GELFAND THEOREM FOR BANACH HYPERALGEBRAS

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Abstract. In this paper, we prove that if A is a commutative Banach hyperalgebra then the maximal w-hyperideal space of A is in one-to-one correspondence with the set of (proper) maximal w-hyperideal in A. Also, we obtain some interesting results in direction.

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1. Introduction

The concept of hyperstructure was first introduced by Marty in [6] and has been studied in the following decades to nowadays by many mathematicians. A short review of the theory of hypergroups appears in [3]. The recent books [3], [4] and [12] contain a wealth of applications. There are applications to the following subjects: geometry, hypergraphs, binary relations, combinatorics, codes, cryptography, probability, groups, rational algebraic functions and etc.

Vougiouklis in the fourth A.H.A. congress (1990)[11], introduced the notion of H_v-structures. The concept of H_v-structures constitute a generalization of the well-known algebraic hyperstructures. The principal notions on the H_v-structures can be founded in [3−5, 12]. In this paper, we introduce the hyperalgebras and the concept of hyperideals of hyperalgebras and we obtain some interesting results.

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The paper is arranged as follow. In this paper, we introduce Banach hyperalgebras and the w-hyperideals. Moreover, we define the multiplicative w-linear functionals on Banach hyperalgebra A and we define the maximal w-hyperideals space of A and give some results in this direction. In addition to, we prove that if A is a commutative Banach hyperalgebra then the maximal w-hyperideal space of A is in one-to-one correspondence with the set of (proper) maximal w-hyperideal in A.

2. Preliminaries

Let A be a non-empty set, we denote by $P^*(A)$ the set of all non-empty subsets of A. Throughout this paper the symbol $\mathbb{K}$ will be used to denote a field that is either the real field $\mathbb{R}$ or the complex field $\mathbb{C}$.

Definition 2.1. [10] Let $\mathbb{K}$ be a field and $(A, +)$ be an abelian group. Then a quadruplet $(A, +, \cdot, \mathbb{K})$ is a hypervector space on the field $\mathbb{K}$ if the map $\cdot : \mathbb{K} \times A \longrightarrow P^*(A)$ satisfies the following conditions

(i) $\forall \lambda \in \mathbb{K}, a, b \in A; \lambda \cdot (a + b) \subseteq \lambda \cdot a + \lambda \cdot b$, (right distributivity);

(ii) $\forall \lambda, \mu \in \mathbb{K}, a \in A; (\lambda + \mu) \cdot a \subseteq \lambda \cdot a + \mu \cdot a$, (left distributivity);

(iii) $\forall \lambda, \mu \in \mathbb{K}, a \in A; \lambda \cdot (\mu \cdot a) = (\lambda\mu) \cdot a$, (associativity);

(iv) $\forall \lambda \in \mathbb{K}, a \in A; \lambda \cdot (-a) = (-\lambda) \cdot a = -(\lambda \cdot a)$;

(v) $\forall a \in A; a \in 1 \cdot a$.

Remark 2.2. In the right side of (i) and (ii) the sum is meant in the sense of Frobenius, that is,

$$\lambda \cdot a + \lambda \cdot b = \{m + n : m \in \lambda \cdot a, n \in \lambda \cdot b\}$$

$$\lambda \cdot a + \mu \cdot a = \{p + q : p \in \lambda \cdot a, q \in \mu \cdot a\}.$$

Moreover the left side of (iii) meant in the set-theoretical union of all the sets $\lambda \cdot b$, where b runs over the set $\mu \cdot a$.

Let A be a hypervector space and S be a nonempty subset of A. Then S is said to be a subhypervector space of A if itself is a hypervector space under hyperoperation " $\cdot$ " restricted to S.

Definition 2.3. [10] A semi-norm $\|\|$ on a hypervector space $(A, +, \cdot, \mathbb{K})$ is a map : $\|\| : A \longrightarrow \mathbb{R}$, of A into the positive real number such that:

(i) $\|0\| = 0$,

(ii) $\|a + b\| \leq \|a\| + \|b\| \quad \forall a, b \in A$,

(iii) $\sup \|\lambda \cdot a\| = |\lambda|\|a\| \quad \forall \lambda \in \mathbb{K}, a \in A$.

