate, leading to a nonlinear incidence rate [27]. On the other hand, there always exists some unknown nonlinear factors in the transmission of infections [28]. As a result, this paper is intended to introduce a generalized nonlinear incidence rate. Furthermore, a susceptible computer could also be infected by an infected removable storage device such as a USB flash disk and a mobile hard disk. Therefore, the influence of the removable storage devices should be incorporated into a computer virus propagation model.

In view of foregoing statements, a new computer virus propagation model (see Figure 1) is proposed based on the $SLBRS$ model reported in Ref. [26].

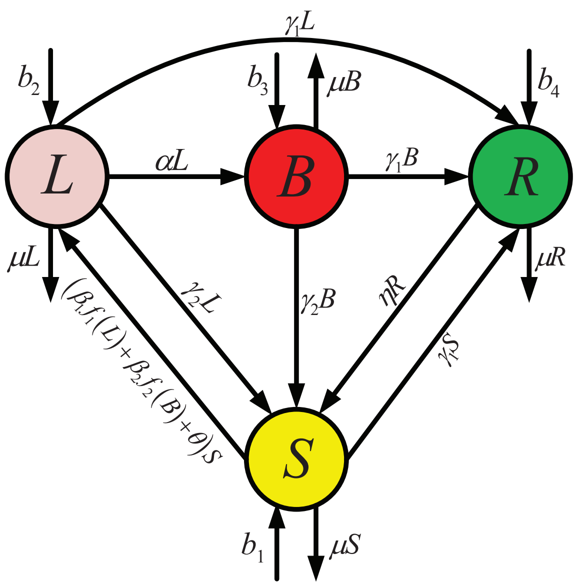

Figure 1: The state transition diagram of the new model.

We shall show by theoretical analysis that, under some moderate conditions, the proposed model admits a globally asymptotically stable viral equilibrium. Numerical simulations will be presented to verify our results, and moreover, the effect of nonlinear incidence rate and removable storage media on viral prevalence is also examined.

The rest of the paper is organized as follows. Section 2 presents the mathematical model. Section 3 proves the existence and global stability of the equilibrium, respectively. Numerical simulations are displayed in Section 4. Section 5 summarizes the work.

2. Model formulation

For the purpose of modeling, the following hypotheses are made.

($H1$) The rates that susceptible, latent, breaking, and recovered computers enter the Internet are b_1, b_2, b_3 and b_4, respectively.

($H2$) Every internal computer leaves the Internet with constant probability μ.

($H3$) Due to the influence of infected removable storage media, every susceptible computer gets infected with constant probability θ.

($H4$) Every latent computer breaks out with constant probability α.

($H5$) Due to the effect of antivirus software, every infected computer is recovered with constant probability γ_1, and every susceptible computer gets immunity with γ_1, too.

($H6$) Every infected computer becomes susceptible with constant probability γ_2 on account of reinstalling operating system.

($H7$) Every recovered computer loses immunity with constant probability η.

($H8$) Due to possible contacts with latent (resp. breaking-out) computers, at time t every susceptible computer gets infected with probability $\beta_1 f_1(L)$ (resp. $\beta_2 f_2(B)$). Here, functions f_i (i = 1, 2) have continuous second derivatives. Clearly, f_i start at the origin (i.e. $f_i(0) = 0$). Furthermore, it is assumed that f_i are strictly increasing (i.e. $f_i' > 0$) and strictly concave (i.e. $f_i'' < 0$).

Based on these hypotheses, we have a new computer virus propagation model which can be represented by the differential equations

$$(1)\quad \begin{cases} \dot{S} = b_1 + \gamma_2 L + \gamma_2 B + \eta R - \gamma_1 S - \mu S - \beta_1 f_1(L)S - \beta_2 f_2(B)S - \theta S, \\ \dot{L} = b_2 - \gamma_1 L - \gamma_2 L - \mu L - \alpha L + \beta_1 f_1(L)S + \beta_2 f_2(B)S + \theta S, \\ \dot{B} = b_3 + \alpha L - \gamma_1 B - \gamma_2 B - \mu B, \\ \dot{R} = b_4 + \gamma_1 S + \gamma_1 L + \gamma_1 B - \mu R - \eta R, \end{cases}$$

where S, L, B, R represent, at time t, the average numbers of susceptible, latent, breaking, and recovered computers, respectively.

