EOQ MODELS FOR NON-INSTANTANEOUS/INSTANTANEOUS DETERIORATING ITEMS WITH CUBIC DEMAND RATE UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENTS

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Abstract. In this article, an attempt is made to develop two inventory models for non-instantaneous deteriorating items and two inventory models for instantaneous deteriorating items with linear deterioration rate and cubic demand rate. That is, the demand rate is a piecewise cubic function of time under inflation and permissible delay in payments. This model supports in minimizing the total inventory cost by finding an optimal replenishment policy; where shortages are allowed, partially backlogged and completely backlogged cases are considered. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Numerical examples are given to establish the analytical results. Sensitivity analysis of the optimal solution with respect to major parameters is carried out and the effects are discussed in detail.

Keywords: inventory, non-instantaneous deterioration, cubic demand, inflation, permissible delay in payments, allowable shortages.

AMS Mathematics Subject Classification: 90B05.

1. Introduction

Inventory system is one of the main streams of the Operation Research, which is essential in business enterprises and industries. Inventory may be considered as accumulation of a product that would be used to satisfy future demands for that product. It needs scientific way of exercising inventory model. Generally, deterioration is defined as the damage, spoilage, dryness, vaporization, etc., that results in the decrease of usefulness of the commodity. Deterioration of goods is a common phenomenon and unavoidable in daily life. Therefore, to control and maintain the inventory of deteriorating items becomes an important factor for decision makers. In all the inventory models for deteriorating items,it is assumed that deterioration starts as soon as the retailer receives the inventory. But most of the goods have a span of maintaining quality or the original condition in real situation. During that period, there was no occurrence of deterioration (e.g., vegetables, fruits, meat, fish and so on). This phenomenon is termed as *non-instantaneous deterioration*. It has been observed that only few researchers considered inventory models for non-instantaneous deteriorating items with inflation and permissible delay in payments simultaneously. They play important role in the optimal order policy and influences the demand of certain products. The inventory model for fashion goods deteriorating at the end of prescribed period was first studied by Whitin [1957]. Then Ghare and Schrader [1963] developed an EOQ model with constant rate of deterioration. Covert and Philip [1973] extended Ghare and Schrader's [1963] model by considering variable rate of deterioration. Further, Shah [1997] extended Covert and Philip's [1973] model by considering shortages.Ouyang et al. [2005] considered an optimal replenishment policy for non-instantaneous deteriorating items with stock dependent and partial backlogging. Again, Ouyang et al. [2006] considered an appropriate inventory model for non- instantaneous deteriorating items with permissible delay in payments. Chung [2008] completed the incomplete proof of Ouyang et al. [2006] model. Geetha and Uthayakumar [2009] proposed an EOQ based model for non-instantaneous deteriorating items with permissible delay in payments. Goyal et al. [2010] proposed an optimal replenishment policies for non-instantaneous deteriorating items stock dependent demand. Soni [2013] extended Goyal et al.[2010] model from two aspects (i) demand rate as multivariate function of price and level of inventory, and (ii) delay in payment permissible. Further Ouyang et al. [2013] extended Soni [2013] model by considering selling those inventories as salvages and all possible replenishment cycle, which may be shorter than the period of non-deterioration. Retailer promotional activity has become prevalent in the business world. Promotional efforts impact the replenishment policy and the sale price of goods. Reza Maihami and Behrooz Karimi [2014] considered the problem of replenishment policy and pricing for non-instantaneous deteriorating items subject to promotional effort and they adopted a price dependent stochastic demand function in which shortages are allowed and partially backlogged. Priyan and Uthayakumar [2015] considered a distributor and a warehouse consisting of a serviceable part and a recoverable part supply chain problem.Chang and Dye[1999] were the first to give a definition for time dependent partial backlogging rate. They considered an EOQ model for deteriorating items with time varying demand and partial backlogging Goyal [1985] was the first to consider the economic order quantity model under conditions of permissible delay in payments. Goyal's[1985] model was extended by Aggarwal and Jaggi [1995] for deteriorating items. Jamal et al. [1997] further extended Aggarwal and Jaggi's [1995] model to consider shortages. Goyal et al.[2005] developed the optimal inventory policies under permissible delay in payments depredating on the ordering quantity. Chang et al. [2015] proposed an inventory system with non-instantaneously deteriorating items in circumstances where the supplier provides the retailer with various trade credits linked to order quantity. Mohsen Lashgari1 et al. [2016] developed an EOQ model with down-stream partial delayed payment and up-stream partial prepayment under three different scenarios: without shortage, with full backordering and with partial backordering. Buzacott [1975] was the first to develop economic order quantity model by considering the effect of inflation. Datta and Pal [1991] studied the effects of inflation and time value of money with linear time dependent demand rate and shortages. Hariga and Ben-Daya [1996] considered optimal time varying lot sizing models underinflationary conditions. Liao et al. [2000] developed an inventory model with deteriorating items under inflation when a delay in payment is permissible. The EOQ model for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effect of inflation and time value of money was considered by Moon et al. [2005]. Yang et al.[2010] developed an inventory model under inflation for deteriorating items with stock dependent consumption rate and partial backlogging shortages. Singh [2011] considered an EOQ model for items having linear demand under inflation and permissible delay in payments. Yashveer Singh et al. [2014] developed an inflation induced stock dependent demand inventory model with permissible delay in payments. An appropriate inventory model for non-instantaneous deteriorating items with cubic demand rate under inflation and permissible delay in payments is proposed in this article. In this model shortages are allowed and partially backlogged. Two inventory models for non- instantaneous deteriorating items and two inventory models for instantaneous deteriorating items with linear deterioration rate and cubic demand rate, that is, the demand rate is a piecewise cubic function of time under inflation and permissible delay in payments are developed. This models supports in minimizing the total inventory cost by finding an optimal replenishment policy. Partially backlogged and completely backlogged cases are considered for all models. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Numerical examples are given to establish the analytical results. Sensitivity analysis of the optimal solution with respect to major parameters is carried out and the effects are discussed in detail.The rest of the article is organized as follows: In section II, the assumptions and notations, which are used throughout this article, are described. In section III, the mathematical formulation and solution of the model to minimize the total inventory cost is established. Numerical examples for all models are provided in section IV. Sensitivity analysis and their observations are discussed in section V. This is followed by conclusion and future research.

2. Assumptions and notations

The following assumptions are made in developing the model:

1. The demand of the product is declining as a cubic function of time.
2. Replenishment rate is infinite and instantaneous.

3. Lead time is zero.
4. Shortages are allowed and are partially backlogged.
5. The deteriorated units can neither be repaired nor replaced during the cycle time.
6. During the time, the account is not settled; generated sales revenue is deposited in an interest bearing account.At the end of the credit period, the account is settled as well as the buyer pays off all units sold and start paying for the interest charges on the items in stocks.

The following notations have been used in developing the model:

1. $D(t)$: $D\,(t) = a + bt{+}ct^2 + dt^3$ is the demand rate, it is a cubic function of time, where a,b,c and d are the positive constants.
2. $I(t)$: Inventory level at any time t, $0 \le t \le T$.
3. Q: Order quantity.
4. Q_1: Inventory level at time $t = 0$.
5. Q_2: Shortage of inventory.
6. C_p: Unit purchase cost of an item.
7. p: Unit selling price of an item.
8. C_2: Shortage cost per unit item.
9. C_3: Cost of lost sales per unit item.
10. δ: Lost sales.
11. I_e: Interest earned per year.
12. I_p: Interest paid in stocks per year.
13. R: Inflation Rate.
14. M: Permissible period of delay in setting the accounts with the supplier.
15. T: The time interval between two successive orders.
16. $\theta\,(t)$: $\theta\,(t) = \theta_1{+}\theta_2 t$ is the deterioration rate of an item, where $\theta_1 > 0$ and $0 < \theta_1 < 1$.
17. HC: Holding Cost per unit (excluding interest charges) is linear function of time $H(t) = \alpha + \beta t$, $\alpha > 0$, $\beta > 0$.
18. TC: Total Cost per unit time.
19. A: Ordering Cost per unit order is known and constant.