A semi-norm in A is called a norm if

$$\|a\| = 0 \quad \Longleftrightarrow a = 0.$$

A hypervector space together with a semi-norm or norm is called a semi-normed or normed hypervector space.

Definition 2.4. [8]) A hyperalgebra is a hypervector space A with a multiplication satisfying the following properties,

(i) $\forall a, b, c \in A; (ab)c = a(bc)$, (associativity);

(ii) $\forall a, b, c \in A;\ (a + b)c = ac + bc$, (left distributivity);

(iii) $\forall a, b, c \in A;\ a(b + c) = ab + ac$, (right distributivity);

(iv) $\forall a, b \in A; (-a)b = a(-b) = -(ab)$,

(v) $\forall a, b \in A$, and $\forall \lambda \in \mathbb{K};\ \lambda \cdot (ab) = (\lambda \cdot a)b = a(\lambda \cdot b)$.

The field $\mathbb{K}$ is called the scalar field of A. If $\mathbb{K} = \mathbb{R}$, A is called a real algebra, and if $\mathbb{K} = \mathbb{C}$, a complex algebra.

Example 2.5. *Let $\mathbb{C}$ be the set of all complex numbers. Then $\mathbb{C}$ is a hyperalgebra on $\mathbb{R}$, with respect usual sum and multiplication, and the following hyperoperation:*

$$\cdot : \mathbb{C} \times \mathbb{C} \longrightarrow P^*(\mathbb{C})$$

$$t \cdot z = \{re^{i\theta} : 0 \leq r \leq |t||z|, 0 \leq \theta \leq 2\pi\}, \qquad \forall t \in \mathbb{R}, z \in \mathbb{C}.$$

Remark 2.6. Let A be a hyperalgebra. An element $e \in A$ is called an identity or unit if for every $a \in A, a = ae = ea$. In this case we say that A is an unital hyperalgebra. And an element a is said to be invertible if it has an inverse in A, that is, there exists an element $a^{-1} \in A$ such that

$$aa^{-1} = a^{-1}a = e,$$

where e is the unit element of A. In this case we say that a is invertible.

It is clear that no $a \in A$ has more than one inverse. For a hyperalgebra A we let $Inv(A)$ denote the set of all invertible elements in A. The compliment of $Inv(A)$ in A is called the set of all non-invertible elements in A, and denote by $Sing(A)$. (See [9]).

Definition 2.7. [8] A semi-norm $\|\|$ on a hyperalgebra A is called a hyperalgebra semi-norm if it satisfies in definition 2.3 and the following condition:

$$\|ab\| \leq \|a\|\|b\| \qquad \forall a, b \in A.$$

A semi-norm in A is called a norm if

$$\|a\| = 0 \quad \Longleftrightarrow a = 0.$$

A hypealgebra together with a hyperalgebra semi-norm or hyperalgebra norm is called a semi-normed or normed hyperalgebra, moreover if $e \in A$ be a unit element of A, we assume that $\|e\| = 1$. A normed hypervector space or normed hyperalgebra which is complete in its norm is called a Banach hypervector space or Banach hyperalgebra, respectively.

Lemma 2.8. [8] *Let A be a hyperalgebra and $a \in A, \lambda \in \mathbb{K}$. Then there exists $z \in \lambda \cdot a$ such that $\|z\| = sup\|\lambda \cdot a\|$.*

Remark 2.9. Throughout this paper, we denote z be obtained in the Lemma 3.11 by $z_{\lambda \cdot a}$, also if $a = e$ we denote by z_λ.

Let A be a hypervector space and S be a nonempty subset of A. Then S is said to be a weak subhypervector space of A if for every $a, b \in S$ and $\lambda \in \mathbb{C}$, $a + b \in S, z_{\lambda \cdot a} \in S$. (See [8])

Definition 2.10. [8] Suppose A is a hyperalgebra and a is in A. the spectrum of a, denoted by $\sigma_A(a)$ or simply by $\sigma(a)$, is the set of all complex numbers λ such that $z_\lambda - a \in Sing(A)$, that is,

$$\sigma(a) = \{\lambda \in \mathbb{C} : z_\lambda - a \in Sing(A)\}.$$

The compliment of $\sigma(a)$ in $\mathbb{C}$ is called the resolvent set of a, and denote by $\rho(a)$.

