ON WEAKLY πg-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract. In this paper, the concepts of weakly πg-continuous functions, weakly πg-compact spaces and weakly πg-connected spaces are introduced and some of their properties are investigated.

Keywords: Topological space, πg-closed set, αg-closed set, $g\alpha$-closed set.
2010 Mathematics Subject Classification: 54C10, 54C08, 54C05.

1. Introduction

Recently many authors have studied various classes of generalized closed sets in general topology. Dontchev and Noiri [9] have introduced the concept of πg-closed sets and studied their most fundamental properties in topological spaces. Also, recently, Ekici and Noiri [10] have introduced a generalization of πg-closed sets and πg-open sets.

In this paper, we study a new class of generalized closed sets in topological spaces. We introduce the notions of weakly πg-closed sets and weakly πg-open sets, which are weaker forms of πg-closed sets and πg-open sets, respectively. Also, the relationships among related generalized closed sets are investigated.

2. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1 A subset A of a space (X, τ) is called

1. semi-open set [18] if A $\subseteq$ cl(int(A));
2. preopen set [21] if A $\subseteq$ int(cl(A));
3. α-open set [23] if A $\subseteq$ int(cl(int(A)));
4. β-open set [1] (= semi-preopen [2]) if A $\subseteq$ cl(int(cl(A)));
5. b-open set [3] if A $\subseteq$ cl(int(A)) $\cup$ int(cl(A));
6. regular open set [32] if A = int(cl(A));
7. π-open set [38] if A is the finite union of regular open sets.

The complements of the above mentioned open sets are called their respective closed sets.

The semi-closure [7] (resp. α-closure [23], preclosure [25]) of a subset A of X, scl(A) (resp. αcl(A), pcl(A)), is defined to be the intersection of all semi-closed (resp. α-closed, preclosed) sets of (X, τ) containing A. It is known that scl(A) (resp. αcl(A), pcl(A)) is a semi-closed (resp. an α-closed, preclosed) set.

Definition 2.2 A subset A of a space (X, τ) is called

1. a generalized closed (briefly g-closed) set [17] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

   The complement of g-closed set is called g-open set;

2. a semi-generalized closed (briefly sg-closed) set [5] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

   The complement of sg-closed set is called sg-open set;

3. an α-generalized closed (briefly αg-closed) set [19] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

   The complement of αg-closed set is called αg-open set;

4. a generalized α-closed (briefly $g\alpha$-closed) set [20] if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is α-open in (X, τ).

   The complement of $g\alpha$-closed set is called $g\alpha$-open set;

5. a $\hat{g}$-closed set [35] (= ω-closed [31]) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

   The complement of $\hat{g}$-closed set is called $\hat{g}$-open set;

6. a $^{\star}g$-closed set [36] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{g}$-open in (X, τ).

   The complement of $^{\star}g$-closed set is called $^{\star}g$-open set;

7. a $^{\#}g$-semi-closed (briefly $^{\#}g$s-closed) set [37] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $^{\star}g$-open in (X, τ).

   The complement of $^{\#}g$s-closed set is called $^{\#}g$s-open set;

8. a $\tilde{g}$-closed set [16] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $^{\#}g$s-open in (X, τ).

   The complement of $\tilde{g}$-closed set is called $\tilde{g}$-open set;

9. a $\tilde{g}$-semi-closed (briefly $\tilde{g}s$-closed) set [34] if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $^{\#}g$s-open in (X, τ).

   The complement of $\tilde{g}s$-closed set is called $\tilde{g}s$-open set;

10. a πg-closed set [9] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is π-open in (X, τ).

    The complement of πg-closed set is called πg-open set.

Definition 2.3 A subset A of a topological space (X, τ) is called

1. a weakly g-closed (briefly wg-closed) set [33] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is open in (X, τ).

2. a weakly $\tilde{g}$-closed (briefly w$\tilde{g}$-closed) set [29] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is $^{\#}g$s-open in (X, τ).

3. a weakly ω-closed set [30] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in (X, τ).

4. a regular weakly generalized closed (briefly rwg-closed) set [22] if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular open in (X, τ).

Definition 2.4 Let X and Y be topological spaces. A mapping f : X $\rightarrow$ Y is called

1. completely continuous [4] (resp. R-map[6]) if f^{-1}(V) is regular open in X for each open (resp. regular open) set in Y.