3. Mathematical formulation and solution of the model

The instantaneous inventory level $I_1(t)$ at any time t during the cycle time $(0, t_1)$ is governed by the following differential equation

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -\left(a + bt + ct^2 + dt^3\right), \quad 0 \le t \le t_1 \tag{1}$$

The solution of above equation with boundary condition $I_1(0) = Q_1$ is

$$I_1(t) = Q_1 - (at + \frac{bt^2}{2} + \frac{ct^3}{3} + \frac{dt^4}{4}) \tag{2}$$

The instantaneous inventory level $I_2(t)$ at any time t during the cycle time (t_1, t_2) is governed by the following differential equation

$$\frac{dI_2(t)}{dt} + (\theta_1 + \theta_2 t) I_2(t) = -\left(a + bt + ct^2 + dt^3\right), \quad t_1 \le t \le t_2 \tag{3}$$

The solution of above equation with boundary condition $I_2(t_1) = 0$ is

$$I_2(t) = \begin{bmatrix} a\{(t_2 - t) + \frac{\theta_1}{2}(t_2^2 - 2tt_2 + t^2) + \frac{\theta_2}{6}(t_2^3 - 3t^2 t_2 + 2t^3)\} \\ + b\{\frac{1}{2}(t_2^2 - t^2) + \frac{\theta_1}{6}(2t_2^3 - 3tt_2^2 + t^3) + \frac{\theta_2}{8}(t_2^4 - 2t^2 t_2^2 + t^4)\} \\ + c\{\frac{1}{3}(t_3^2 - t^3) + \frac{\theta_1}{12}(3t_2^4 - 4tt_3^2 + t^4) + \frac{\theta_2}{30}(3t_2^5 - 5t^2 t_2^3 + 2t^5)\} \\ + d\{\frac{1}{4}(t_2^4 - t^4) + \frac{\theta_1}{20}(4t_2^5 - 5tt_2^4 + t^5) + \frac{\theta_2}{24}(2t_2^6 - 3t^2 t_2^4 + t^6)\} \end{bmatrix} \tag{4}$$

Due to continuity of $I(t)$ at $t = t_1$, it follows from equation (2) and (4), which implies that $I_1(t_1) = I_1(t_2)$, we get

$$Q_1 = \begin{bmatrix} a\{(t_2) + \frac{\theta_1}{2}(t_2^2 - 2t_1 t_2 + t_1^2) + \frac{\theta_2}{6}(t_2^3 - 3t_1^2 t_2 + 2t_1^3)\} \\ + b\{\frac{1}{2}(t_2^2) + \frac{\theta_1}{6}(2t_2^3 - 3t_1 t_2^2 + t_1^3) + \frac{\theta_2}{8}(t_2^4 - 2t_1^2 t_2^2 + t_1^4)\} \\ + c\{\frac{1}{3}(t_3^2) + \frac{\theta_1}{12}(3t_2^4 - 4t_1 t_3^2 + t_1^4) + \frac{\theta_2}{30}(3t_2^5 - 5t_1^2 t_2^3 + 2t_1^5)\} \\ + d\{\frac{1}{4}(t_2^4) + \frac{\theta_1}{20}(4t_2^5 - 5t_1 t_2^4 + t_1^5) + \frac{\theta_2}{24}(2t_2^6 - 3t_1^2 t_2^4 + t_1^6)\} \end{bmatrix} \tag{5}$$

$$I_1(t) = \begin{bmatrix} a\{(t_2 - t) + \frac{\theta_1}{2}(t_2^2 - 2t_1 t_2 + t_1^2) + \frac{\theta_2}{6}(t_2^3 - 3t_1^2 t_2 + 2t_1^3)\} \\ + b\{\frac{1}{2}(t_2^2 - t^2) + \frac{\theta_1}{6}(2t_2^3 - 3t_1 t_2^2 + t_1^3) + \frac{\theta_2}{8}(t_2^4 - 2t_1^2 t_2^2 + t_1^4)\} \\ + c\{\frac{1}{3}(t_3^2 - t^3) + \frac{\theta_1}{12}(3t_2^4 - 4t_1 t_3^2 + t_1^4) + \frac{\theta_2}{30}(3t_2^5 - 5t_1^2 t_2^3 + 2t_1^5)\} \\ + d\{\frac{1}{4}(t_2^4 - t^4) + \frac{\theta_1}{20}(4t_2^5 - 5t_1 t_2^4 + t_1^5) + \frac{\theta_2}{24}(2t_2^6 - 3t_1^2 t_2^4 + t_1^6)\} \end{bmatrix} \tag{6}$$

During the shortage period (t_2, T), the demand rate at time "t" is partially backlogged at rate of $e^{-\delta(T-t)}(a + bt + ct^2 + dt^3)$.

The instantaneous inventory level $I_3(t)$ at any time t during the cycle time (t_2, T) is governed by the following differential equation

$$\frac{dI_3(t)}{dt} = -e^{-\delta(T-t)}\left(a + bt + ct^2 + dt^3\right), \quad t_2 \le t \le T. \tag{7}$$

The solution of the above equation with the boundary conditions $I_3(t_2) = 0$ and $I_3(T) = -Q_2$ is

$$I_3\left(t\right) = \begin{bmatrix} a\{(t_2 - t)\left(1 - \delta T\right) + \frac{\delta}{2}\left(t_2^2 - t^2\right)\} \\ + \, b\{\frac{1}{2}\left(t_2^2 - t^2\right)\left(1 - \delta T\right) + \frac{\delta}{3}\left(t_2^3 - t^3\right)\} \\ + \, c\{\frac{1}{3}\left(t_3^2 - t^3\right)\left(1 - \delta T\right) + \frac{\delta}{4}\left(t_2^4 - t^4\right)\} \\ + \, d\{\frac{1}{4}\left(t_2^4 - t^4\right)\left(1 - \delta T\right) + \frac{\delta}{5}\left(t_2^5 - t^5\right)\} \end{bmatrix}, \quad t_2 \le t \le T \tag{8}$$

$$Q_2 = \begin{bmatrix} a\{(T - t_2)\left(1 - \delta T\right) + \frac{\delta}{2}\left(T^2 - t_2^2\right)\} \\ + \, b\{\frac{1}{2}\left(T^2 - t_2^2\right)\left(1 - \delta T\right) + \frac{\delta}{3}\left(T^3 - t_2^3\right)\} \\ + \, c\{\frac{1}{3}\left(T^3 - t_2^3\right)\left(1 - \delta T\right) + \frac{\delta}{4}\left(T^4 - t_2^4\right)\} \\ + \, d\{\frac{1}{4}\left(T^4 - t_2^4\right)\left(1 - \delta T\right) + \frac{\delta}{5}\left(T^5 - t_2^5\right)\} \end{bmatrix}, \quad t_2 \le t \le T. \tag{9}$$

The optimum order quantity is given by

$$I(0) = Q = Q_1 + Q_2. \tag{10}$$

The Total Cost (TC) per unit time consists of the following costs:

1. *Ordering Cost:*

$$OC = \frac{A}{T} \tag{11}$$

2. *Holding Cost*:

$$HC = \frac{1}{T}\int_0^{t_1} (\alpha + \beta t) I\left(t\right) dt \text{ (see appendix 1)} \tag{12}$$

3. *Deterioration Cost*: $DC = \frac{C_p}{T}\int_0^{t_2} D(t)e^{-RT}dt$

$$DC = \frac{C_p}{T}\begin{bmatrix} a\left\{ \begin{array}{c} t_1 + \frac{\theta_1}{2}\left(t_2^2 - 2t_1t_2 + t_1^2\right) \\ + \, \frac{\theta_2}{6}\left(t_2^3 - 3t_1^2t_2 + 2t_1^3\right) + \frac{R}{2}\left(t_2^2 - t_1^2\right) \end{array} \right\} \\ + \, b\left\{ \begin{array}{c} \frac{t_2^2}{2} + \frac{\theta_1}{6}\left(2t_2^3 - 3t_1t_2^2 + t_1^3\right) \\ + \, \frac{\theta_2}{8}\left(t_2^4 - 2t_1^2t_2^2 + t_1^4\right) + \frac{R}{2}\left(t_2^3 - t_1^3\right) \end{array} \right\} \\ + \, c\left\{ \begin{array}{c} \frac{t_3^2}{3} + \frac{\theta_1}{12}\left(3t_2^4 - 4t_1t_3^2 + t_1^4\right) \\ + \, \frac{\theta_2}{30}\left(3t_2^5 - 5t_1^2t_2^3 + 2t_1^5\right) + \frac{R}{2}\left(t_2^4 - t_1^4\right) \end{array} \right\} \\ + \, d\left\{ \begin{array}{c} \frac{t_2^4}{4} + \frac{\theta_1}{20}\left(4t_2^5 - 5t_1t_2^4 + t_1^5\right) \\ + \, \frac{\theta_2}{24}\left(2t_2^6 - 3t_1^2t_2^4 + t_1^6\right) + \frac{R}{2}\left(t_2^5 - t_1^5\right) \end{array} \right\} \end{bmatrix} \tag{13}$$