Definition 2.11. [8] Let A be a hyperalgebra. A mapping $\varphi : A \longrightarrow \mathbb{C}$ is said to be weak linear functional (or simply, w-linear functional) if

(i) $\varphi(a + b) = \varphi(a) + \varphi(b),\ \forall a, b \in A$,

(ii) $\lambda\varphi(a) = \varphi(z_{\lambda \cdot a}),\ \forall a \in A, \lambda \in \mathbb{K}$.

Definition 2.12. [8] A w-linear functional φ on a hyperalgebra A is multiplicative if φ is nontrivial and $\varphi(ab) = \varphi(a)\varphi(b)$ for all a and b in A. We let M_A^w denote the set of all multiplicative w-linear functional on A. M_A^w is called the maximal w-hyperideal space of A for reasons that will become clear a little later.

Lemma 2.13. [8] *Let φ be a multiplicative w-linear functional on hyperalgebra A. Then $\varphi(e) = 1$.*

Definition 2.14. [8] Let A be a normed hyperalgebra, we say A is normal hyperalgebr if

(i) $\forall \lambda \in \mathbb{K}, a, b \in A; z_{\lambda \cdot (a+b)} = z_{\lambda \cdot a} + z_{\lambda \cdot b}$.

(ii) $\forall \lambda, \mu \in \mathbb{K}, a \in A; z_{(\lambda+\mu) \cdot a} = z_{\lambda \cdot a} + z_{\mu \cdot a}$.

3. Main results

Definition 2.15. Let A be a hyperalgebra over field $\mathbb{K}$. A nonempty set I of A is called

(i) a left (res. right) weak hyperideal (or simply, w-hyperideal) of hyperalgebra A if and only if

   (a) $(I, +, \cdot, \mathbb{K})$ is a weak subhypervector space of $(A, +, \cdot, \mathbb{K})$; and

   (b) $ab \in I$ (res. $ba \in I$), for all $a \in A$ and for all $b \in I$.

(ii) a w-hyperideal of hyperalgebra A if it is both left and right w-hyperideal of A. The hyperideal I of A denoted by $I \lhd_h^w A$.

If $I \neq A$, I is a proper w-hyperideal. Maximal w-hyperideals are proper w-hyperideals which are not contained in any larger proper w-hyperideals.

If I is a left hyperideal in A, $a \in I$, and a is let invertible, then $I = A$.

In fact, if $ba = e$, then $e \in I$ since I is a left w-hyperideal. Thus for $x \in A$, $x = xe \in I$. This forms a link between ideals and invertibility.

Lemma 2.16. *If A is a Banach hyperalgebra with identity,*

$$\begin{aligned} G_l(A) &= \{a \in A : a \textit{ is left invertible}\}, \\ G_r(A) &= \{a \in A : a \textit{ is right invertible}\}, \textit{ and} \\ G(A) &= \{a \in A : a \textit{ is invertible}\}, \end{aligned}$$

then G_l, G_l and $G(A)$ are open subsets of A.

Proof. The proof of the preceding theorem is similar case that A is a Banach algebra (see [2]). ∎

Lemma 2.17. *If A is a Banach hyperalgebra with identity, then*

(a) *the closure of a proper left, right, or two-side w-hyperideal is a proper left, right, or two-side w-hyperideal;*

(b) *a maximal left, right, or two-side w-hyperideal is closed.*

Proof. (a) Let I be a proper left w-hyperideal and let G_l be the set of left invertible elements in A. It follows that $I \cap G_l = \emptyset$. Thus $I \subseteq A \backslash G_l$. By Theorem 3.2, $A \backslash G_l$ is closed. Hence $\overline{I} \subseteq A \backslash G_l$ and thus $\overline{I} \neq A$. To see that $\overline{I}$ is a w-hyperideal, let $a \in \overline{I}, b \in A$. Therefore there is a sequence $\{a_n\}$ in I such that $\lim\limits_{n \to \propto} a_n = a$, it follows that $\lim\limits_{n \to \propto} ba_n = ba$, so $ba \in \overline{I}$ since I is a w-hyperideal. For every $\lambda \in K$ we have

$$\|z_{\lambda \cdot a_n} - z_{\lambda \cdot a}\| = \|z_{\lambda \cdot (a_n - a)}\| = |\lambda| \|a_n - a\|.$$

Hence $\lim\limits_{n \to \propto} z_{\lambda \cdot a_n} = z_{\lambda \cdot a}$ and since $z_{\lambda \cdot a_n} \in I$ for $n \in N$ we obtain $z_{\lambda \cdot a} \in \overline{I}$.