2. perfectly continuous [24] if f^{-1}(V) is both open and closed in X for each open set V in Y.

3. $\hat{g}$-irresolute [35] if f^{-1}(V) is $\hat{g}$-open in X for each $\hat{g}$-open subset V in Y.

4. πg-continuous [9] if f^{-1}(V) is πg-closed in X for every closed subset V of Y.

5. π-irresolute [12] if f^{-1}(V) is π-closed in X for every π-closed subset V of Y.
6. irresolute [8] if f^{-1}(V) is semi-open in X for every semi-open subset V in Y.

Definition 2.5 [15] A space (X, τ) is called πg-$T_{1/2}$ if every πg-closed set is closed.

Definition 2.6 [12] A mapping f :X $\rightarrow$ Y is called contra πg-continuous if f^{-1}(V) is πg-closed in X for every open set V of Y.

Definition 2.7 [12] A topological space (X, τ) is said to be locally πg-indiscrete if every πg-open set of X is closed in X.

Definition 2.8 [26] A subset A of (X, τ) is said to be πgp-closed if pcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is π-open in X.

Definition 2.9 [26] A space (X, τ) is called πgp-$T_{1/2}$ if every πgp-closed set is preclosed.

Definition 2.10 [14] A mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ is said to be almost πgp-continuous if f^{-1}(int(cl(V))) is a πgp-open set in X for every V $\in \sigma$.

Definition 2.11 [13] A mapping f : X $\rightarrow$ Y is called (πg, s)-continuous if the inverse image of each regular open set of Y is πg-closed in X.

Definition 2.12 [27] A mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ is said to be w$\tilde{g}$-continuous if the inverse image of every open set in (Y, σ) is w$\tilde{g}$-open in X.

Definition 2.13 [28] A mapping f : $(X, \tau) \rightarrow (Y, \sigma)$ is said to be $\tilde{g}$-continuous if the inverse image of every open set in (Y, σ) is $\tilde{g}$-open in X.

Definition 2.14 [15] A topological space X is called πg-compact if every cover of X by πg-open sets has finite subcover.

Definition 2.15 [11] A space X is said to be almost connected if X cannot be written as a disjoint union of two non-empty regular open sets.

Definition 2.16 [12, 13] A space X is called πg-connected if X is not the union of two disjoint nonempty πg-open sets.

Definition 2.17 [12] A mapping f : X $\rightarrow$ Y is called πg-open if image of each πg-open set is πg-open.

Definition 2.18 [15] A mapping f : X $\rightarrow$ Y is said to be πg-irresolute if f^{-1}(V) is πg-closed in (X, τ) for every πg-closed set V of (Y, σ).

Lemma 2.19 [26] *Let Y be open in X. Then*

(1) *If A is π-open in Y, then there exists a π-open set B in X such that A=B$\cap$Y.*

(2) *If A is π-open in X, then A $\cap$ Y is π-open in Y.*

3. Weakly πg-closed sets

We introduce the definition of weakly πg-closed sets in a topological space and study the relationships of such sets.

Definition 3.1 A subset A of a topological space (X, τ) is called a weakly πg-closed (briefly wπg-closed) set if cl(int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is π-open in (X, τ).

Proposition 3.2 *Every πg-closed set is wπg-closed. But the converse of this implication is not true in general.*

Example 3.3 Let X = {a, b, c, d} and τ = {ϕ, {a}, {b, c}, {a, b, c}, X}. Then the set {c} is wπg-closed but not πg-closed in (X, τ).

Theorem 3.4 *Every wπg-closed set is rwg-closed but not conversely.*

Proof. Let A be any wπg-closed set and let U be regular open set containing A. Then U is a π-open set containing A. We have cl(int(A)) $\subseteq$ U. Thus A is rwg-closed.

Example 3.5 Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Then the set {a, b} is rwg-closed but not a wπg-closed.

Theorem 3.6 *Every wg-closed set is wπg-closed but not conversely.*

Proof. Let A be any wg-closed set and let U be π-open set containing A. Then U is an open set containing A. We have cl(int(A)) $\subseteq$ U. Thus A is wπg-closed.

Example 3.7 Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Then the set {a, c} is wπg-closed but not a wg-closed.