4. *Shortage Cost*:

$$SC = \frac{-C_2}{T}\int_{t_2}^{T} I_3(t)e^{-RT}dt \quad \text{(see appendix 2)} \tag{14}$$

5. *Cost due to lost sales*: $CLS = \frac{C_3}{T}\int_{t_2}^{T} D(t)\left(1-e^{-\delta(T-t)}\right)e^{-RT}dt$

$$CLS = \frac{C_3}{T}\begin{bmatrix} a\left(T^2 - Tt_2\right) \\ +\frac{1}{2}\begin{Bmatrix} bT^3 - bTt_2^2 - aT^2 \\ +at_2^2 - aRT^3 + aRt_2^2T \end{Bmatrix} \\ +\frac{1}{3}\begin{Bmatrix} cT^4 - cTt_2^3 - bT^3 + bt_2^3 \\ -bRT^4 + bRTt_2^3 + aRT^3 - aRt_2^3 \end{Bmatrix} \\ +\frac{1}{4}\begin{Bmatrix} dT^5 - dTt_2^4 - cT^4 + ct_2^4 \\ -cRT^5 + cRTt_2^4 + bRT^4 - bRt_2^4 \end{Bmatrix} \\ +\frac{1}{5}\left(dRT^6 - dRt_2^6\right) \end{bmatrix} \tag{15}$$

To determine the interest earned and interest payable, there will be three cases as follows:

Case I: $0 \le M < t_1$

In this case, the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to t_1.

1. *Interest earned per cycle:* $InE_1 = \frac{pI_e}{T}\int_0^M D(t)e^{-RT}dt$

$$InE_1 = \frac{pI_e}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}+\frac{cM^4}{4}+\frac{dM^5}{5}-R\left(\frac{aM^3}{3}+\frac{bM^4}{4}+\frac{cM^5}{5}+\frac{dM^6}{6}\right)\right] \tag{16}$$

2. *Interest payable per cycle for the inventory not sold after the due period M is*

$$InP_1 = \frac{C_pI_p}{T}\int_M^{t_2} I(t)e^{-RT}dt \quad \text{(see appendix 3)} \tag{17}$$

The Total Cost per unit time is given by

$$TC_1 = OC + HC + DC + SC + CLS + InP_1 - InE_1.$$

Our objective is to minimize the total cost.

The necessary condition for total cost to be minimized are

(i) $\dfrac{\partial(TC_1)}{\partial t_2} = 0$ and

(ii) $\dfrac{\partial^2(TC_1)}{\partial t_2^2} > 0.$

The optimal value of t_2 can be obtained by using the condition (i). Condition (ii) is also satisfied for the value of t_2 obtained from condition (i). The value of t_2 is used to find the optimal values Q and TC_1. Since equation (i) is nonlinear, it is solved by using MATLAB software.

Case II: $t_1 \leq M < t_2$

In this case, the retailer can earn interest on revenue generated from the sales up to M. Although, he has to settle the accounts at M, for that he has to arrange money at some specified rate of interest in order to get his remaining stocks financed for the period M to t_2.

1. *Interest earned per cycle*: $InE_2 = \frac{pI_e}{T}\int_0^M D(t)e^{-RT}dt$

$$InE_2=\frac{pI_e}{T}\left[\frac{aM^2}{2}+\frac{bM^3}{3}+\frac{cM^4}{4}+\frac{dM^5}{5}-R\left(\frac{aM^3}{3}+\frac{bM^4}{4}+\frac{cM^5}{5}+\frac{dM^6}{6}\right)\right] \tag{18}$$

2. *Interest payable per cycle for the inventory not sold after the due period M is*

$$InP_2 = \frac{C_pI_p}{T}\int_M^{t_2} I(t)e^{-RT}dt \quad \text{(see appendix 4)} \tag{19}$$

The Total Cost per unit time is given by

$$TC_2 = OC + HC + DC + SC + CLS + InP_2 - InE_2.$$

Our objective is to minimize the total cost.

The necessary condition for total cost to be minimized are

(i) $\dfrac{\partial(TC_2)}{\partial t_2} = 0$ and

(ii) $\dfrac{\partial^2(TC_2)}{\partial t_2^2} > 0.$

The optimal value of t_2 can be obtained by using condition (i). Condition (ii) is also satisfied for the value of t_2 obtained from condition (i). The value of t_2 is used to find the optimal values Q and TC_2. Since equation (i) is nonlinear, it is solved by using MATLAB software.

Case III: $t_2 \leq M < T$

In this case, the retailer earns interest on the sales revenue up to the permissible delay period and no interest is payable during this period.

1. *Interest earned per cycle*: $InE_3 = \frac{pI_e}{T}\int_0^M D(t)e^{-RT}dt$

$$InE_3 = \frac{pI_e}{T}\left[\begin{array}{l}\left\{\begin{array}{l}\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}+\frac{dt_2^5}{5}\\ -R\left(\frac{at_2^3}{3}+\frac{bt_2^4}{4}+\frac{ct_2^5}{5}+\frac{dt_2^6}{6}\right)\end{array}\right\}\\ \\ +(M-t_2)\left\{\begin{array}{l}\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}+\frac{dt_2^5}{5}\\ -R\left(\frac{at_2^3}{3}+\frac{bt_2^4}{4}+\frac{ct_2^5}{5}+\frac{dt_2^6}{6}\right)\end{array}\right\}\end{array}\right] \tag{20}$$

2. *Interest payable per cycle for the inventory not sold after the due period M is*

$$InP_3 = 0 \tag{21}$$

The Total Cost per unit time is given by

$$TC_3 = OC + HC + DC + SC + CLS + InP_3 - InE_3$$

Our objective is to minimize the total cost.

The necessary condition for total cost to be minimized are

$$\text{(i)} \quad \frac{\partial(TC_3)}{\partial t_2} = 0 \text{ and } \quad \text{(ii)} \quad \frac{\partial^2(TC_3)}{\partial t_2^2} > 0.$$

The optimal value of t_2 can be obtained by using condition (i). Condition (ii) is also satisfied for the value of t_2 obtained from condition (i). The value of t_2 is used to find the optimal values Q and TC_3. Since equation (i) is non-linear, it is solved by using MATLAB software.

4. Numerical analysis

MODEL I: Inventory Model for Non Instantaneous Deteriorating Items with partial backlogging

Case I: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 0.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05,\ C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 0.8,\ t_1 = 1$ *year,* $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.8240$*, the optimal total cost* $TC_1 = Rs.24,049$ *and the optimum order quantity* $Q = 7475.9$

Case II: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 1.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 0.8,\ t_1 = 1$ *year,* $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.7601$*, the optimal total cost* $TC_2 = Rs.18,818$ *and the optimum order quantity* $Q = 7248.3$

Case III: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 2.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 0.8,\ t_1 = 1$ *year,* $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.5756$*, the optimal total cost* $TC_1 = Rs.14,054$ *and the optimum order quantity* $Q = 6601.5$

MODEL II: Inventory Model for Instantaneous Deteriorating Items with partial backlogging

Case I: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 0.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 0.8,\ t_1 = 0,\ T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.8484$, *the optimal total cost* $TC_1 = Rs.14,868$ *and the optimum order quantity* $Q = 7737.7$

Case II: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 1.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 0.8,\ t_1 = 0,\ T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.7913$, *the optimal total cost* $TC_2 = Rs.9,535.5$ *and the optimum order quantity* $Q = 7521.8$

Case III: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 2.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 0.8,\ t_1 = 0,\ T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.6393$, *the optimal total cost* $TC_3 = Rs.4,478.7$ *and the optimum order quantity* $Q = 6955.8$

MODEL III: Inventory Model for Non Instantaneous Deteriorating Items with complete backlogging

Case I: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 0.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 1,\ t_1 = 1$ *year,* $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.8480$, *the optimal total cost* $TC_1 = Rs.23,124$, *and the optimum order quantity* $Q = 6920.4$

