SOME INCLUSION PROPERTIES OF STARLIKE AND CONVEX FUNCTIONS ASSOCIATED WITH HOHLOV OPERATOR. II

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Abstract. A new subclass $\mathbf{K}(\lambda, \alpha)$ involving Hohlov Operator is introduced and some inclusion relations and distortion bounds are obtained for $f \in \mathbf{K}(\lambda, \alpha)$.

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1. Introduction

Let $\mathcal{A}$ be the class of functions f normalized by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}.$$

As usual, we denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions which are also univalent in $\mathbb{U}$. The well known subclasses of $\mathcal{S}$ are the class of starlike functions($\mathcal{ST}$) and convex functions($\mathcal{CV}$). A function $f(z) \in \mathcal{S}$ is starlike of order $\alpha(0 \leq \alpha < 1)$ denoted by $\mathcal{ST}(\alpha)$, if $\text{Re}\left(\frac{zf'(z)}{f(z)}\right) > \alpha$ and it is convex of order $\alpha(0 \leq \alpha < 1)$ denoted by $\mathcal{CV}(\beta)$, if $\text{Re}\left(1 + \frac{zf''(z)}{f'(z)}\right) > \beta$. It is an established fact that $f \in \mathcal{CV}(\alpha) \Longleftrightarrow zf' \in \mathcal{ST}(\alpha)$.

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For functions $f \in \mathcal{A}$ given by (1.1) and $g \in \mathcal{A}$ given by $g(z) = z + \sum_{n=2}^{\infty} b_n z^n$, we define the Hadamard product (or convolution) of f and g by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n, \quad (z \in \mathbb{U}). \tag{1.2}$$

Let $\mathcal{T}$ denote the subclass of $\mathcal{A}$ consisting of functions of the form

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0;\ z \in \mathbb{U}). \tag{1.3}$$

The class $\mathcal{T}$ was introduced by Silverman [10]. We denote by $\mathcal{T}^*(\alpha)$ and $\mathcal{C}(\alpha)$ denote the class of functions of the form (1.3) which are, respectively, starlike of order α and convex of order α with $0 \leq \alpha < 1$.

The Gaussian hypergeometric function $F(a, b; c, z)$ given by

$$F(a, b; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} z^n \quad (z \in \mathbb{U}) \tag{1.4}$$

where, a, b, c are complex numbers such that $c \neq 0, -1, -2, -3, \ldots,$ $(a)_0 = 1$ for $a \neq 0$ and for each positive integer n, $(a)_n = a(a+1)(a+2)\ldots(a+n-1)$ is the Pochhammer symbol, and is the solution of the homogenous hypergeometric differential equation

$$z(1-z)w''(z) + [c - (a+b+1)z]w'(z) - abw(z) = 0$$

has rich applications in various fields such as conformal mappings, quasi conformal theory, continued fractions and so on. The Gauss Summation theorem

$$F(a, b; c; 1) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n (1)_n} = \frac{\Gamma(c-a-b)\Gamma(c)}{\Gamma(c-a)\Gamma(c-b)} \text{ for } \mathrm{Re}(c-a-b) > 0 \tag{1.5}$$

and the function $F(a, b; c; 1)$ is bounded if $\mathrm{Re}(c-a-b) > 0$ and has a pole at $z = 1$ if $\mathrm{Re}(c-a-b) \leq 0$.

For $f \in \mathcal{A}$, we recall the operator $I_{a,b,c}(f)$ of Hohlov [5] which maps $\mathcal{A}$ into itself defined by means of Hadamard product as

$$I_{a,b,c}(f)(z) = zF(a, b; c; z) * f(z) \tag{1.6}$$

Therefore, for a function f defined by (1.1), we have

$$I_{a,b,c}(f)(z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1} (b)_{n-1}}{(c)_{n-1} (1)_{n-1}} a_n z^n. \tag{1.7}$$

$$\Phi(n) = \frac{(a)_{n-1} (b)_{n-1}}{(c)_{n-1} (1)_{n-1}} \quad (a, b > 0;\ n \geq 2). \tag{1.8}$$

A function $f\in\mathcal{A}$ is said to be in the class $\mathcal{R}^{\tau}(A,B)$, $(\tau\in\mathbb{C}\backslash\{0\},\ -1\leq B<A\leq 1)$, if it satisfies the inequality

$$\left|\frac{f'(z)-1}{(A-B)\tau-B[f'(z)-1]}\right|<1\quad(z\in\mathbb{U}).$$

The class $\mathcal{R}^{\tau}(A,B)$ was introduced earlier by Dixit and Pal [3]. If we put

$$\tau=1,\ A=\beta\ \text{ and }\ B=-\beta\ (0<\beta\leq 1),$$

we obtain the class of functions $f\in\mathcal{A}$ satisfying the inequality

$$\left|\frac{f'(z)-1}{f'(z)+1}\right|<\beta\quad(z\in\mathbb{U};0<\beta\leq 1)$$

which was studied by (among others) Padmanabhan [8] and Caplinger and Causey [2], (see also [12]). We recall the following lemma relevant for our discussions.