Theorem 3.8 *If a subset A of a topological space (X, τ) is both closed and αg-closed, then it is wπg-closed in (X, τ).*

Proof. Let A be an αg-closed set in (X, τ) and U be an π-open set containing A. Then U is open containing A and so U $\supseteq$ αcl(A) = A $\cup$ cl(int(cl(A))). Since A is closed, U $\supseteq$ cl(int(A)) and hence A is wπg-closed in (X, τ).

Theorem 3.9 *If a subset A of a topological space (X, τ) is both π-open and wπg-closed, then it is closed.*

Proof. Since A is both π-open and wπg-closed, A $\supseteq$ cl(int(A)) = cl(A) and hence A is closed in (X, τ).

Corollary 3.10 *If a subset A of a topological space (X, τ) is both π-open and wπg-closed, then it is both regular open and regular closed in (X, τ).*

Theorem 3.11 *Let (X, τ) be a πg-$T_{1/2}$ space and A $\subseteq$ X be π-open. Then, A is wπg-closed if and only if A is πg-closed.*

Proof. Let A be πg-closed. By Proposition 3.2, it is wπg-closed. Conversely, let A be wπg-closed. Since A is π-open, by Theorem 3.9, A is closed. Since X is πg-$T_{1/2}$, A is πg-closed.

Theorem 3.12 *A set A is wπg-closed if and only if cl(int(A)) $-$ A contains no non-empty π- closed set.*

Proof. *Necessity.* Let F be a π-closed set such that F $\subseteq$ cl(int(A)) $-$ A. Since F^c is π- open and A $\subseteq F^c$, from the definition of wπg-closed set it follows that cl(int(A)) $\subseteq F^c$. ie. F $\subseteq$ (cl(int(A)))^c. This implies that F $\subseteq$ (cl(int(A))) $\cap$ (cl(int(A)))$^c = \phi$.

Sufficiency. Let A$\subseteq$ G, where G is π-open set in X. If cl(int(A)) is not contained in G, then cl(int(A)) $\cap$ G^c is a non-empty π-closed subset of cl(int(A)) $-$ A, we obtain a contradiction. This proves the sufficiency and hence the theorem.

Corollary 3.13 *A wπg-closed set A is regular closed if and only if cl(int(A))$-$A is π-closed and cl(int(A)) $\supseteq$ A.*

Proof. *Necessity.* Since the set A is regular closed, cl(int(A)) $-$ A $= \phi$ is regular closed and hence π-closed.

Sufficiency. By Theorem 3.12, cl(int(A)) $-$ A contains no non-empty π-closed set. That is cl(int(A)) $-$ A $= \phi$. Therefore A is regular closed.

Theorem 3.14 *Let (X, τ) be a topological space and $B \subseteq A \subseteq X$. If B is w$\pi g$-closed set relative to A and A is both open and wπg-closed subset of X then B is wπg-closed set relative to X.*

Proof. Let B $\subseteq$ U and U be a π-open in (X, τ). Then B $\subseteq$ A $\cap$ U. Since B is wπg-closed relative to A, $cl_A(int_A$(B)) $\subseteq$ A $\cap$ U. That is A $\cap$ cl(int(B)) $\subseteq$ A $\cap$ U. We have A $\cap$ cl(int(B)) $\subseteq$ U and then [A $\cap$ cl(int(B))] $\cup$ (cl(int(B)))^c $\subseteq$ U $\cup$ (cl(int(B)))^c. Since A is wπg-closed in (X, τ), we have cl(int(A)) $\subseteq$ U $\cup$ (cl(int(B)))^c. Therefore cl(int(B)) $\subseteq$ U since cl(int(B)) is not contained in (cl(int(B)))^c. Thus B is wπg-closed set relative to (X, τ).

Corollary 3.15 *If A is both open and wπg-closed and F is closed in a topological space (X, τ), then A $\cap$ F is wπg-closed in (X, τ).*

Proof. Since F is closed, we have A $\cap$ F is closed in A. Therefore cl_A(A $\cap$ F) $=$ A $\cap$ F in A. Let A $\cap$ F $\subseteq$ G, where G is π-open in A. Then $cl_A(int_A$(A $\cap$ F)) $\subseteq$ G and hence A $\cap$ F is wπg-closed in A. By Theorem 3.14, A$\cap$ F is wπg-closed in (X, τ).