Let $N = S + L + B + R, b = b_1 + b_2 + b_3 + b_4,\ N^* = \frac{b}{\mu},\ R^* = \frac{\gamma_1 N^* + b_4}{\gamma_1 + \mu + \eta}$. Then, adding up the four equations of system (1) and simplifying, one can get $\frac{\mathrm{d}N(t)}{\mathrm{d}t} = b - \mu N(t)$, implying $\lim_{t\to\infty} N(t) = N^*$. Likewise, $\lim_{t\to\infty} R(t) = R^*$. Thus, system (1) can be written as the following limiting system [29]:

$$(2)\qquad \begin{cases} \dot{L} = b_2 - (\gamma_1 + \gamma_2 + \alpha + \mu)L + (\beta_1 f_1(L) + \beta_2 f_2(B) + \theta)(N^* - R^* - L - B), \\ \dot{B} = b_3 + \alpha L - (\gamma_1 + \gamma_2 + \mu)B. \end{cases}$$

Bellow we mainly consider the existence, uniqueness and global stability of the equilibrium point in regard to the positively invariant region: $\Omega = \{(L, B) \in R_+^2 : L + B \le N^*\}$.

3. Theoretical analysis

3.1. Equilibrium

Theorem 1 *System (2) has a unique (viral) equilibrium $E^*(L^*, B^*)$, where $L^*=x^*$, $B^* = \frac{b_3 + \alpha x^*}{\gamma_1 + \gamma_2 + \mu}$, and x^* is the unique positive zero of the function*

$$\begin{aligned} g(x) &= b_2 - (\gamma_1 + \gamma_2 + \alpha + \mu)x \\ &\quad + \left[\beta_1 f_1(x) + \beta_2 f_2\left(\frac{b_3 + \alpha x}{\gamma_1 + \gamma_2 + \mu}\right) + \theta\right]\left(N^* - R^* - x - \frac{b_3 + \alpha x}{\gamma_1 + \gamma_2 + \mu}\right), \end{aligned}$$

where $x \in \left[0, \frac{(\gamma_1 + \gamma_2 + \mu)N^* - b_3}{\gamma_1 + \gamma_2 + \alpha + \mu}\right]$.

Proof. First of all, we would show that function g has at least one zero. As

$$g(0) = b_2 + \left[\beta_1 f_1(0) + \beta_2 f_2\left(\frac{b_3}{\gamma_1 + \gamma_2 + \mu}\right) + \theta\right]\left(N^* - R^* - \frac{b_3}{\gamma_1 + \gamma_2 + \mu}\right) > 0$$

and

$$g\left(\frac{(\gamma_1 + \gamma_2 + \mu)N^* - b_3}{\gamma_1 + \gamma_2 + \alpha + \mu}\right) = -(\gamma_1 + \gamma_2)N^* - b_1 - b_4 < 0,$$

it follows that g does have at least one (positive) zero. Furthermore, we have

$$\begin{aligned} &g'\left(\frac{(\gamma_1 + \gamma_2 + \mu)N^* - b_3}{\gamma_1 + \gamma_2 + \alpha + \mu}\right) = -(\gamma_1 + \gamma_2 + \alpha + \mu) \\ &-\frac{\gamma_1+\gamma_2+\alpha + \mu}{\gamma_1+\gamma_2+\mu}\left[\beta_1 f_1\left(\frac{(\gamma_1+\gamma_2+\mu)N^* - b_3}{\gamma_1+\gamma_2+\alpha+\mu}\right) + \beta_2 f_2\left(\frac{b_3+\alpha N^*}{\gamma_1+\gamma_2+\mu+\alpha}\right) + \theta\right] < 0 \end{aligned}$$

and

$$\begin{aligned} g''(x) = &\left[\beta_1 f_1''(x) + \beta_2\left(\frac{\alpha}{\gamma_1+\gamma_2+\mu}\right)^2 f_2''\left(\frac{b_3+\alpha x}{\gamma_1+\gamma_2+\mu}\right)\right]\left(N^* - R^* - x - \frac{b_3+\alpha x}{\gamma_1+\gamma_2+\mu}\right) \\ &- 2\frac{\gamma_1 + \gamma_2 + \alpha + \mu}{\gamma_1 + \gamma_2 + \mu}\left[\beta_1 f_1'(x) + \beta_2 \frac{\alpha}{\gamma_1 + \gamma_2 + \mu} f_2'\left(\frac{b_3 + \alpha x}{\gamma_1 + \gamma_2 + \mu}\right)\right] < 0, \end{aligned}$$

where $x \in \left[0, \dfrac{(\gamma_1+\gamma_2+\mu)N^*-b_3}{\gamma_1+\gamma_2+\alpha+\mu}\right]$.