Case II: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 1.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 1,\ t_1 = 1year,\ T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.7933$, *the optimal total cost* $TC_2 = Rs.17,775$ *and the optimum order quantity* $Q = 6679.9$

Case III: *Considering* $A = Rs.1000,\ C_p = Rs.25,\ p = Rs.0.15,\ I_p = Rs.0.15,\ I_e = Rs.0.12,\ M = 2.5$ *years,* $\theta_1 = 0.04,\ \theta_2 = 0.04,\ a = 1000,\ b = 500,\ c = 250,\ d = 125,\ \alpha = 5,\ \beta = 0.05, C_2 = Rs.8,\ C_3 = Rs.2,\ R = 0.1,\ \delta = 1,\ t_1 = 1$ *year,* $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.6568$, *the optimal total cost* $TC_3 = Rs.12,666$, *and the optimum order quantity* $Q = 6084.5$

MODEL IV: Inventory Model for Instantaneous Deteriorating Items with complete backlogging

Case I: *Considering* $A = Rs.1000$, $C_p = Rs.25$, $p = Rs.0.15$, $I_p = Rs.0.15$, $I_e = Rs.0.12$, $M = 0.5$ *years,* $\theta_1 = 0.04$, $\theta_2 = 0.04$, $a = 1000$, $b = 500$, $c = 250$, $d = 125$, $\alpha = 5$, $\beta = 0.05$, $C_2 = Rs.8$, $C_3 = Rs.2$, $R = 0.1$, $\delta = 1$, $t_1 = 0$, $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.8695$, *the optimal total cost* $TC_1 = Rs.13,986$ *and the optimum order quantity* $Q = 7194.1$

Case II: *Considering* $A = Rs.1000$, $C_p = Rs.25$, $p = Rs.0.15$, $I_p = Rs.0.15$, $I_e = Rs.0.12$, $M = 1.5$ *years,* $\theta_1 = 0.04$, $\theta_2 = 0.04$, $a = 1000$, $b = 500$, $c = 250$, $d = 125$, $\alpha = 5$, $\beta = 0.05, C_2 = Rs.8$, $C_3 = Rs.2$, $R = 0.1$, $\delta = 1$, $t_1 = 0$, $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.8195$, *the optimal total cost* $TC_2 = Rs.8,550.4$ *and the optimum order quantity* $Q = 6963.4$

Case III: *Considering* $A = Rs.1000$, $C_p = Rs.25$, $p = Rs.0.15$, $I_p = Rs.0.15$, $I_e = Rs.0.12$, $M = 2.5$ *years,* $\theta_1 = 0.04$, $\theta_2 = 0.04$, $a = 1000$, $b = 500$, $c = 250$, $d = 125$, $\alpha = 5$, $\beta = 0.05, C_2 = Rs.8$, $C_3 = Rs.2$, $R = 0.1$, $\delta = 1$, $t_1 = 0$, $T = 3$ *years in appropriate units. Then the optimal value of* $t_2 = 1.6756$, *the optimal total cost* $TC_3 = Rs.3,207.7$ *and the optimum order quantity* $Q = 6411.9$

5. Sensitivity analysis

On the basis of the data given in above examples, it is studied the sensitivity analysis by changing the parameters one at a time by and keeping the rest fixed. From Table 1, the following points are observed:

1. with increase in parameters a, β, θ_1 and θ_2 there is corresponding increase in t_2, total cost and total quantity for all cases;

2. with decrease in parameters a, β, θ_1 and θ_2 there is corresponding decrease in t_2, total cost and total quantity for all cases;

3. with increase in parameter α, there is corresponding decrease in t_2, and total quantity for cases I and II and increase in t_2, and total quantity for case III. Also there is corresponding increase in total cost for all cases;

4. with decrease in parameter α, there is corresponding increase in t_2, and total quantity for cases I and II and decrease in t_2, and total quantity for case III. Also there is corresponding decrease in total cost for all cases;

5. with increase in parameters M and R there is corresponding decrease in t_2, total cost and total quantity for all cases;

6. with decrease in parameters M and R there is corresponding increase in t_2, total cost and total quantity for all cases;
7. with increase in parameter δ, there is corresponding increase in t_2 for all cases and decrease in total cost and total quantity for all cases;
8. with decrease in parameter δ, there is corresponding decrease in t_2 for all cases and increase in total cost and total quantity for all cases.

From Tables 2, 3 and 4, the following points are observed:

1. with increase in parameters a,β, θ_1, θ_2 and δ there is corresponding increase in t_2 and total quantity for all cases and decrease in total cost for all cases;
2. with decrease in parameters a,β, θ_1, θ_2 and δ there is corresponding decrease in t_2 and total quantity for all cases and decrease in total cost for all cases;
3. with increase in parameter α, there is corresponding decrease in t_2, and total quantity for all cases and decrease in total cost for all cases;
4. with decrease in parameter α, there is corresponding increase in t_2, and total quantity for all cases and decrease in total cost for all cases;
5. with increase in parameters M and R there is corresponding decrease in t_2, and total quantity for all cases and decrease in total cost for all cases;
6. with decrease in parameters M and R there is corresponding increase in t_2, and total quantity for all cases and decrease in total cost for all cases.

From the solutions of numerical examples for all models and their cases, the following points are observed:

1. Total cost of Inventory Model for Instantaneous Deteriorating Items with partial backlogging 38.17% for case I, 49.32% for case II and 68.13% for case III are less than the total cost of Inventory Model for Non-Instantaneous Deteriorating Items with partial backlogging.
2. Total cost of Inventory Model for Instantaneous Deteriorating Items with complete backlogging 39.51% for case I, 51.89% for case II and 74.67% for case III are less than the total cost of Inventory Model for Non-Instantaneous Deteriorating Items with complete backlogging.
3. Total cost of Inventory Model for Non-Instantaneous Deteriorating Items with partial backlogging 3.84% for case I, 5.54% for case II and 9.87% for case III are more than the total cost of Inventory Model for Non-Instantaneous Deteriorating Items with complete backlogging.
4. Total cost of Inventory Model for Instantaneous Deteriorating Items with partial backlogging 5.93% for case I, 10.32% for case II and 28.37% for case III are more than the total cost of Inventory Model for Instantaneous Deteriorating Items with complete backlogging.

5. The value of Q for Inventory Model for Instantaneous Deteriorating Items with partial backlogging 3.38% for case I, 3.63% for case II and 5.09% for case III are more than the value of Q for Inventory Model for Non-Instantaneous Deteriorating Items with partial backlogging.
6. The value of Q for Inventory Model for Instantaneous Deteriorating Items with complete backlogging 3.8% for case I, 4.06% for case II and 5.10% for case III are more than the value of Q for Inventory Model for Non-Instantaneous Deteriorating Items with complete backlogging.
7. The value of Q for Inventory Model for Non-Instantaneous Deteriorating Items with partial backlogging 7.43% for case I, 7.84% for case II and 7.83% for case III are more than the value of Q for Inventory Model for Non-Instantaneous Deteriorating Items with complete backlogging.
8. The value of Q for Inventory Model for Instantaneous Deteriorating Items with partial backlogging 7.02% for case I, 7.42% for case II and 7.81% for case III are more than the value of Q for Inventory Model for Instantaneous Deteriorating Items with complete backlogging.
9. The value of for Inventory Model for Instantaneous Deteriorating Items with partial backlogging 1.32% for case I, 1.74% for case II and 3.88% for case III are more than the value of for Inventory Model for Non-Instantaneous Deteriorating Items with partial backlogging.
10. The value of for Inventory Model for Instantaneous Deteriorating Items with complete backlogging 1.15% for case I, 1.43% for case II and 1.12% for case III are more than the value of for Inventory Model for Non-Instantaneous Deteriorating Items with complete backlogging.
11. The value of for Inventory Model for Non-Instantaneous Deteriorating Items with partial backlogging 1.29% for case I, 1.85% for case II and 4.9% for case III are less than the value of for Inventory Model for Non-Instantaneous Deteriorating Items with complete backlogging.
12. The value of for Inventory Model for Instantaneous Deteriorating Items with partial backlogging 1.12% for case I, 1.54% for case II and 2.16% for case III are more than the value of for Inventory Model for Instantaneous Deteriorating Items with complete backlogging.