Theorem 3.16 *If A is wπg-closed and A $\subseteq$ B $\subseteq$ cl(int(A)), then B is wπg-closed.*

Proof. Since A$\subseteq$B, cl(int(B))$-$B$\subseteq$cl(int(A))$-$A. By Theorem 3.12, cl(int(A))$-$A contains no non-empty π-closed set and so cl(int(B)) $-$ B. Again by Theorem 3.12, B is wπg-closed.

Theorem 3.17 *Let (X, τ) be a topological space and $A \subseteq Y \subseteq X$ and Y be open. If A is wπg-closed in X, then A is wπg-closed relative to Y.*

Proof. Let A $\subseteq$ Y $\cap$ G where G is π-open in (X, τ). Since A is wπg-closed in (X, τ), A $\subseteq$ G implies cl(int(A)) $\subseteq$ G. That is Y $\cap$ (cl(int(A))) $\subseteq$ Y $\cap$ G where Y $\cap$ cl(int(A)) is closure of interior of A in (Y, σ). Thus A is wπg-closed relative to (Y, σ).

Theorem 3.18 *If a subset A of a topological space (X, τ) is nowhere dense, then it is wπg-closed.*

Proof. Since int(A) $\subseteq$ int(cl(A)) and A is nowhere dense, int(A) = ϕ. Therefore cl(int(A)) = ϕ and hence A is wπg-closed in (X, τ).

The converse of Theorem 3.18 need not be true as seen in the following Example.

Example 3.19 Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X}. Then the set {a} is wπg-closed in (X, τ) but not nowhere dense in (X, τ).

Remark 3.20 If any subsets A and B of topological space X are wπg-closed, then their intersection need not be wπg-closed.

Example 3.21 Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. In this topological space the subsets {a, c} and {a, d} are wπg-closed but their intersection {a} is not wπg-closed in (X, τ).

Proposition 3.22 *Every $g\alpha$-closed set is wπg-closed but not conversely.*

Proof. Let A be any $g\alpha$-closed subset of (X, τ) and let U be an π-open set containing A. Then U is α-open set containing A. Now G $\supseteq$ αcl(A) $\supseteq$ cl(int(cl(A))) $\supseteq$ cl(int(A)). Thus A is wπg-closed in (X, τ).

The converse of Proposition 3.22 need not be true as seen in the following Example.

Example 3.23 Let X = {a, b, c, d} and τ = {ϕ, {a}, {a, b}, X}. Then the set {a, b, c} is wπg-closed but not $g\alpha$-closed in (X, τ).

Remark 3.24 wπg-closedness is independent of semi-closedness, β-closedness, b-closedness, sg-closedness and $\tilde{g}s$-closedness in (X, τ).

Example 3.25 Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X}. Then the set {a, b, c} is wπg-closed in (X, τ) but not semi-closed, β-closed, b-closed, sg-closed and $\tilde{g}s$-closed in (X, τ).

Example 3.26 Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a, b}, {a, c}, {a, b, c}, X}. Then the set {a, c} is semi-closed, β-closed, b-closed, sg-closed and $\tilde{g}s$-closed in (X, τ) but not wπg-closed in (X, τ).

Remark 3.27 The following diagram shows the relationships established between wπg-closed sets and some other sets. A $\rightarrow$ B represents A implies B but not conversely.

Diagram

$$\begin{array}{ccccc} \text{closed} & \longrightarrow & \text{w}\tilde{g}\text{-closed} & \longrightarrow & \text{weakly } \omega\text{-closed} \\ \downarrow & & & & \downarrow \\ \text{rwg-closed} & \longleftarrow & \text{w}\pi g\text{-closed} & \longleftarrow & \text{wg-closed} \end{array}$$

Definition 3.28 A subset A of a topological space X is called wπg-open set if A^c is wπg-closed in X.

Proposition 3.29

1. *Every πg-open set is wπg-open;*
2. *Every g-open set is wπg-open.*

Theorem 3.30 *A subset A of a topological space X is wπg-open if G $\subseteq$ int(cl(A)) whenever G $\subseteq$ A and G is π-closed.*

Proof. Let A be any wπg-open. Then A^c is wπg-closed. Let G be a π-closed set contained in A. Then G^c is a π-open set in X containing A^c. Since A^c is wπg-closed, we have cl(int(A^c)) $\subseteq G^c$. Therefore G $\subseteq$ int(cl(A)).

Conversely, we suppose that G $\subseteq$ int(cl(A)) whenever G $\subseteq$ A and G is π-closed. Then G^c is a π-open set containing A^c and $G^c \supseteq$ (int(cl(A)))^c. It follows that $G^c \supseteq$ cl(int(A^c)). Hence A^c is wπg-closed and so A is wπg-open.