Then, we make the following discussion.

Case 1. $g'(0) \le 0$. Then, g is strictly decreasing and, thus, has a unique (positive) zero.

Case 2. $g'(0) > 0$. Let $\overline{x}{=}max\left\{x \in \left[0, \frac{(\gamma_1+\gamma_2+\mu)N^*-b_3}{\gamma_1+\gamma_2+\alpha+\mu}\right] : g'(x) \ge 0\right\}$. Then, g is strictly increasing in $[0, \overline{x}]$ and strictly decreasing in $\left[\overline{x}, \frac{(\gamma_1+\gamma_2+\mu)N^*-b_3}{\gamma_1+\gamma_2+\alpha+\mu}\right]$, implying that g has a unique (positive) zero in $\left[\overline{x}, \dfrac{(\gamma_1+\gamma_2+\mu)N^*-b_3}{\gamma_1+\gamma_2+\alpha+\mu}\right]$.

In conclusion, g has a unique (positive) zero. The proof is complete. ∎

3.2. The local stability of the equilibrium

Lemma 1 *E^* is locally asymptotically stable with respect to Ω.*

Proof. Let $S^* = N^* - R^* - L^* - B^*$. The Jacobian matrix of system (2) evaluated at E^* is

$$J_{E^*} = \begin{pmatrix} k_1 & k_2 \\ \alpha & -(\gamma_1+\gamma_2+\mu) \end{pmatrix},$$

where

$$\begin{aligned} k_1 &= -(\gamma_1+\gamma_2+\alpha+\mu) + \beta_1 f_1'(L^*)S^* - [\beta_1 f_1(L^*) + \beta_2 f_2(B^*) + \theta], \\ k_2 &= \beta_2 f_2'(B^*)S^* - [\beta_1 f_1(L^*) + \beta_2 f_2(B^*) + \theta]. \end{aligned}$$

The corresponding characteristic equation is

$$\lambda^2 + a_1\lambda + a_2 = 0,$$

where

$$\begin{aligned} a_1 &= (\gamma_1+\gamma_2+\mu) - k_1, \\ a_2 &= -k_1(\gamma_1+\gamma_2+\mu) - \alpha k_2. \end{aligned}$$

Let $F(x) = f_1'(x)x - f_1(x)$. As $F(0) = 0$ and $F'(x) = f_1''(x)x \le 0$, we have $F(L^*) < 0$, namely $f_1'(L^*)L^* < f_1(L^*)$. Furthermore, from the second equation of system (1), we get

$$S^* < \frac{(\gamma_1+\gamma_2+\mu+\alpha)L^*}{\beta_1 f_1(L^*) + \beta_2 f_2(B^*) + \theta}.$$

Therefore, we gain

$$\begin{aligned} f_1'(L^*)S^* &< \frac{(\gamma_1+\gamma_2+\mu+\alpha)L^* f_1'(L^*)}{\beta_1 f_1(L^*) + \beta_2 f_2(B^*) + \theta} < \frac{(\gamma_1+\gamma_2+\mu+\alpha)f_1(L^*)}{\beta_1 f_1(L^*) + \beta_2 f_2(B^*) + \theta} \\ &< \frac{\gamma_1+\gamma_2+\mu+\alpha}{\beta_1}. \end{aligned}$$

Thus, $\beta_1 f_1'(L^*)S^* < \gamma_1 + \gamma_2 + \mu + \alpha$, implying $a_1 > 0$. What's more, the proof of Theorem 1 implies that $g'(L^*) < 0$, which is equivalent to $a_2 > 0$.

Hence, it follows from the Hurwitz criterion [30] that the two roots of equation (9) have negative real parts. The claimed result follows. ∎

3.3. The global stability of the equilibrium

In this section, we are about to show the global stability of the equilibrium. First, we have the following two lemmas.