(b) If M is a maximal left w-hyperideal, $\overline{M}$ is a proper left w-hyperideal by (a). Hence $M = \overline{M}$ by maximality. ∎

Lemma 2.18. [8] *If A is a hyperalgebra then every proper w-hyperideals of A is contained in a maximal w-hyperideals of A.*

Theorem 2.19. [9] *Let A be a hyperalgebra and I be a w-hyperideals in A. Then $(A/I, +, \odot, \mathbb{K})$ is a hypervector space, with respect to the following hyperoperation:*

$$\odot : \mathbb{K}K \times A/I \longrightarrow P^*(A/I)$$

$$\forall \lambda \in \mathbb{K}, \forall [a] \in A/I; \lambda \odot [a] = \{[t] : t \in \lambda \cdot a\}.$$

On the other hand,

$$\forall \lambda \in \mathbb{K},\ \forall (a + I) \in A/I;\ \lambda \odot (a + I) = \lambda \cdot a + I.$$

Lemma 2.20. *If A is a normed hyperalgebra and I is a closed w-hyperideal in A, then the quotient hypervector space A/I is a normed hypervector space with the following (quotient) norm.*

$$\|[a]\| = \inf\{\|a - b\| : b \in I\}, \quad [a] \in A/I.$$

Proof. It is clear that $\|[\underline{0}]\| = 0$ and $\|[a] + [b]\| \leq \|[a]\| + \|[b]\|$ for every $a, b \in A$. Since for every $a \in A, \lambda \in \mathbb{K}$ we have $\lambda \cdot a \subseteq \lambda \cdot a + I$, we obtain

$$|\lambda|\|[a]\| \leq |\lambda|\|a\| = \sup \|\lambda \cdot a\| \leq \sup \|\lambda \cdot a + I\| = sup\|\lambda \odot [a]\|.$$

Hence

$$|\lambda|\|[a]\| \leq \sup \|\lambda \odot [a]\|. \tag{1}$$

On the other hand, for every $t \in \lambda \cdot a$ we have

$$\|[t]\| \leq \|t\| \leq |\lambda|\|a\| = \sup \|\|\lambda \cdot a\|$$

so

$$\sup \|\lambda \odot [a]\| = \sup\{\|[t]\| : t \in \lambda \cdot a \leq \sup \|\|\lambda \cdot a\|\}. \tag{2}$$

By (1) and (2)

$$\sup \|\lambda \odot [a]\| = |\lambda|\|[a]\|.$$ ∎

Theorem 2.21. *If A is a Banach hyperalgebra and I be a closed w-hyperideal in A. Then A/I is a Banach hyperalgebra. If A has an identity, so does A/I.*

The proof of the preceding theorem is similar to the case where A is a Banach algebra (see [2]).

Definition 2.22. An unital hyperalgebra in which each non-zero element is invertible is a divison hyperalgebra.

Lemma 2.23. *Let A be a division normed hyperalgebra. Then $\lambda \in \sigma(a)$ if and only if $a \in \lambda \cdot e$ and $\|a\| = |\lambda|$.*

Proof. If $\lambda \in \sigma(a)$ then $(z_\lambda - a) \in Sing(A)$, it follows that $z_\lambda - a = \underline{0}$, so that $z_\lambda = a$, it follow that z_λ is unique and $a \in \lambda \cdot e$. Hence $\lambda \in \sigma(a)$ if and only if $a \in \lambda \cdot e$. ∎

Lemma 2.24. *Let A be a division normed normal hyperalgebra. Then $\sigma(a)$ has only one point.*