4. Weakly πg-continuous mappings

Definition 4.1 Let X and Y be topological spaces. A mapping f : X $\rightarrow$ Y is called weakly πg-continuous (briefly wπg-continuous) if f^{-1}(U) is a wπg-open set in X for each open set U in Y.

Example 4.2 Let X=Y=$\{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. The mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is wπg-continuous, because every subset of X is wπg-closed.

Proposition 4.3 *Every πg-continuous mapping is wπg-continuous.*

Proof. It follows from Proposition 3.29 (1).

The converse of Proposition 4.3 need not be true as per the following example.

Example 4.4 Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, Y}. Let the mapping f : $(X, \tau) \to (Y, \sigma)$ be the identity map. Then f is wπg-continuous but it is not πg-continuous.

Theorem 4.5 *A mapping f : X → Y is called wπg-continuous if and only if f^{-1}(U) is a wπg-closed set in X for each closed set U in Y.*

Proof. Let U be any closed set in Y. According to the assumption $f^{-1}((U^c))$= X \ f^{-1}(U) is wπg-open in X, so f^{-1}(U) is wπg-closed in X.

The converse can be proved in a similar manner.

Theorem 4.6 *Suppose that X and Y are spaces and the family of πg-open sets of X is closed under arbitrary unions. If a mapping f : X → Y is contra πg-continuous and Y is regular, then f is wπg-continuous.*

Proof. Let f : X → Y be contra πg-continuous and Y be regular. By Theorem 16 of [12], f is πg-continuous. Hence, f is wπg-continuous.

Theorem 4.7 *Let f : $(X, \tau) \to (Y, \sigma)$ be a mapping. If f is contra πg-continuous and (X, τ) is locally πg-indiscrete, then f is wπg-continuous.*

Proof. Let f : X → Y be contra πg-continuous and (X, τ) be locally πg-indiscrete. By Theorem 21 of [12], f is continuous. Hence, f is wπg-continuous.

Theorem 4.8 *Suppose that a topological space (X, τ) is πgp-$T_{1/2}$ and submaximal and Y is regular. If f is almost πgp-continuous, then f is wπg-continuous.*

Proof. Let f be almost πgp-continuous. By Theorem 30 of [14], f is almost πg-continuous. Also, by Theorem 38 of [14], f is πg-continuous. Hence, f is wπg-continuous.

Theorem 4.9 *Let Y be a regular space and f : X → Y be a mapping. Suppose that the collection of πg-closed sets of X is closed under arbitrary intersections. Then if f is (πg, s)-continuous, f is wπg-continuous.*

Proof. Let f be $(\pi g, s)$-continuous. By Theorem 24 of [13], f is πg-continuous. Thus, f is wπg-continuous.

Remark 4.10 Every $\tilde{g}$-continuous mapping is w$\tilde{g}$-continuous and every w$\tilde{g}$-continuous mapping is wπg-continuous but not conversely as shown in [27] and in the below example.

Example 4.11 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is wπg-continuous but not w$\tilde{g}$-continuous.

Proposition 4.12 *If f : X → Y is perfectly continuous and π-irresolute, then it is R-map.*

Proof. Let V be any regular open subset of Y. According to the assumption, f^{-1}(V) is both π-open and closed in X. Since f^{-1}(V) is closed it is wπg-closed. Then f^{-1}(V) is both π-open and wπg-closed. Hence by Corollary 3.10 it is regular open in X, so f is R-map.

Definition 4.13 A topological space X is weakly πg-compact (briefly wπg-compact) if every wπg-open cover of X has a finite subcover.

Remark 4.14 Every wπg-compact space is πg-compact.

Theorem 4.15 *Let $f : X \to Y$ be a surjective wπg-continuous mapping. If X is wπg-compact, then Y is compact.*

Proof. Let $\{A_i$: i $\in$ I$\}$ be an open cover of Y. Then $\{f^{-1}(A_i)$: i $\in$ I$\}$ is a wπg-open cover of X. Since X is wπg-compact, it has a finite subcover, say $\{ f^{-1}(A_1)$, f^{-1} (A_2),...., $f^{-1}(A_n)\}$. Since f is surjective $\{A_1$, A_2, ..., $A_n\}$ is a finite subcover of Y and hence Y is compact.