Lemma 2 *System* (2) *has no periodic orbit in the interior of* Ω.

Proof. Let

$$\begin{aligned} h_1(L,B) &= b_2 - (\gamma_1+\gamma_2+\alpha+\mu)L \\ &\quad +[\beta_1 f_1(L) + \beta_2 f_2(B) + \theta](N^* - R^* - L - B), \\ h_2(L,B) &= b_3 + \alpha L - (\gamma_1+\gamma_2+\mu)B, \\ D(L,B) &= 1/L. \end{aligned}$$

Then, as the proof of Lemma 1 implies that $f_1'(L)L < f_1(L)$, we get

$$\begin{aligned} \frac{\partial(Dh_1)}{\partial L} + \frac{\partial(Dh_2)}{\partial B} &= -\frac{b_2}{L^2} - \frac{\gamma_1+\gamma_2+\mu}{L} \\ &\quad +\frac{1}{L^2}\{\beta_1(N^*-R^*-B)[Lf_1'(L) - f_1(L)] \\ &\quad -(\beta_2 f_2(B)+\theta)(N^*-R^*-B) - \beta_1 L^2 f_1'(L)\} \\ &< 0. \end{aligned}$$

According to the Bendixson-Dulac criterion [30], system (2) admits no periodic orbit in the interior of Ω. ∎

Lemma 3 *System* (2) *has no periodic orbit that passes through a point on* $\partial\Omega$, *the boundary of* Ω.

Proof. Suppose there is a periodic orbit Γ that passes through a point $(\overline{L}, \overline{B})$ on $\partial\Omega$. Considering the smoothness of all orbits, we infer that $(\overline{L}, \overline{B})$ can not be any point of $(0,0), (0,N^*)$ and $(N^*,0)$. Then, $(\overline{L}, \overline{B})$ must be a noncorner point on $\partial\Omega$. There are three possibilities. By the truth that Γ must be tangent to $\partial\Omega$ at $(\overline{L}, \overline{B})$ if it exists, we deny all of the following possibilities with reduction to absurdity.

(1) $0 < \overline{L} < N^*, \overline{B} = 0$. Then $\mathrm{d}B/\mathrm{d}t|_{(\overline{L},\overline{B})} = b_3 + \alpha\overline{L} > 0$, implying that Γ is not tangent to $\partial\Omega$ at this point.

(2) $0 < \overline{B} < N^*, \overline{L} = 0$. Then $\mathrm{d}L/\mathrm{d}t|_{(\overline{L},\overline{B})} = b_2 + (\beta_2 f_2(\overline{B}) + \theta)(N^* - R^* - \overline{B}) > 0$, implying that Γ is not tangent to $\partial\Omega$ at this point.

(3) $\overline{L} + \overline{B} = N^*, \overline{L} \neq 0, \overline{B} \neq 0$. Then $\mathrm{d}(L+B)/\mathrm{d}t|_{(\overline{L},\overline{B})} = -b_1 - b_4 - (\gamma_1+\gamma_2)N^* < 0$, implying that Γ is not tangent to $\partial\Omega$ at this point.

On account of the above discussions, we come to a conclusion that system (2) has no periodic orbit that passes through a point on $\partial\Omega$. ■

The main result of this paper is displayed as follows.

Theorem 2 *E^* is globally asymptotically stable with respect to Ω.*

Proof. On the basis of the generalized *Poincaré-Bendixson* theorem [30], one can reach the claimed result easily by combining Lemmas 1-3. ■

Remark 1 Theorem 2 implies that it would be practically impossible to eradicate computer viruses on the Internet. Figures 2-3 verify the obtained result.

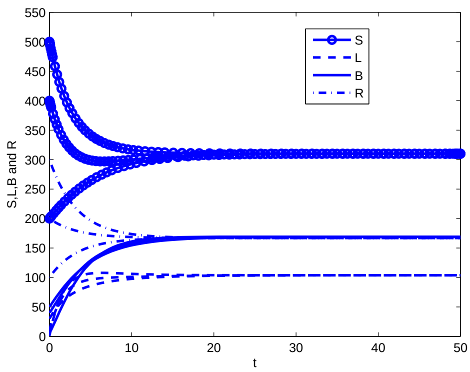

Figure 2: Time plots of S, L, B, R for a common system with three different initial conditions, where $f_1(L) = L/(1+L)$, $f_2(B) = B/(1+B)$.