6. Conclusion

An inventory model for non-instantaneous deteriorating items with cubic demand rate under inflation and permissible delay in payments is proposed in this article. In this model shortages are allowed and partially backlogged. Two inventory models for non- instantaneous deteriorating items and two inventory models for instantaneous deteriorating items with linear deterioration rate and cubic demand rate under inflation and permissible delay in payments are developed. This model supports in minimizing the total inventory cost by finding an optimal replenishment policy. Partially backlogged and completely backlogged cases are considered for all models. From the numerical examples and sensitivity analysis, the following conclusions are obtained for all cases:

(i) *Total cost of Inventory Model for Instantaneous Deteriorating Items with partial/complete backlogging is less than the total cost of Inventory Model for Non- Instantaneous Deteriorating Items with partial /complete backlogging.*

(ii) *Total cost of Inventory Model for Non-instantaneous/Instantaneous Deteriorating Items with partial backlogging is more than the total cost of Inventory Model for Non-instantaneous/Instantaneous Deteriorating Items with complete backlogging.*

(iii) *The value of Q for Inventory Model for Instantaneous Deteriorating Items with partial /complete backlogging is more than the value of Q for Inventory Model for Non- Instantaneous Deteriorating Items with partial /complete backlogging.*

(iv) *The value of Q for Inventory Model for Non-instantaneous/Instantaneous Deteriorating Items with partial backlogging is more than the value of Q for Inventory Model for Non-instantaneous/ Instantaneous Deteriorating Items with complete backlogging.*

(v) *The value of occurrence of shortage period for Inventory Model for Instantaneous Deteriorating Items with partial/complete backlogging is more than the value of occurrence of shortage period for Inventory Model for Non-Instantaneous Deteriorating Items with partial/complete backlogging.*

(vi) *The value of occurrence of shortage period for Inventory Model for Non-instantaneous/Instantaneous Deteriorating Items with partial backlogging is more than the value of occurrence of shortage period for Inventory Model for Noninstantaneous/Instantaneous Deteriorating Items with complete backlogging.*

In future, the proposed model can be extended in several ways. For instance, this inventory model may be extended incorporating with various considerations like fuzzy environment, probabilistic demand rates, probabilistic deterioration rate including shortages, price discount, quantity discount and others.

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Appendix 1

$$HC=\frac{1}{T}\left[\begin{array}{c}
\alpha\left[\begin{array}{c}
a\left\{\begin{array}{c}\frac{t_2^2}{2}+\frac{\theta_1}{6}\left(t_2^3-3t_1^2t_2+2t_1^3\right)\\+\frac{\theta_2}{12}\left(t_2^4-4t_1^3t_2+3t_1^4\right)\end{array}\right\}\\
b\left\{\begin{array}{c}\frac{t_2^3}{3}+\frac{\theta_1}{8}\left(t_2^4-2t_1^2t_2^2+t_1^4\right)\\+\frac{\theta_2}{30}\left(2t_2^5-5t_1^3t_2^2+3t_1^5\right)\end{array}\right\}\\
c\left\{\begin{array}{c}\frac{t_2^4}{4}+\frac{\theta_1}{30}\left(3t_2^5-5t_1^2t_2^3+2t_1^5\right)\\+\frac{\theta_2}{18}\left(t_2^6-2t_1^3t_2^3+t_1^6\right)\end{array}\right\}\\
d\left\{\begin{array}{c}\frac{t_2^5}{5}+\frac{\theta_1}{24}\left(2t_2^6-3t_1^2t_2^4+t_1^6\right)\\+\frac{\theta_2}{84}\left(4t_2^7-7t_1^3t_2^4+3t_1^7\right)\end{array}\right\}
\end{array}\right]\\
+\left(\beta-\alpha R\right)\left[\begin{array}{c}
a\left\{\begin{array}{c}\frac{t_2^3}{6}+\frac{\theta_1}{24}\left(t_2^4-4t_1^3t_2+3t_1^4\right)\\+\frac{\theta_2}{40}\left(t_2^5-5t_1^4t_2+4t_1^5\right)\end{array}\right\}\\
b\left\{\begin{array}{c}\frac{t_2^4}{8}+\frac{\theta_1}{60}\left(2t_2^5-5t_1^3t_2^2+3t_1^5\right)\\+\frac{\theta_2}{48}\left(t_2^6-3t_1^4t_2^2+2t_1^6\right)\end{array}\right\}\\
c\left\{\begin{array}{c}\frac{t_2^5}{10}+\frac{\theta_1}{36}\left(t_2^6-2t_1^3t_2^3+t_1^6\right)\\+\frac{\theta_2}{168}\left(3t_2^7-7t_1^5t_2^2+5t_1^7\right)\end{array}\right\}\\
d\left\{\begin{array}{c}\frac{t_2^6}{12}+\frac{\theta_1}{168}\left(4t_2^7-7t_1^3t_2^4+3t_1^7\right)\\+\frac{\theta_2}{64}\left(t_2^8-2t_1^4t_2^4+t_1^8\right)\end{array}\right\}
\end{array}\right]\\
-\beta R\left[\begin{array}{c}
a\left\{\begin{array}{c}\frac{t_2^4}{12}+\frac{\theta_1}{60}\left(t_2^5-5t_1^4t_2+4t_1^5\right)\\+\frac{\theta_2}{90}\left(t_2^6-6t_1^5t_2+5t_1^6\right)\end{array}\right\}\\
b\left\{\begin{array}{c}\frac{t_2^5}{15}+\frac{\theta_1}{72}\left(t_2^6-3t_1^4t_2^2+2t_1^6\right)\\+\frac{\theta_2}{210}\left(2t_2^7-7t_1^5t_2^2+5t_1^7\right)\end{array}\right\}\\
c\left\{\begin{array}{c}\frac{t_2^6}{18}+\frac{\theta_1}{252}\left(3t_2^7-7t_1^4t_2^3+4t_1^7\right)\\+\frac{\theta_2}{360}\left(3t_2^8-8t_1^5t_2^3+5t_1^8\right)\end{array}\right\}\\
d\left\{\begin{array}{c}\frac{t_2^7}{21}+\frac{\theta_1}{96}\left(t_2^8-2t_1^4t_2^4+t_1^7\right)\\+\frac{\theta_2}{540}\left(4t_2^9-9t_1^5t_2^4+5t_1^9\right)\end{array}\right\}
\end{array}\right]
\end{array}\right]$$

Appendix 2

$$SC=\frac{C_2}{T}\left[\begin{array}{c}
-\left[\begin{array}{c}
\frac{a}{2}\left\{\begin{array}{c}\left(2Tt_2-t_2^2-T^2\right)\\-\delta\left(2T^2t_2-2T^2t_2-T^3\right)\end{array}\right\}\\
\frac{1}{6}\left\{\begin{array}{c}\left(a\delta+b\right)\left(3Tt_2^2-2t_2^3-T^3\right)\\-b\delta\left(3T^2t_2^2-2Tt_2^3-T^4\right)\end{array}\right\}\\
\frac{1}{12}\left\{\begin{array}{c}\left(b\delta+c\right)\left(4Tt_2^3-3t_2^4-T^4\right)\\-c\delta\left(4T^2t_2^3-3Tt_2^4-T^5\right)\end{array}\right\}\\
\frac{1}{20}\left\{\begin{array}{c}\left(c\delta+d\right)\left(3Tt_2^2-2t_2^6-T^6\right)\\-d\delta\left(3T^3t_2^4-2Tt_2^6-T^7\right)\end{array}\right\}\\
+\frac{d\delta}{70}\left(7T^2t_2^5-5t_2^7-2T^7\right)
\end{array}\right]\\
+R\left[\begin{array}{c}
\frac{a}{6}\left\{\begin{array}{c}\left(3T^2t_2-t_2^3-2T^3\right)\\-\delta\left(3T^3t_2-T^3t_2-2T^4\right)\end{array}\right\}\\
\frac{1}{8}\left\{\begin{array}{c}\left(a\delta+b\right)\left(2T^2t_2^2-t_2^4-T^4\right)\\-b\delta\left(2T^3t_2^2-Tt_2^4-T^5\right)\end{array}\right\}\\
\frac{1}{30}\left\{\begin{array}{c}\left(b\delta+c\right)\left(5T^2t_2^3-3t_2^5-2T^5\right)\\-c\delta\left(5T^3t_2^3-3Tt_2^5-2T^6\right)\end{array}\right\}\\
\frac{1}{24}\left\{\begin{array}{c}\left(c\delta+d\right)\left(3T^2t_2^4-2t_2^6-T^6\right)\\-d\delta\left(3T^3t_2^4-2Tt_2^6-T^7\right)\end{array}\right\}\\
+\frac{d\delta}{70}\left(7T^2t_2^5-5t_2^7-2T^7\right)
\end{array}\right]
\end{array}\right]$$