Definition 4.16 A topological space X is weakly πg-connected (briefly wπg-connected) if X cannot be written as the disjoint union of two non-empty wπg-open sets.

Theorem 4.17 *If a topological space X is wπg-connected, then X is almost connected and πg-connected.*

Proof. It follows from the fact that each regular open set and each πg-open set is wπg-open.

Theorem 4.18 *For a topological space X the following statements are equivalent:*

(1) *X is wπg-connected.*

(2) *The empty set ϕ and X are only subsets which are both wπg-open and wπg-closed.*

(3) *Each wπg-continuous mapping from X into a discrete space Y which has at least two points is a constant function.*

Proof. (1) $\Rightarrow$ (2). Let S $\subseteq$ X be any proper subset, which is both wπg-open and wπg-closed. Its complement X $\setminus$ S is also wπg-open and wπg-closed. Then X = S $\cup$ (X $\setminus$ S) is a disjoint union of two non-empty wπg-open sets which is a contradiction with the fact that X is wπg-connected. Hence, S = ϕ or X.

(2) $\Rightarrow$ (1). Let X = A $\cup$ B where A $\cap$ B = ϕ, A $\neq \phi$, B $\neq \phi$ and A, B are wπg-open. Since A = X $\setminus$ B, A is wπg-closed. According to the assumption A = ϕ, which is a contradiction.

(2) $\Rightarrow$ (3). Let f : X $\to$ Y be a wπg-continuous mapping where Y is a discrete space with at least two points. Then $f^{-1}(\{y\})$ is wπg-closed and wπg-open for each y $\in$Y and X = $\cup\{ f^{-1}(\{y\}) \mid y \in Y\}$. According to the assumption,

f^{-1}({y})=ϕ or f^{-1}({y}) = X. If f^{-1}({y}) = ϕ for all y ∈ Y, f will not be a mapping. Also there is no exist more than one y ∈Y such that f^{-1}({y}) = X. Hence, there exists only one y ∈Y such that f^{-1}({y}) = X and f^{-1}({y_1}) = ϕ where y ≠ y_1 ∈ Y. This shows that f is a constant mapping.

(3) ⇒ (2). Let S≠ϕ be both wπg-open and wπg-closed in X. Let f : X → Y be a wπg-continuous mapping defined by f(S) = {a} and f(X \ S) = {b} where a ≠ b. Since f is constant mapping we get S = X.

Theorem 4.19 *Let $f : X \to Y$ be a $w\pi g$-continuous surjective mapping. If X is $w\pi g$-connected, then Y is connected.*

Proof. We suppose that Y is not connected. Then Y = A ∪ B where A ∩ B = ϕ, A ≠ ϕ, B ≠ ϕ and A, B are open sets in Y. Since f is wπg-continuous surjective mapping X = f^{-1}(A) ∪f^{-1}(B) are disjoint union of two non-empty wπg-open subsets. This is contradiction with the fact that X is wπg-connected.

5. Weakly πg-open mappings and weakly πg-closed mappings

Definition 5.1 Let X and Y be topological spaces. A mapping f : X → Y is called weakly πg-open (briefly wπg-open) if f(V) is a wπg-open set in Y for each open set V in X.

Remark 5.2 Every πg-open mapping is wπg-open but not conversely.

Example 5.3 Let $X = Y = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{a, b, d\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is $w\pi g$-open but not πg-open.

Definition 5.4 Let X and Y be topological spaces. A mapping f : X → Y is called weakly πg-closed (briefly wπg-closed) if f(V) is a wπg-closed set in Y for each closed set V in X.

It is clear that an open mapping is wπg-open and a closed mapping is wπg-closed.

Theorem 5.5 *Let X and Y be topological spaces. A mapping $f : X \to Y$ is $w\pi g$-closed if and only if for each subset B of Y and for each open set G containing $f^{-1}(B)$ there exists a $w\pi g$-open set F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$.*

Proof. Let B be any subset of Y and let G be an open subset of X such that f^{-1}(B) ⊆ G. Then F = Y \ f(X \ G) is wπg-open set containing B and f^{-1}(F) ⊆ G. Conversely, let U be any closed subset of X. Then f^{-1}(Y \ f(U)) ⊆ X \ U and $X \setminus U$ is open. According to the assumption, there exists a wπg-open set F of Y such that Y \ f(U) ⊆ F and f^{-1}(F) ⊆ X \ U. Then U ⊆ X \ f^{-1}(F). From Y \ F ⊆f(U) ⊆ f(X \ f^{-1}(F)) ⊆Y \ F follows that f(U) = Y \ F, so f(U) is wπg-closed in Y. Therefore f is a wπg-closed mapping.