Lemma 1.1 [3] *If $f\in\mathcal{R}^{\tau}(A,B)$ is of form* (1.1)*, then*

$$|a_n|\leq(A-B)\frac{|\tau|}{n},\quad n\in\mathbb{N}\setminus\{1\}.\tag{1.9}$$

The result is sharp for the function

$$f(z)=\int_0^z\left(1+\frac{(A-B)\tau z^{n-1}}{1+Bz^{n-1}}\right)dz,\qquad(n\geq 2;\ z\in\mathbb{U}).$$

In this paper, we consider the following subclass of $\mathcal{S}$ due to Kamali et al. [7] as given below:

For some $\alpha(0\leq\alpha<1)$ and $\lambda(0\leq\lambda<1)$, we let $\mathbf{K}(\lambda,\alpha)$ be a new subclass of $\mathcal{S}$ consisting of functions of the form (1.1) satisfying the analytic criteria

$$\operatorname{Re}\left(\frac{\lambda z^3f'''(z)+(2\lambda+1)z^2f''(z)+zf'(z)}{\lambda z^2f''(z)+zf'(z)}\right)>\alpha,\ z\in\mathbb{U}.$$

We recall the following lemma due to Kamali et al. [7] to prove the main results.

Lemma 1.2 *A function $f\in\mathcal{T}$ belongs to the class $\mathbf{K}(\lambda,\alpha)$ if and only if*

$$\sum_{n=2}^{\infty}n(n-\alpha)(1+n\lambda-\lambda)|a_n|\leq 1-\alpha.\tag{1.10}$$

Lemma 1.3 [10] *A function f of the form* (1.3) *is in $\mathcal{T}^*(\alpha)$ if and only if*

$$\sum_{n=2}^{\infty}(n-\alpha)a_n\leq 1-\alpha\quad(0\leq\alpha<1)$$

and is in $\mathcal{C}(\alpha)$ if and only if

$$\sum_{n=2}^{\infty}n(n-\alpha)a_n\leq 1-\alpha\quad(0\leq\alpha<1).$$

Motivated by the earlier works on hypergeometric functions studied recently in [9], [11]–[14], we will study the action of the hypergeometric function on the class $\mathbf{K}(\lambda,\alpha)$.

2. Main results

Theorem 2.1 [14] *Let $a,b\in\mathbb{C}\setminus\{0\}$, and c be a real number. If $f\in\mathcal{ST}$ and the inequality*

$$\begin{aligned}(2.11)\quad &\lambda\frac{|a||b|(1+|a|)(1+|b|)(2+|a|)(2+|b|)(3+|a|)(3+|b|)}{c(1+c)(2+c)(3+c)}\ {}_2F_1(4+|a|,4+|b|;4+c,1)\\ &+[1-\lambda(\alpha-9)]\frac{|a||b|(1+|a|)(1+|b|)(2+|a|)(2+|b|)}{c(1+c)(2+c)}\ {}_2F_1(3+|a|,3+|b|;3+c,1)\\ &+[6-\lambda(5\alpha-19)-\alpha]\frac{|a||b|(1+|a|)(1+|b|)}{c(1+c)}\ {}_2F_1(2+|a|,2+|b|;2+c,1)\\ &+(1-\alpha)\frac{|ab|}{c}\ {}_2F_1(1+|a|,1+|b|;1+c,1)\le 1-\alpha\end{aligned}$$

is satisfied, then $I_{a,b,c}(f)\in\mathbf{K}(\lambda,\alpha)$.

Theorem 2.2 [14] *Let $a,b\in\mathbb{C}\setminus\{0\}$ and let c be a real number. If $f\in\mathcal{CV}$ and the inequality*

$$\begin{aligned}(2.12)\quad &\lambda\frac{|a||b|(1+|a|)(1+|b|)(2+|a|)(2+|b|)}{c(1+c)(2+c)}\ {}_2F_1(3+|a|,3+|b|;3+c,4;1)\\ &+[1-\lambda(\alpha-5)]\frac{|a||b|(1+|a|)(1+|b|)}{c(1+c)}\ {}_2F_1(2+|a|,2+|b|;2+c,3;1)\\ &+[3-2\lambda(\alpha-2)-\alpha]\frac{|ab|}{c}\ {}_2F_1(1+|a|,1+|b|;1+c,2;1)\\ &+(1-\alpha)\ {}_2F_1(|a|,|b|;c,1;1)\le 2(1-\alpha)\end{aligned}$$

is satisfied, then $I_{a,b,c}(f)\in\mathbf{K}(\lambda,\alpha)$.