Appendix 3

$$InP_1 = \frac{C_p I_p}{T}\left[\begin{array}{c}
\left[\begin{array}{c}
a\left\{\begin{array}{c}
\frac{1}{2}\left(-2Mt_2+M^2+t_2^2\right)\\
+\frac{\theta_1}{6}\left(t_2^3-3t_1^2t_2+2t_1^3-3M\left(t_2^2-2t_1t_2+t_1^2\right)\right)\\
+\frac{\theta_2}{12}\left(t_2^4-4t_1^3t_2+3t_1^4-2M\left(t_2^3-3t_1^2t_2+2t_1^3\right)\right)
\end{array}\right\}\\
+b\left\{\begin{array}{c}
\frac{1}{6}\left(-3Mt_2^2+M^3+2t_2^3\right)\\
+\frac{\theta_1}{24}\left(3t_2^4-6t_1^2t_2^2+3t_1^4-4M\left(2t_2^3-3t_1t_2^2+t_1^3\right)\right)\\
+\frac{\theta_2}{120}\left(8t_2^5-20t_1^3t_2^2+12t_1^5-15M\left(t_2^4-2t_1^2t_2+t_1^4\right)\right)
\end{array}\right\}\\
+c\left\{\begin{array}{c}
\frac{1}{12}\left(-4Mt_2^3+M^4+3t_2^4\right)\\
+\frac{\theta_1}{60}\left(6t_2^5-10t_1^2t_2^3+4t_1^4-5M\left(3t_2^4-4t_1t_2^3+t_1^4\right)\right)\\
+\frac{\theta_2}{90}\left(5t_2^6-10t_1^3t_2^3+5t_1^6-3M\left(3t_2^5-5t_1^2t_2^2+2t_1^5\right)\right)
\end{array}\right\}\\
+d\left\{\begin{array}{c}
\frac{1}{20}\left(-5Mt_2^4+M^5+4t_2^5\right)\\
+\frac{\theta_1}{120}\left(10t_2^6-15t_1^2t_2^4+5t_1^4-6M\left(4t_2^5-5t_1t_2^4+t_1^5\right)\right)\\
+\frac{\theta_2}{168}\left(8t_2^7-14t_1^3t_2^4+6t_1^7-7M\left(2t_2^6-3t_1^2t_2^4+t_1^6\right)\right)
\end{array}\right\}
\end{array}\right]\\
-R\left[\begin{array}{c}
a\left\{\begin{array}{c}
\frac{1}{6}\left(-3M^2t_2+2M^3+t_2^3\right)\\
+\frac{\theta_1}{24}\left(t_2^4-4t_1^3t_2+3t_1^4-6M^2\left(t_2^2-2t_1t_2+t_1^2\right)\right)\\
+\frac{\theta_2}{120}\left(3t_2^5-15t_1^4t_2+12t_1^5-10M^2\left(t_2^3-3t_1^2t_2+2t_1^3\right)\right)
\end{array}\right\}\\
+b\left\{\begin{array}{c}
\frac{1}{8}\left(-2M^2t_2^2+M^4+t_2^4\right)\\
+\frac{\theta_1}{120}\left(4t_2^5-10t_1^3t_2^2+6t_1^5-10M^2\left(2t_2^3-3t_1t_2^2+t_1^3\right)\right)\\
+\frac{\theta_2}{48}\left(t_2^6-3t_1^4t_2^2+2t_1^6-3M^2\left(t_2^4-2t_1^2t_2^2+t_1^4\right)\right)
\end{array}\right\}\\
+c\left\{\begin{array}{c}
\frac{1}{30}\left(-5M^2t_2^3+2M^5+3t_2^5\right)\\
+\frac{\theta_1}{216}\left(6t_2^6-12t_1^3t_2^3+6t_1^6-9M^2\left(3t_2^4-4t_1t_2^3+t_1^4\right)\right)\\
+\frac{\theta_2}{840}\left(15t_2^7-35t_1^4t_2^3+20t_1^6-14M^2\left(3t_2^5-5t_1^2t_2^3+2t_1^5\right)\right)
\end{array}\right\}\\
+d\left\{\begin{array}{c}
\frac{1}{24}\left(-3M^2t_2^4+M^6+2t_2^6\right)\\
+\frac{\theta_1}{840}\left(20t_2^7-35t_1^3t_2^4+15t_1^7-21M^2\left(4t_2^5-5t_1t_2^4+t_1^5\right)\right)\\
+\frac{\theta_2}{192}\left(3t_2^8-6t_1^4t_2^4+3t_1^8-4M^2\left(2t_2^6-3t_1^2t_2^4+t_1^6\right)\right)
\end{array}\right\}
\end{array}\right]
\end{array}\right]$$

Appendix 4

$$InP_2 = \frac{C_p I_p}{T}\left[\begin{array}{c}
\left[\begin{array}{c}
a\left\{\begin{array}{c}
\frac{1}{2}\left(-2Mt_2+M^2+t_2^2\right)\\
+\frac{\theta_1}{6}\left(t_2^3-3Mt_2^2+3M^2t_2-M^3\right)\\
+\frac{\theta_2}{12}\left(t_2^4-2Mt_2^3+2M^3t_2-M^4\right)
\end{array}\right\}\\
+b\left\{\begin{array}{c}
\frac{1}{6}\left(-3Mt_2^2+M^3+2t_2^3\right)\\
+\frac{\theta_1}{24}\left(3t_2^4-8Mt_2^3+6M^2t_2^2-M^4\right)\\
+\frac{\theta_2}{120}\left(8t_2^5-15Mt_2^4+10M^3t_2^2-3M^5\right)
\end{array}\right\}\\
+c\left\{\begin{array}{c}
\frac{1}{12}\left(-4Mt_2^3+M^4+3t_2^4\right)\\
+\frac{\theta_1}{60}\left(6t_2^5-15Mt_2^4+10M^2t_2^3-M^5\right)\\
+\frac{\theta_2}{90}\left(5t_2^6-9Mt_2^5+5M^3t_2^3-M^6\right)
\end{array}\right\}\\
+d\left\{\begin{array}{c}
\frac{1}{20}\left(-5Mt_2^2+M^5+4t_2^5\right)\\
+\frac{\theta_1}{120}\left(10t_2^6-24Mt_2^5+15M^2t_4-M^6\right)\\
+\frac{\theta_2}{168}\left(8t_2^7-14Mt_2^6+7M^3t_2^4-M^7\right)
\end{array}\right\}
\end{array}\right]\\
-R\left[\begin{array}{c}
a\left\{\begin{array}{c}
\frac{1}{6}\left(-3M^2t_2+2M^3+t_2^3\right)\\
+\frac{\theta_1}{24}\left(t_2^4-6M^2t_2^2+8M^3t_2-3M^4\right)\\
+\frac{\theta_2}{120}\left(3t_2^5-10M^2t_2^3+15M^4t_2-8M^5\right)
\end{array}\right\}\\
+b\left\{\begin{array}{c}
\frac{1}{8}\left(-2M^2t_2^2+M^4+t_2^4\right)\\
+\frac{\theta_1}{30}\left(t_2^5-5M^2t_2^3+5M^3t_2^2-M^5\right)\\
+\frac{\theta_2}{48}\left(t_2^6-3M^2t_2^4+3M^4t_2^2-M^6\right)
\end{array}\right\}\\
+c\left\{\begin{array}{c}
\frac{1}{30}\left(-5M^2t_2^3+2M^5+3t_2^5\right)\\
+\frac{\theta_1}{216}\left(6t_2^6-27M^2t_2^4+24M^3t_2^3-M^6\right)\\
+\frac{\theta_2}{840}\left(15t_2^7-42M^2t_2^5+35M^4t_2^3-8M^7\right)
\end{array}\right\}\\
+d\left\{\begin{array}{c}
\frac{1}{24}\left(-3M^2t_2^4+M^6+2t_2^6\right)\\
+\frac{\theta_1}{420}\left(10t_2^7-42M^2t_2^5+35M^3t_2^4-3M^7\right)\\
+\frac{\theta_2}{192}\left(3t_2^8-8M^2t_2^6+6M^4t_2^4-M^8\right)
\end{array}\right\}
\end{array}\right]
\end{array}\right]$$