Remark 5.6 The composition of two wπg-closed mappings need not be wπg-closed as we can see from the following example.

Example 5.7 Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$, $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$ and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, Z\}$. We define $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be the identity mappings. Hence both f and g are $w\pi g$-closed mappings. For a closed set $U = \{a\}, (gof)(U) = g(f(U)) = g(\{a\}) = \{a\}$ which is not $w\pi g$-closed in Z. Hence the composition of two $w\pi g$-closed mappings need not be $w\pi g$-closed.

Theorem 5.8 *Let X, Y and Z be topological spaces. If $f : X \to Y$ be a closed mapping and $g : Y \to Z$ be a $w\pi g$-closed map, then $g\ o\ f : X \to Z$ is a $w\pi g$-closed mapping.*

Definition 5.9 A mapping f : X $\to$ Y is called a weakly πg-irresolute (briefly wπg-irresolute) mapping if f^{-1}(U) is a wπg-open set in X for each wπg-open set U in Y.

Example 5.10 Let $X = Y = \{a, b, c\}, \tau = \{\phi, \{b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is wπg-irresolute.

Remark 5.11 The following examples show that irresoluteness and wπg-irresoluteness are independent.

Example 5.12 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{c\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is wπg-irresolute but not irresolute.

Example 5.13 Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a, b}, Y}. Let f : $(X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is irresolute but not wπg-irresolute.

Remark 5.14 Every πg-irresolute function is wπg-continuous but not conversely. Also, the concepts of πg-irresoluteness and wπg-irresoluteness are independent of each other.

Example 5.15 Let X = Y = {a, b, c, d}, τ = {ϕ, {a}, {b, c}, {a, b, c}, X} and σ = {ϕ, {a}, {a, b, d}, Y}. Let f : $(X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is wπg-continuous but not πg-irresolute.

Example 5.16 Let $X{=}Y{=}\{a, b, c, d\}$, $\tau{=}\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, \{a, b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity mapping. Then f is πg-irresolute but not wπg-irresolute.

Example 5.17 Let $X = \{a, b, c, d\}$, $Y = \{p, q\}$, $\tau = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, X\}$ and $\sigma = \{\phi, \{p\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be defined as $f(a) = f(c) = f(d) = p$ and $f(b) = q$. Then f is wπg-irresolute but not πg-irresolute.

Theorem 5.18 *The composition of two wπg-irresolute mappings is also wπg-irresolute.*

Theorem 5.19 *Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be mappings such that $g \circ f : X \rightarrow Z$ is wπg-closed mapping. Then the following statements hold:*

(1) *if f is continuous and injective, then g is wπg-closed.*

(2) *if g is wπg-irresolute and injective, then f is wπg-closed.*

Proof. (1) Let F be a closed set of Y. Since f^{-1}(F) is closed in X, we can conclude that (g o f)(f^{-1}(F)) is wπg-closed in Z. Hence g(F) is wπg-closed in Z. Thus g is a wπg-closed mapping.

(2) It can prove in a similar manner as (1).

Theorem 5.20 *If $f : X \rightarrow Y$ is an wπg-irresolute mapping, then it is wπg-continuous.*

Remark 5.21 The converse of the above Theorem need not be true in general. Let X = Y = {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b}, {a, c}, {a, b, c}, X} and σ = {ϕ, {d}, Y} The mapping f : X $\rightarrow$ Y defined as f(a) = d, f(b) = c, f(c) = b and f(d) = a. Then f is wπg-continuous but not wπg-irresolute. Since f^{-1}({a}) = {d} is not wπg-open in X.

Theorem 5.22 *If $f : X \rightarrow Y$ is a surjective wπg-irresolute mapping and X is wπg-compact, then Y is wπg-compact.*

Theorem 5.23 *If $f : X \rightarrow Y$ is surjective wπg-irresolute mapping and X is wπg-connected, then Y is wπg-connected.*

Acknowledgement. We thank the referee(s) for his/her/ their valuable suggestions and comments for improvement of this paper.

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Accepted: 25.03.2016