Proof. Note that $\sigma(a)$ is non-empty (see [8]). If $\{\lambda, \mu\} \subseteq \sigma(a)$, then

$$\|a\| = |\lambda| = |\mu|.$$

On the other hand,

$$z_{\frac{\lambda+\mu}{2}} = z_{\frac{\lambda}{2}} + z_{\frac{\mu}{2}}.$$

It follows that

$$\left|\frac{\lambda+\mu}{2}\right| \leq |\lambda| = |\mu| = \|a\|.$$

$$\begin{aligned}(\lambda+\mu)(\bar{\lambda}+\bar{\mu}) \leq 4\lambda\bar{\lambda} &\Rightarrow \lambda\bar{\mu} + \mu\bar{\lambda} \leq \lambda\bar{\lambda} + \mu\bar{\mu} \\ &\Rightarrow \lambda(\bar{\mu} - \bar{\lambda}) - \mu(\bar{\mu} - \bar{\lambda}) \leq 0 \\ &\Rightarrow (\bar{\mu} - \bar{\lambda})(\lambda - \mu) \leq 0 \\ &\Rightarrow |\lambda - \mu|^2 \leq 0 \\ &\Rightarrow |\lambda - \mu| = 0 \\ &\Rightarrow \lambda = \mu.\end{aligned}$$

This implies that $\sigma(a)$ contains only one point, which we call λ_a. ∎

Theorem 2.25. *If A is a normed normal hyperalgebra in which every non-zero element is invertible, Then A is isometrically isomorphic to $\mathbb{C}$.*

Proof. Let $a \in A$. By Lemma 3.10, $\sigma(a)$ is non-empty and contains only one point, which we call λ_a. Define $\phi : A \longrightarrow \mathbb{C}$ by $\phi(a) = \lambda_a$. If $a, b \in A$, then there exists $\lambda_a, \lambda_b \in \mathbb{C}$ such that $a \in \lambda_a \cdot e, a = z_{\lambda_a}$ and $b \in \lambda_b \cdot e, b = z_{\lambda_b}$ (see Lemma 3.10). Since $z_{\lambda_a+\lambda_b} = z_{\lambda_a} + z_{\lambda_b}$, it follows $\lambda_a + \lambda_b \in \sigma(a+b)$, that is, $\varphi(a+b) = \lambda_a + \lambda_b = \varphi(a) + \varphi(b)$.

For every $\lambda \in K$, we have $\lambda \cdot a \subseteq (\lambda\lambda_a) \cdot e$ and obviously $z_{\lambda \cdot a} = z_{\lambda\lambda_a}$, hence $\varphi(z_{\lambda \cdot a}) = \varphi(z_{\lambda\lambda_a}) = \lambda\lambda_a = \lambda\varphi(a)$.

Since $\sigma(ab)$ is non- empty then there exist $z_{\lambda_{ab}} \in \lambda_{ab} \cdot e$ such that $z_{\lambda_{ab}} = ab$, $\|ab\| = |\lambda_{ab}|$. It follows that $e \in \lambda_{ab}^{-1} \cdot (ab)$.

On the other hand, $z_{\lambda_a} z_{\lambda_b} = ab \in (\lambda_a\lambda_b) \cdot e$ and $|\lambda_a||\lambda_b| = \|ab\|$. Hence $\lambda_a\lambda_b \in \sigma(ab)$. Since in division Banach hyperalgebra z_λ is unique, therefore, $z_{\lambda_{ab}} = z_{\lambda_a} z_{\lambda_b}$. It follows that $\varphi(ab) = \varphi(a)\varphi(b)$. Obviously ϕ is one-to-one and onto. Thus ϕ is an isometrical isomorphism. ∎

Theorem 2.26. *If A is a commutative normal Banach hyperalgebra. Then M_A^w is in one-to-one correspondence with the set of (proper) maximal w-hyperideals in A.*

Proof. First assume that φ is a multiplicative w-linear functional on A. We show that $ker\varphi$ is a proper maximal w-hyperideal in A. In fact, it is clear that $ker\varphi$ is a proper w-hyperideal because if $a \in ker\varphi$ and $\lambda \in \mathbb{C}$ then $\varphi(z_{\lambda\cdot a}) = \lambda\varphi(a) = 0$. To see that it is maximal, let $a \in A \ker\varphi$. We have

$$e \in (e - z_{(\varphi(a))^{-1}\cdot a}) + z_{(\varphi(a))^{-1}\cdot a}.$$

The set in the parentheses contain in $ker\varphi$. We see that the linear span of a and $ker\varphi$ contains e, and hence any w-hyperideal containing both $ker\varphi$ and a must be the whole hyperalgebra. Thus $ker\varphi$ is maximal.