Table 1: MODEL I

		Case I			Case II			Case III		
Parameter	%	t_2	TC	Q	t_2	TC	Q	t_2	TC	Q
a	+50%	1.8606	31,071	8864.2	1.7976	24,484	8607.0	1.6342	18,177	7940.7
	+25%	1.8430	27,546	8168.1	1.7796	21,640	7925.5	1.6067	16,105	7269.1
	-25%	1.8034	20,581	6788.3	1.7388	16,018	6575.7	1.5397	12,027	5938.0
	-50%	1.7807	17,145	6105.1	1.7153	13,243	5908.3	1.4970	10,027	5278.4
α	+50%	1.7646	25,249	7264.3	1.7156	20,230	7090.7	1.5925	15,605	6659.9
	+25%	1.7891	24,613	7351.4	1.7336	19,509	7154.4	1.5862	14,828	6638.1
	-25%	1.8788	23,631	7672.2	1.8030	18,190	7401.0	1.5544	13,285	6528.4
	-50%	1.9803	23,601	8037.1	1.8866	17,740	7700.2	1.4892	12,533	6305.5
β	+50%	1.8406	24,287	7535.3	1.7764	18,987	7306.2	1.5916	14,115	6656.8
	+25%	1.8322	24,166	7505.3	1.7681	18,901	7276.7	1.5835	14,084	6628.8
	-25%	1.8162	23,937	7448.1	1.7523	18,738	7220.6	1.5680	14,026	6575.2
	-50%	1.8086	23,830	7420.9	1.7448	18,662	7194.0	1.5621	13,998	6554.9
θ_1	+50%	1.8332	24,281	7528.9	1.7697	18,986	7298.9	1.5915	14,143	6665.4
	+25%	1.8286	24,164	7502.3	1.7649	18,901	7273.5	1.5836	14,098	6633.4
	-25%	1.8194	23,936	7449.9	1.7552	18,735	7223.0	1.5675	14,012	6569.5
	-50%	1.8148	23,824	7424.1	1.7503	18,654	7198.0	1.5593	13,971	6537.5
θ_2	+50%	1.8367	24,339	7547.5	1.7734	19,033	7316.8	1.5962	14,161	6683.7
	+25%	1.8304	24,193	7511.6	1.7667	18,924	7282.1	1.5859	14,107	6642.3
	-25%	1.8177	23,908	7441.1	1.7533	18,714	7214.3	1.5652	14,004	6560.8
	-50%	1.8113	23,771	7406.5	1.7466	18,612	7181.1	1.5548	13,956	6520.6
M	+50%	1.8091	22,891	7422.7	1.7101	12,392	7071.3	0.3743	10,645	3016.3
	+25%	1.8166	23,475	7449.5	1.7352	16,070	7160.0	1.3963	11,767	5993.0
	-25%	1.8314	24,618	7502.4	1.7845	20,995	7335.0	1.6654	16,637	6914.1
	-50%	1.8386	25,184	7528.1	1.8084	22,873	7420.2	1.7282	19,412	7135.2
δ	+50%	1.8683	22,241	6385.1	1.8200	16,792	6132.8	1.7078	11,423	5548.8
	+25%	1.8480	23,124	6920.4	1.7933	17,775	6679.9	1.6568	12,666	6084.5
	-25%	1.7948	25,026	8056.9	1.7156	19,940	7841.4	-0.7380	17,693	2884.9
	-50%	1.7569	26,070	8670.0	1.6450	21,183	8461.1	-1.4823	30,308	4632.2
R	+50%	1.7320	24,435	8623.0	1.6795	17,948	6963.6	1.5539	13,822	6526.7
	+25%	1.7711	25,148	8697.0	1.7134	18,336	7082.0	1.5625	13,937	6556.3
	-25%	1.9123	26,897	8970.0	1.8304	19,484	7498.8	1.5980	14,178	6679.0
	-50%	2.0372	28,517	9218.5	1.9552	20,640	7946.8	1.6464	14,321	6847.5

Table 2: MODEL II

		Case I			Case II			Case III		
Parameter	%	t_2	TC	Q	t_2	TC	Q	t_2	TC	Q
a	+50%	1.8846	14,809	7875.3	1.8283	9,488	7661.5	1.6920	4,431.2	7150.4
	+25%	1.8672	14,852	7809.1	1.8105	9,523.0	7594.2	1.6671	4,466.2	7058.3
	-25%	1.8280	14,851	7604.1	1.7704	9,521.7	7443.2	1.6076	4,464.4	6839.7
	-50%	1.8055	14,796	7575.3	1.7473	9,477.7	7356.5	1.5708	4,416.3	6705.8
α	+50%	1.7849	14,717	7497.7	1.7404	9,459.2	7330.7	1.6332	4,478.2	6933.4
	+25%	1.8111	14,813	7596.5	1.7612	9,507.6	7408.6	1.6356	4,478.5	6942.3
	-25%	1.9062	14,712	7957.7	1.8394	9,453.8	7703.6	1.6460	4,478.0	6980.5
	-50%	2.0119	13,357	8362.6	1.9307	8,720.7	8051.3	1.6619	4,470.5	7039.1
β	+50%	1.8650	14,856	7800.8	1.8077	9,526.4	7583.6	1.6551	4,474.8	7014.0
	+25%	1.8566	14,865	7768.8	1.7994	9,533.1	7552.3	1.6471	4,477.8	6984.5
	-25%	1.8405	14,865	7707.7	1.7835	9,533.3	7492.4	1.6317	4,477.9	6927.9
	-50%	1.8329	14,858	7678.9	1.7760	9,527.9	7464.2	1.6244	4,475.5	6901.2
θ_1	+50%	1.8642	14,857	7797.7	1.8094	9,524.4	7590.1	1.6692	4,464.1	7066.0
	+25%	1.8564	14,865	7768.1	1.8005	9,532.5	7556.5	1.6546	4,475.0	7012.1
	-25%	1.8402	14,865	7706.6	1.7819	9,532.5	7486.4	1.6232	4,474.9	6896.8
	-50%	1.8319	14,857	7675.2	1.7723	9,524.0	7450.3	1.6062	4,463.1	6834.6
θ_2	+50%	1.8643	14,857	7798.1	1.8085	9,525.5	7586.7	1.6654	4,467.7	7052.0
	+25%	1.8564	14,865	7768.1	1.8000	9,532.8	7554.6	1.6525	4,476.0	7004.4
	-25%	1.8403	14,865	7707.0	1.7825	9,532.8	7488.7	1.6257	4,476.0	6905.9
	-50%	1.8321	14,857	7675.9	1.7737	9,525.6	7455.6	1.6117	4,467.7	6854.7
M	+50%	1.8338	14,859	7682.4	1.7479	9,479.2	7358.8	0.9880	2,431.0	4741.4
	+25%	1.8411	14,865	7710.0	1.7696	9,520.7	7440.2	1.5316	4,335.8	6564.3
	-25%	1.8556	14,865	7765.0	1.8130	9,519.5	7603.7	1.7093	4,391.9	7214.7
	-50%	1.8626	14,859	7791.6	1.8345	9,470.1	7685.0	1.7624	4,178.7	7413.1
δ	+50%	1.8877	14,798	7887.1	1.8430	9,440.4	7717.2	1.7407	4,284.2	7331.8
	+25%	1.8695	14,848	7817.9	1.8195	9,508.3	7628.2	1.6990	4,416.9	7176.4
	-25%	1.8234	14,843	7643.0	1.7554	9,496.4	7386.9	1.5222	4,312.9	6530.6
	-50%	1.7923	14,748	7525.6	1.7050	9,330.0	7198.7	-1.4720	-8,289.8	-975.533
R	+50%	1.7537	14,550	7380.5	1.7066	9,337.3	7204.7	1.5998	4,456.7	6811.3
	+25%	1.7940	14,755	7532.0	1.7423	9,464.5	7337.8	1.6158	4,470.7	6869.7
	-25%	1.9283	14,558	8042.1	1.8645	9,337.3	7798.9	1.6774	4,454.6	7096.3
	-50%	2.0655	11,925	8568.7	1.9929	7,615.0	8289.7.0	1.7528	4,228.8	7377.1