Theorem 2.3 *Let $a,b\in\mathbb{C}\setminus\{0\}$ and let c be a real number such that $c>|a|+|b|+1$. If $f\in\mathcal{R}^{\tau}(A,B)$ and if the inequality*

$$\begin{aligned}(2.13)\quad &\lambda\frac{|a||b|(1+|a|)(1+|b|)}{c(1+c)}F(2+|a|,2+|b|,2+c;1)\\ &+[1-\lambda(\alpha-2)]\frac{|ab|}{c}F(1+|a|,1+|b|,1+c;1)+(1-\alpha)F(|a|,|b|,c;1)\\ &\le(1-\alpha)\left(\frac{1}{(A-B)|\tau|}+1\right)\end{aligned}$$

is satisfied, then $I_{a,b,c}(f)\in\mathbf{K}(\lambda,\alpha)$.

Proof. Let f be of the form (1.1) belong to the class $\mathcal{R}^{\tau}(A,B)$. By virtue of Lemma 1.1, it suffices to show that

$$\sum_{n=2}^{\infty} n(n-\alpha)(1+n\lambda-\lambda)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \leq 1-\alpha. \tag{2.14}$$

Taking into account the inequality (1.9) and the relation $|(a)_{n-1}| \leq (|a|)_{n-1}$, we deduce that

$$\begin{aligned}
&\sum_{n=2}^{\infty} n(n-\alpha)(1+n\lambda-\lambda)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}a_n\right| \\
&\leq (A-B)|\tau|\left(\lambda\sum_{n=2}^{\infty}(n-1)(n-2)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\right|\right. \\
&\left.+[1-\lambda(\alpha-2)]\sum_{n=2}^{\infty}(n-1)\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\right| + (1-\alpha)\sum_{n=2}^{\infty}\left|\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\right|\right) \\
&\leq (A-B)|\tau|\left(\lambda\sum_{n=2}^{\infty}\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-3}} + [1-\lambda(\alpha-2)]\sum_{n=2}^{\infty}\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-2}}\right. \\
&\left.+(1-\alpha)\sum_{n=2}^{\infty}\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}}\right) \\
&= (A-B)|\tau|\left(\lambda\frac{(|a|_2)(|b|_2)}{(c)_2}\sum_{n=2}^{\infty}\frac{(2+|a|)_{n-3}(2+|b|)_{n-3}}{(2+c)_{n-3}(1)_{n-3}}\right. \\
&\left.+[1-\lambda(\alpha-2)]\frac{|ab|}{c}\sum_{n=2}^{\infty}\frac{(1+|a|)_{n-2}(1+|b|)_{n-2}}{(1+c)_{n-2}(1)_{n-2}} + (1-\alpha)\sum_{n=2}^{\infty}\frac{(|a|)_{n-1}(|b|)_{n-1}}{(c)_{n-1}(1)_{n-1}}\right) \\
&= (A-B)|\tau|\left(\lambda\frac{|a||b|(1+|a|)(1+|b|)}{c(1+c)}F(2+|a|,2+|b|,2+c;1)\right. \\
&\left.+[1-\lambda(\alpha-2)]\frac{|ab|}{c}F(1+|a|,1+|b|,1+c;1)+(1-\alpha)\left(F(|a|,|b|,c;1)-1\right)\right),
\end{aligned}$$

where we use the relation $(a)_n = a(a+1)_{n-1}$.

The proof now follows by an application of the Gauss summation theorem and (1.5). ∎

Next, we prove the following properties for the operator $I_{a,b;c}(f)$, when a function f belongs to the class $\mathbf{K}(\lambda,\alpha)$.

Theorem 2.4 *Let $a,b>0$, $c \geq \max\{0, a+b-1, (1/2)(ab+a+b-1)\}$ and let a function f of the form* (1.3) *be in $\mathbf{K}(\lambda,\alpha)$. Then*

$$|z| - \frac{(1-\alpha)}{2(2-\alpha)(1+\lambda)}\,\frac{ab}{c}\,|z|^2 \leq |I_{a,b;c}f(z)| \leq |z| + \frac{(1-\alpha)}{2(2-\alpha)(1+\lambda)}\,\frac{ab}{c}\,|z|^2 \tag{2.15}$$

and

$$1-\frac{(1-\alpha)}{(2-\alpha)(1+\lambda)}\,\frac{ab}{c}\,|z|\le|(I_{a,b;c}f(z))'|\le 1+\frac{(1-\alpha)}{(2-\alpha)(1+\lambda)}\,\frac{ab}{c}\,|z|. \tag{2.16}$$

The results are sharp.