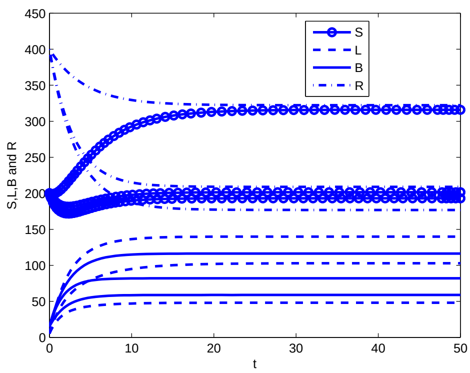

Figure 3: Time plots of S, L, B, R for three different systems with a common initial condition, where $f_1(L) = L/(1+L)$, $f_2(B) = B/(1+B)$.

Remark 2 From Figure 3, we can clearly see that the steady number of the infected computers varies with model parameters, implying that viral spread could be controlled below an acceptable level by adjusting the corresponding parameters. Specifically, in real life, we should update our antivirus software timely, run a virus scan when some removable storage devices are connected to our computers and do not click unknown links etc.

4. Numerical Simulations

This section examines the effect of nonlinear incidence rate as well as the removable storage media on the spread of computer viruses. Figure 4 describes the impact of some specific nonlinear incidence rates on viral spreading, from which it can be seen that computer viruses are suppressed in the case of the nonlinear incidence rate. It is logical to draw that a relatively flat incidence rate contributes to the containment of computer viruses. Figure 5 shows that computer viruses transmission levels are increasing with θ. This implies that the removable storage devices had better be scanned when they are connected to the computer.

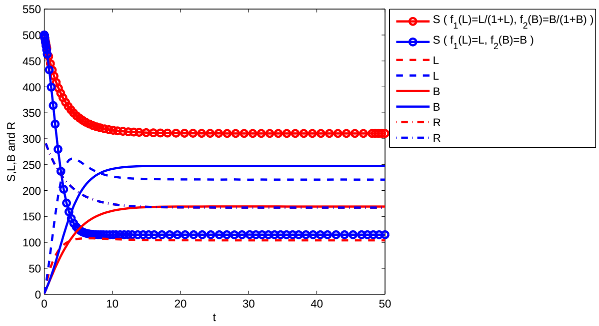

Figure 4: The comparison between the nonlinear and linear incidence rate.

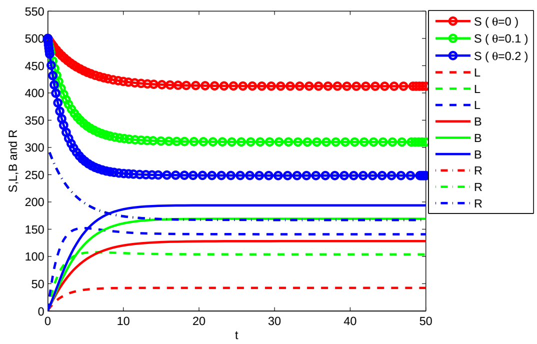

Figure 5: The effect of the infected removable storage media on viral prevelance, where $f_1(L) = L/(1+L)$, $f_2(B) = B/(1+B)$.

Actually, there are quite a few nonlinear functions satisfying the assumption (H8). For example, $f(x) = kx^m$ $(k > 0,\ 0 < m < 1)$ also fits the postulate. So carrying out more numerical simulations to get more accurate results would be a part of our next work. In addition, it is definitely undeniable that the existing incidence rates don't necessarily meet the mentioned condition. In other words, we need to consider those incidence rates which don't satisfy the hypothesis in subsequent research work.

5. Conclusions

This paper has examined the impact of nonlinear incidence rate as well as the removable storage media on viral prevalence based on an improved $SLBRS$ model.

It has been shown that the new model has a globally asymptotically stable viral equilibrium. And by setting a specific nonlinear incidence rate we have deduced that a relatively flat incidence rate conduces to the containment of computer viruses. The influence of removable storage media has also been illustrated. On this basis, several useful suggestions have been given.

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