Table 3: MODEL III

		Case I			**Case II**			**Case III**		
Parameter	%	t_2	TC	Q	t_2	TC	Q	t_2	TC	Q
a	+50%	1.8827	23,068	7073.2	1.8283	17,730	6833.7	1.7038	12,621	6288.6
	+25%	1.8660	23,109	6999.7	1.8115	17,763	6759.9	1.6815	12,654	6191.6
	-25%	1.8284	23,108	6834.2	1.7734	17,762	6592.7	1.6292	12,652	5965.2
	-50%	1.8069	23,055	6739.7	1.7516	17,720	6497.2	1.5977	12,607	5829.6
α	+50%	1.7857	22,972	6646.6	1.7430	17,696	6459.6	1.6440	12,663	6029.1
	+25%	1.8116	23,070	6760.3	1.7637	17,746	6550.2	1.6491	12,665	6051.2
	-25%	1.9038	22,974	7166.2	1.8396	17,695	6883.4	1.6698	12,663	6140.8
	-50%	2.0048	21,710	7611.2	1.9258	17,010	7263.2	1.6963	12,635	6255.9
β	+50%	1.8639	23,113	6990.4	1.8088	17,766	6748.0	1.6712	12,662	6146.9
	+25%	1.8558	23,121	6954.7	1.8009	17,773	6713.3	1.6639	12,665	6115.2
	-25%	1.8404	23,121	6887.0	1.7859	17,773	6647.5	1.6500	12,665	6055.1
	-50%	1.8331	23,115	6854.8	1.7787	17,768	6615.9	1.6435	12,663	6026.1
θ_1	+50%	1.8562	23,121	6956.5	1.8016	17,772	6716.4	1.6682	12,663	6133.9
	+25%	1.8521	23,123	6938.5	1.7974	17,774	6697.9	1.6625	12,665	6109.2
	-25%	1.8438	23,123	6901.9	1.7891	17,774	6661.5	1.6511	12,665	6059.8
	-50%	1.8396	23,121	6883.4	1.7848	17,772	6642.6	1.6454	12,663	6035.2
θ_2	+50%	1.8595	23,118	6971.0	1.8050	17,770	6731.3	1.6719	12,661	6149.9
	+25%	1.8537	23,122	6945.5	1.7991	17,774	6705.4	1.6644	12,665	6117.4
	-25%	1.8422	23,122	6894.9	1.7874	17,774	6654.0	1.6493	12,663	6052.0
	-50%	1.8364	23,118	6869.4	1.7815	17,770	6628.2	1.6417	12,662	6019.2
M	+50%	1.8349	23,117	6862.8	1.7528	17,723	6502.5	1.4443	12,088	5179.3
	+25%	1.8415	23,122	6891.8	1.7729	17,761	6590.5	1.5747	12,558	5731.0
	-25%	1.8544	23,122	6948.6	1.8138	17,760	9770.0	1.7176	12,589	6348.7
	-50%	1.8608	23,117	6976.8	1.8343	17,713	6860.1	1.7661	12,393	6560.7
δ	+50%	1.8942	23,023	7123.9	1.8525	17,641	6940.2	1.7613	12,419	6539.7
	+25%	1.8730	23,095	7030.5	1.8260	17,736	6823.6	1.7181	12,588	6350.9
	-25%	1.8173	23,085	6785.4	1.7503	17,716	6491.5	1.5438	12,472	5599.1
	-50%	1.7773	22,931	6609.8	1.6856	17,445	6209.4	-1.3511	-8.6484	-3.3961
R	+50%	1.7532	22,791	6504.2	1.7076	17,558	6305.1	1.6088	12,627	5877.3
	+25%	1.7939	23,008	6682.6	1.7442	17,699	6464.8	1.6289	12,652	5963.9
	-25%	1.9263	22,817	7265.4	1.8650	17,574	6995.3	1.6991	12,630	6268.1
	-50%	2.0588	20,307	7848.4	1.9880	15,922	7537.3	1.7748	12,342	6598.8

Table 4: MODEL IV

		Case I			Case II			Case III		
Parameter	%	t_2	TC	Q	t_2	TC	Q	t_2	TC	Q
a	+50%	1.9041	13,925	7354.2	1.8544	8,500.5	7124.4	1.7441	3,157.5	6617.4
	+25%	1.8874	13,970	7276.9	1.8376	8,537.4	7046.8	1.7226	3,194.5	6519.3
	-25%	1.8500	13,969	7104.0	1.7999	8,536.0	6873.2	1.6727	3,192.7	6292.7
	-50%	1.8287	13,912	7005.8	1.7783	8,489.2	6774.0	1.6427	3,142.6	6157.2
α	+50%	1.8038	13,801	6891.1	1.7644	8,443.7	6710.3	1.6748	3,195.0	6302.2
	+25%	1.8312	13,921	7017.3	1.7872	8,512.2	6814.9	1.6846	3,203.1	6346.6
	-25%	1.9279	13,805	7464.5	1.8698	8,443.8	7195.5	1.7226	3,194.5	6519.3
	-50%	2.0327	12,288	7950.7	1.9620	7,540.4	7622.7	1.7693	3,079.9	6732.8
β	+50%	1.8855	13,974	7268.1	1.8353	8,540.5	7036.2	1.7136	3,202.7	6478.3
	+25%	1.8774	13,983	7230.7	1.8273	8,548.0	6999.3	1.7062	3,206.5	6444.7
	-25%	1.8618	13,983	7158.5	1.8121	8,548.2	6929.3	1.6921	3,206.6	6380.6
	-50%	1.8544	13,976	7124.4	1.8049	8,542.3	6896.2	1.6853	3,203.5	6349.8
θ_1	+50%	1.8838	13,976	7260.3	1.8354	8,540.4	7036.7	1.7214	3,195.8	6513.8
	+25%	1.8767	13,984	7227.4	1.8275	8,547.9	7000.3	1.7104	3,204.7	6463.8
	-25%	1.8621	13,984	7159.9	1.8114	8,547.8	6926.1	1.6873	3,204.6	6358.8
	-50%	1.8546	13,976	7125.3	1.8031	8,540.2	6887.9	1.6752	3,195.4	6304.0
θ_2	+50%	1.8840	13,976	7261.2	1.8348	8,541.1	7033.9	1.7192	3,198.1	6503.8
	+25%	1.8768	13,984	7227.9	1.8272	8,548.1	6998.9	1.7092	3,205.3	6458.3
	-25%	1.8621	13,984	7159.9	1.8118	8,548.1	6927.9	1.6887	3,205.3	6365.2
	-50%	1.8546	13,976	7125.3	1.8040	8,541.3	6892.1	1.6782	3,198.2	6317.6
M	+50%	1.8630	13,978	7134.1	1.7831	8,502.2	6796.0	1.5376	2,762.4	5688.0
	+25%	1.8630	13,984	7164.1	1.8011	8,537.6	6878.7	1.6322	3,117.8	6110.0
	-25%	1.8759	13,984	7223.7	1.8383	8,536.3	7050.0	1.7512	3,139.6	6649.9
	-50%	1.8822	13,978	7252.9	1.8571	8,492.2	7136.8	1.7942	2,962.2	6847.0
δ	+50%	1.9113	13,896	7387.6	1.8721	8,433.4	7206.1	1.7869	3,001.3	6813.5
	+25%	1.8919	13,961	7297.7	1.8483	8,516.8	7096.2	1.7494	3,144.4	6641.7
	-25%	1.8426	13,953	7069.9	1.7832	8,502.5	6796.5	1.6189	3,081.5	6050.2
	-50%	1.8088	13,827	6914.1	1.7328	8,300.2	6565.8	-1.3243	-2,003.5	-3441.5
R	+50%	1.7726	13,605	6747.9	1.7308	8,289.4	6556.7	1.6428	3,142.9	6157.7
	+25%	1.8142	13,853	6939.0	1.7688	8,459.3	6730.5	1.6664	3,184.9	6264.2
	-25%	1.9493	13,636	7563.8	1.8936	8,309.0	7305.6	1.7479	3,148.3	6634.8
	-50%	2.0841	10,768	8188.8	2.0200	6,327.1	7891.8	1.8348	2,668.3	7033.9

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