Proof. We note that

$$I_{a,b;c}f(z)=\Big(zF(a,b;c;z)*f\Big)(z)=z-\sum_{n=2}^{\infty}\Phi(n)a_nz^n,$$

where

$$\Phi(n)=\frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}}\quad(a,b>0;\ n\ge 2)$$

and $0<\Phi(n+1)\le\Phi(n)$ $(n\ge 2)$ under the assumption for c. Since $f\in\mathbf{K}(\lambda,\alpha)$, by Lemma 1.2, we have

$$2(2-\alpha)(1+\lambda)\sum_{n=2}^{\infty}a_n\le\sum_{n=2}^{\infty}n(n-\alpha)(1+n\lambda-\lambda)a_n\le 1-\alpha. \tag{2.17}$$

Therefore, by using (2.17), we obtain

$$\begin{aligned}|I_{a,b;c}(f)|&\le|z|+\sum_{n=2}^{\infty}\Phi(n)a_n|z|^n\\&\le|z|+\Phi(2)|z|^2\sum_{n=2}^{\infty}a_n\\&\le|z|+\frac{(1-\alpha)}{2(2-\alpha)(1+\lambda)}\frac{ab}{c}|z|^2\end{aligned}$$

and

$$\begin{aligned}|I_{a,b;c}(f)|&\ge|z|-\sum_{n=2}^{\infty}\Phi(n)a_n|z|^n\\&\ge|z|-\Phi(2)|z|^2\sum_{n=2}^{\infty}a_n\\&\ge|z|-\frac{(1-\alpha)}{2(2-\alpha)(1+\lambda)}\frac{ab}{c}|z|^2.\end{aligned}$$

From (2.17), we note that

$$\sum_{n=2}^{\infty}na_n\le\frac{(1-\alpha)}{(2-\alpha)(1+\lambda)}. \tag{2.18}$$

By using (2.18), we obtain (2.16). The results are sharp for the function $f(z)=z-\frac{1-\alpha}{2(2-\alpha)(1+\lambda)}z^2$. ∎

Now, we find the order β $(0\le\beta<1)$ for which the operator $I_{a,b;c}(f)$ belongs to the classes $\mathcal{T}^*(\beta)$ and $\mathcal{C}(\beta)$ when a function f belongs to the class $\mathbf{K}(\lambda,\alpha)$.

Theorem 2.5 *Let* $a, b > 0$, $\max\{2ab/3, a+b-1, (1/2)(ab+a+b-1)\} \le c \le ab$ *and let a function* f *of the form* (1.3) *be in* $\mathbf{K}(\lambda, \alpha)$. *Then* $I_{a,b;c}(f) \in \mathcal{T}^*(\beta)$, *where*

$$\beta = 1 - \frac{\Phi(2)(1-\alpha)}{2(2-\alpha)(1+\lambda) - \Phi(2)(1-\alpha)}. \tag{2.19}$$

Proof. Let $f \in \mathbf{K}(\lambda, \alpha)$. Consider the operator

$$I_{a,b;c}f(z) = z + \sum_{n=2}^{\infty} \Phi(n) a_n z^n,$$

where

$$\Phi(n) = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}} \quad (a, b > 0;\ n \ge 2).$$

Since $\Phi(n)$ is a decreasing function for n, by Lemma 1.3, we need to find β $(0 \le \beta < 1)$ that

$$\Phi(2) \sum_{n=2}^{\infty} \frac{n-\beta}{1-\beta} a_n \le 1.$$

Since $f \in \mathbf{K}(\lambda, \alpha)$, by Lemma 1.2, we have

$$\sum_{n=2}^{\infty} n(n-\alpha)(1+n\lambda-\lambda) a_n \le 1-\alpha.$$

To complete the proof, it suffices to find β such that

$$\frac{n-\beta}{1-\beta}\Phi(2) \le \frac{n(n-\alpha)(1+n\lambda-\lambda)}{1-\alpha}. \tag{2.20}$$