Next, assume that M be a maximal w-hyperideal of A. Then A/M is a hyperalgebra. Choose $x \in A, x \notin M$, and put

$$J = \{ax + y : a \in A, y \in M\}.$$

It is clear that J is a w-hyperideal in A which is larger than M, since $x \in J$ (take a=e, y=0).

Thus $J = A$. Then $e \in J$, and $e = ax + y$ for some $a \in A$ and $y \in M$.

If $\pi : A \to A/M$ is the quotient map. It follows $\pi(e) = \pi(a)\pi(x)$. Therefore every nonzero element $\pi(x)$ of the Banach hyperalgebra A/M is invertible in A/M. By the Theorem 3.11, there is an isomorphism ϕ of A/M onto $\mathcal{C}$. Put $\varphi = \phi \circ \pi$. Then $\varphi \in M_A$, and M is the null space of φ.

Finally, to see that the above correspondence between multiplicative w-linear functionals and maximal w-hyperideals is one-to-one, let φ_1 and φ_2 be multiplicative w-linear functionals on A with a common kernel M. We must show that $\varphi_1 = \varphi_2$. For any a in A, we can write

$$z_{\varphi_1(a)-\varphi_2(a)} = z_{\varphi_1(a)} - z_{\varphi_2(a)} = (a - z_{\varphi_2(a)}) - (a - z_{\varphi_1(a)})$$

The first term on the right is in $ker\varphi_2 = M$ and the second term is in $ker\varphi_1 = M$. Thus $z_{\varphi_1(a)-\varphi_2(a)} \in M$ and we must have $\varphi_1(a) = \varphi_2(a)$, completing the proof of the theorem. ∎

Theorem 2.27. *Let A be a commutative normal Banach hyperalgebra. Then $a \in A$ is invertible if and only if $\varphi(a) \neq 0$ for every $\varphi \in M_A^w$.*

Proof. Let $a \in A$ be invertible, then for every $\varphi \in M_A^w$ we have

$$1 = \varphi(e) = \varphi(a)\varphi(a^{-1})$$

so $\varphi(a)\varphi(a^{-1}) \neq 0$ and $\varphi(a) \neq 0$. Conversely, if a is not invertible, then the set $J = \{xa \mid x \in A\}$ dose not contain e, hence is a proper w-hyperideal in A, since

$$(\lambda \cdot x)a = \{ta : t \in \lambda \cdot x\} \quad \forall x \in A.$$

Thus $J \subseteq M$ for some maximal w-hyperideal M. By theorem 3.12, $M = ker\varphi$ for some $\varphi \in M_A^w$, then $\varphi(a) = 0$. ∎

Theorem 2.28. *Let A be a commutative normal Banach hyperalgebra and $a \in A$. Then*

$$\sigma(a) = \{\varphi(a) : \varphi \in M_A^w\}.$$

Proof. If $\varphi \in M_A^w$, then $(z_{\varphi(a)} - a) \in Sing(A)$ because, if $(z_{\varphi(a)} - a) \in Inv(A)$, then there exists $b \in A$ such that $b(z_{\varphi(a)} - a) = (z_{\varphi(a)} - a)b = e$ which implies $0 = \varphi(b)0 = 0\varphi(z) = \varphi(e) = 1$, this is a contradiction.

Conversely, if $(z_\lambda - a) \in Sing(A)$, then $(z_\lambda - a)A$ is a w-hyperideal which is, by Lemma 3.4, contained in some maximal w-hyperideal M. By Theorem 3.12, there exists $\varphi \in M_A^w$ such that $M = ker\varphi$ so $0 = \varphi(z_\lambda - a)e = \varphi(z_\lambda - a) = \varphi(z_\lambda) - \varphi(a)$, so we obtain $\varphi(z_\lambda) = \lambda$ that is $\varphi(a) = \lambda$. ∎

Remark 2.29. In particular, Theorem 3.14 implies that the set all multiplicative linear functionals of A is not empty.

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