From (2.20), we obtain

$$\beta \le \Psi(n),$$

where

$$\Psi(n) = \frac{n(n-\alpha)(1+n\lambda-\lambda) - n\Phi(2)(1-\alpha)}{n(n-\alpha)(1+n\lambda-\lambda) - \Phi(2)(1-\alpha)}. \tag{2.21}$$

By the assumption of the theorem, it is easy to see that $\Psi(n)$ is an increasing function for n $(n \ge 2)$. Setting $n = 2$ in (2.21), we have

$$\beta = \frac{2(2-\alpha)(1+\lambda) - 2\Phi(2)(1-\alpha)}{2(2-\alpha)(1+\lambda) - \Phi(2)(1-\alpha)},$$

hence we get(2.19). Therefore we complete the proof of Theorem 2.5. ∎

Theorem 2.6 *Let* $a, b > 0$, $\max\{2ab/3, a+b-1, (1/2)(ab+a+b-1)\} \le c \le ab$ *and let a function* f *of the form* (1.3) *be in* $\mathbf{K}(\lambda, \alpha)$. *Then* $I_{a,b;c}(f) \in \mathcal{C}^*(\beta)$, *where*

$$\beta = 1 - \frac{\Phi(2)(1-\alpha)}{(2-\alpha)(1+\lambda) - \Phi(2)(1-\alpha)}. \tag{2.22}$$

Proof. Let $f \in \mathbf{K}(\lambda, \alpha)$. Consider the operator

$$I_{a,b;c}f(z) = z + \sum_{n=2}^{\infty} \Phi(n) a_n z^n,$$

where

$$\Phi(n) = \frac{(a)_{n-1}(b)_{n-1}}{(c)_{n-1}(1)_{n-1}} \quad (a, b > 0;\ n \geq 2).$$

Since $\Phi(n)$ is a decreasing function for n, by Lemma1.3, we need to find β $(0 \leq \beta < 1)$ that

$$\Phi(2) \sum_{n=2}^{\infty} n \frac{n-\beta}{1-\beta} a_n \leq 1.$$

Since $f \in \mathbf{K}(\lambda, \alpha)$, by Lemma 1.2, we have

$$\sum_{n=2}^{\infty} n(n-\alpha)(1+n\lambda-\lambda)a_n \leq 1-\alpha.$$

To complete the proof, it suffices to find β such that

$$n\frac{n-\beta}{1-\beta}\Phi(2) \leq \frac{n(n-\alpha)(1+n\lambda-\lambda)}{1-\alpha}. \tag{2.23}$$

From (2.23), we obtain

$$\beta \leq \Upsilon(n),$$

where

$$\Upsilon(n) = \frac{(n-\alpha)(1+n\lambda-\lambda) - n\Phi(2)(1-\alpha)}{(n-\alpha)(1+n\lambda-\lambda) - \Phi(2)(1-\alpha)}. \tag{2.24}$$

By the assumption of the theorem, it is easy to see that $\Psi(n)$ is an increasing function for n $(n \geq 2)$. Setting $n = 2$ in (2.21), we have

$$\beta = \frac{(2-\alpha)(1+\lambda) - 2\Phi(2)(1-\alpha)}{(2-\alpha)(1+\lambda) - \Phi(2)(1-\alpha)},$$

hence we get (2.19). Therefore we complete the proof of Theorem 2.6. ∎

3. Concluding remarks

If $a = 1, b = 1 + \delta$, $c = 2 + \delta$ with $\mathrm{Re}(\delta) > -1$, then the convolution operator $I_{a,b,c}(f)$ turns into a Bernardi operator

$$B_f(z) = [I_{a,b,c}(f)](z) = \frac{1+\delta}{z^\delta} \int_0^1 t^{\delta-1} f(t) \mathrm{d}t.$$

Further, $I_{1,1,2}(f)$ and $I_{1,2,3}(f)$ are known as Alexander and Libera operators, respectively. Further, note that, when $|b| = 1$, we get $I_{a,1,c}(f) = \mathcal{L}(a,c)f(z) = \left(z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} z^n\right) * f(z) = z + \sum_{n=2}^{\infty} \frac{(a)_{n-1}}{(c)_{n-1}} a_n z^n$, the Carlson-Shaffer operator and also for $a = \delta + 1 (\delta > -1),\ b = 1,\ c = 1$ the Ruscheweyh derivative operator

$$\mathcal{D}^{\delta} f(z) = \frac{z}{(1-z)^{\delta+1}} * f(z) = z + \sum_{n=2}^{\infty} \begin{pmatrix} \delta + n - 1 \\ n-1 \end{pmatrix} a_n z^n,$$

hence one can deduce various interesting results for the function class defined by these operator as a corollary, we omit the details involved.

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